Power system state estimation based on Iterative Extended Kalman Filtering and bad data detection using normalized residual test

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Abstract— **This paper proposed an enhanced real-time state estimation using Iterative Extended Kalman Filtering (IEKF). The IEKF estimated state variables based on past state variables. Largest Normalized Residual Test (LNRT) was integrated with IEKF for bad data detection. A comparison with the conventional Weighted Least Squares (WLS) was also investigated using the IEEE 14 bus test system simulated in MATLAB. Based on the results, the merits and limitations of IEKF were summarized.**

Keywords— Bad data, iterative extended kalman filter, normalized residual test, power systems, sensitivity threshold, state estimation, weighted least squares

I. Introduction

Determining the operational states of modern power systems demands a real-time state estimation. Technically, a state estimation is a mathematical tool for determining the best estimates of the system states through processing and interpretation of measurements dispersed across the power system network [1]. Measurements include power injections at buses, power flows in transmission lines, and voltage magnitudes buses. State variables are computed by using the collected measurements with a set of predefined differential equations [2].

Since the introduction of state estimation to power systems by Schweppe et al [3]-[5], the methodologies used for state estimation have evolved substantially to address the growing demands of modern power grids. State estimation could be regarded as one of the key Energy Management Systems (EMS) functions used in grid planning and control operations [1]. Estimation of a state vector at k-th time instant could be calculated by the measurement set corresponding of the k-th time instant is known as static state estimator (SSE) [2]. In SSE, state vectors are obtained from calculations whose measurement data set represents the respective time step. Past estimations and measurements are not considered. Among developed state estimation algorithms, weighted least squares (WLS) is widely used for SSE [6].

Since power system is a quasi-static system that changes in a slow manner, one could assume the previous estimated states would be similar to the present values if the reporting/updating time is relatively small. Hence, the concept of utilizing previous state values to predict the next states is plausible. Similar to SSE, a dynamic state estimation (DSE) have also been a topic of interest. Several studies discussed ways to manipulate state estimates and measurements in a dynamic manner [2], [7]-[12]. Among them, Iterative Extended Kalman Filter (IEKF) from [13] is a nonlinear technique, which provides better estimation of states under quasi-steady-state.

The motivation of this paper was to examine the performance of dynamic state estimators. Comparisons between WLS and IEKF were conducted to study the merits of static and dynamic state estimators, respectively. Furthermore, the operational robustness of IEKF was extended by equipping with LNRT.

The paper was presented as follows. Section II summarized the problem formulations of IEKF and WLS. Section III discussed the simulation results and observations. Finally, conclusions were drawn in Section IV.

II. Problem formulation

A. IEKF problem formulation

In recent years, discrete-time dynamic state estimation was proposed for estimating static state variables [13]. Since power system is characterized by a quasi-static system which changes slowly and steadily, a transition relationship between consecutive states could be formulated and utilized in predefined algorithms to enhance the estimation results and predict the upcoming states as well [12]. In [14], a state transition matrix with identity matrix was formulated and the work was later extended using a diagonal matrix in [10]. Both works assumed the states do not change abruptly, and would fail when large changes occur. To resolve this issue, an improvement was proposed in [15], where a pseudo-dynamic model was formulated using the incremental change in power flow injections of the buses. Kalman Filter was applied to solve the problem, in which similar approach is followed in this paper. The state vector x is the voltage magnitudes and phase angles of each bus. If there are N number of buses, then we have active and reactive power injections u described as:

$$u = [P_1 \cdots P_N Q_1 \cdots Q_N]^T \qquad (1)$$

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Consider a function g representing the active and reactive power balance equations at each bus. The net power injection at each bus terminals was assumed to be equal to zero. Here, g would be a function of u and x as shown in (2).

$$g_P = P_i - V_i \sum_{j=1}^{N} V_j [G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij} = 0$$
$$g_Q = Q_i - V_i \sum_{j=1}^{N} V_j [G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij} = 0 \quad (2)$$

Linearization of (2) would result to (3), where J is the Jacobian expressed in (4), (5) and I is an identity matrix.

$$-J\Delta x + I\Delta u + e = 0 \quad (3)$$

$$J = \begin{cases} \frac{\partial P_i}{\partial \theta_i} = \sum_{j=1}^{N} V_i V_j (-G_{ij} \sin\theta_{ij} + B_{ij} \cos\theta_{ij}) - V_i^2 B_{ii} \\ \frac{\partial P_i}{\partial \theta_j} = V_i V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) \\ \frac{\partial P_i}{\partial V_i} = \sum_{j=1}^{N} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) + V_i G_{ii} \\ \frac{\partial P_i}{\partial V_j} = V_i (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) \\ \frac{\partial Q_i}{\partial \theta_i} = \sum_{j=1}^{N} V_i V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) - V_i^2 G_{ii} \\ \frac{\partial Q_i}{\partial \theta_j} = V_i V_j (-G_{ij} \cos\theta_{ij} - B_{ij} \sin\theta_{ij}) \\ \frac{\partial Q_i}{\partial V_i} = \sum_{j=1}^{N} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) - V_i B_{ii} \\ \frac{\partial Q_i}{\partial V_j} = V_i (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) \end{cases} \quad (4)$$

$$J = -\begin{bmatrix} \frac{\partial g_P}{\partial \theta} & \frac{\partial g_P}{\partial V} \\ \frac{\partial g_Q}{\partial \theta} & \frac{\partial g_Q}{\partial V} \end{bmatrix} \quad (5)$$

Re-arranging and solving for x gave:

$$x_k = x_{k-1} + J_k^{-1}[u_k - u_{k-1}] + J_k^{-1} e \quad (6)$$

This generated a discrete relationship between consecutive states. Equation (6) is the pseudo-dynamic model of the power system linking state variables at time step k with k-1 [16]. The term $J^{-1}e$ is the process noise, which for this specific problem was due to the linearization error. Equation (6) outlined the dynamics of state x due to an applied input u. Once the pseudo-dynamic model was formulated, we could apply the prediction-correction cycles of the non-linear version of Kalman Filter known as Extended Kalman Filter (EKF).

$$\text{Prediction} \begin{cases} x_k = x_{k-1} + J^{-1}[u_k - u_{k-1}] \\ P_k = P_{k-1} + Q \end{cases} \quad (7)$$

$$\text{Correction} \begin{cases} K_{k,i} = P_k H'_{k,i-1} (H_{k,i-1} P_k H'_{k,i-1} + R)^{-1} \\ x_{k,i} = x_{k,i-1} + K_{k,i}[z_k - h_{k,i}] \\ P_{k,i} = (I - K_{k,i} H_{k,i-1}) P_k \end{cases} \quad (8)$$

Where

- Pk is the state error covariance matrix at time step k
- Q is process noise caused due to linearization errors
- K is the Kalman gain
- R is the measurement covariance matrix
- H is the Jacobian matrix of the measurement function h
- z is measurement vector and h is the measurement estimations

In the prediction step, the *a-priori* states were predicted using previous information of states and the transition model. For example, as shown in (7), the state and error covariance matrix at time step k can be predicted using the state variable information at time step k-1 plus the dynamics between k and k-1. Subsequently, in the correction step, the prediction errors were corrected using a Kalman gain $K_{k,i}$ and the measurement innovation term z_k-$h_{k,i}$. Updated P was also calculated at this step. The correction step (8) was an iterative process that would terminate when the process converges to a predefined threshold value. The whole process summarized is shown in Fig. 3.

B. Bad data detection

The bad data detection scheme was developed using normalized residual test from [6] as applied for WLS. Referring to (9), S is the covariance matrix of the residual r. The residual could be normalized using equation (10).

$$S = (R - HP^{-1}H^T) \quad (9)$$

$$r_i^N = \frac{|r_i|}{\sqrt{S_{ii}}} \quad (10)$$

Bad data detection was carried out after the convergence of the estimation of the correction step. If an element appeared to be greater than the predefined threshold value in the residual vector, then the measurement corresponding to that residual would be suspected to contain bad data. This is known as the Largest Normalized Residual Test (LNRT). Although the LNRT could detect the existence of bad data, it often failed to identify the measurements containing bad data.

C. Weighted Least Squares (WLS)

Similarly in WLS, if we have a measurement vector z with normally distributed errors e due to limited accuracy of measurement devices, then the state vector x could be related to measurements as:

$$z = h(x) + e \quad (11)$$

where h is the non-linear measurement function linking the states with measurements. This could be regarded as an optimization problem, whose solutions were computed by minimizing J as shown in (12), or equivalently in (13).

$$J(x) = \sum_{i=1}^{m} \frac{(z_i - h_i(x))^2}{R} \quad (12)$$

$$J(x) = \frac{1}{2}[z - h(x)]^T R^{-1} [z - h(x)] \quad (13)$$

Note σ_i refers to the standard deviation of the measurement covariance matrix such that:

$$R = \begin{bmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_m^2 \end{bmatrix} \quad (14)$$

This is an iterative problem that began with a flat start, i.e. angles set as zero and voltages as one. Hence, at iteration k, the solution would be:

$$x^{k+1} = x^k + \Delta x \quad (15)$$

$$x^{k+1} = x^k + G(x^k)^{-1} H^T R^{-1} (z - h(x)) \quad (16)$$

Where H and G were defined as:

$$H(x) = \frac{\partial h(x)}{\partial x} \quad (17)$$

$$G(x) = H(x)^T R^{-1} H(x) \quad (18)$$

III. Simulation and Observations

Simulations were carried out in MATLAB to evaluate WLS and IEKF algorithms using IEEE 14 bus test system. Measurements obtained from power flow analysis were corrupted with normally distributed errors. The standard deviations of voltage and power measurements are 0.01 and 0.02, respectively. The procedure was done over 20 time steps and the load profile of Bus 4 is shown in Fig.2. By inspection, the process noise Q has a constant 0.1 in its diagonal entries while the state vector x was initialized with a flat start condition. Similarly, the state covariance matrix P was also initialized with 10 in its diagonal entries. Fig. 3 shows the flow chart followed in executing IEKF state estimations. During the correction step, iterations were terminated when a threshold of 0.001 was reached. Subsequently, the results were tested for bad data. The sensitivity threshold for detection was set at 3. Hence, any residuals larger than 3 exists would indicate the existence of bad data. Therefore it should be identified, corrected and then re-estimate before proceeding to the prediction step. The performance of IEFK is illustrated in Fig.4. Comparing the results of WLS in Fig. 5, it is evident that IEKF was less accurate. Unlike WLS, IEKF consisted of more sensitive parameters and demanded heavier computation.

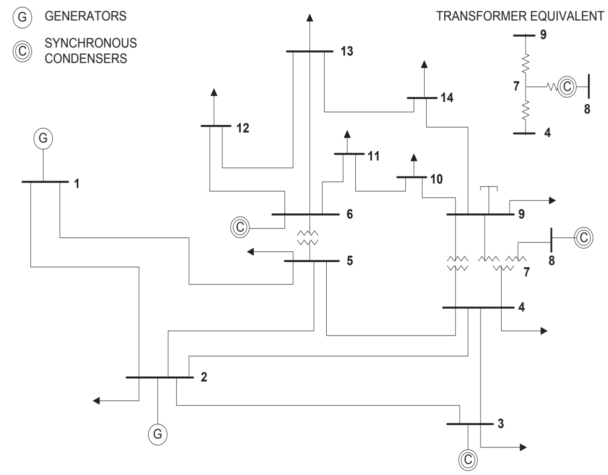

Fig. 1 IEEE 14 bus test system

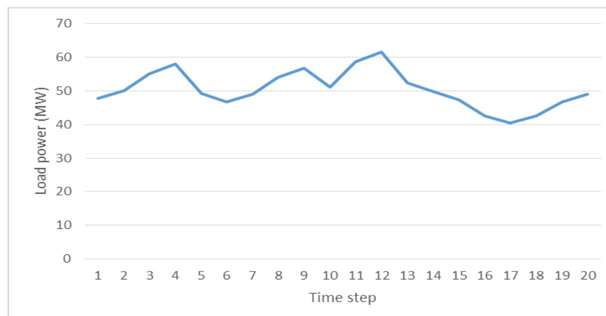

Fig. 2 Bus 4 Load profile

However, IEKF still achieved results within acceptable limits. In fact, IEKF could be more accurate than WLS when the system was perturbed by small disturbances. This is due to its recursive Kalman function.

The corresponding predictions at Bus 4 for IEKF is shown in Fig.6.

To evaluate the integrated bad data detection scheme, a faulty data that was 10 times the standard deviation was added into the active power injection measurement at bus 4 (P4) at time step 5. TABLE I and TABLE II summarized the results of the state estimation under normal and bad data situations, respectively. It could be observed in TABLE II that errors were introduced into state estimation results. In actual system operation, this could lead to inappropriate control decisions. Although bad data detection scheme identified the faulty measurement, IEKF was not able to accurately identify the location. The normalized residual results of the bad data is shown in Table III.

IV. Conclusions

Two state estimation algorithms were reviewed and evaluated in this paper. The weighted least square (WLS) was a static estimator that extracted state variables by taking a snapshot of

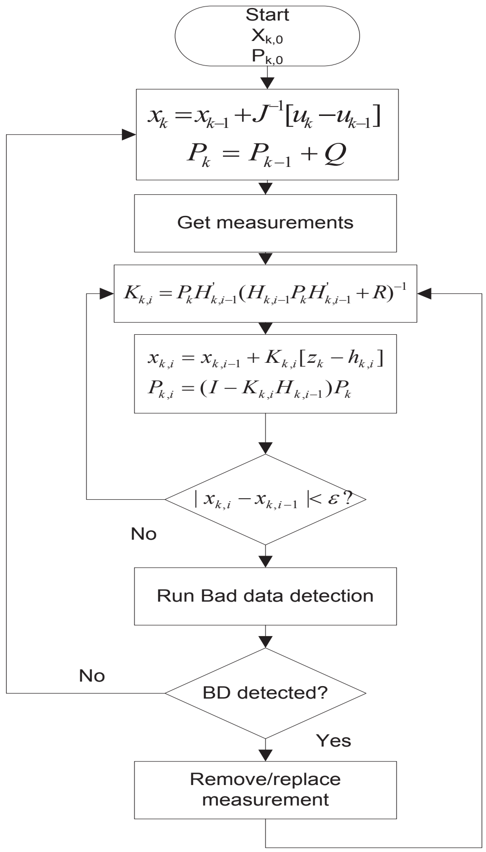

Fig. 3 Flow chart implemented IEKF preocedures

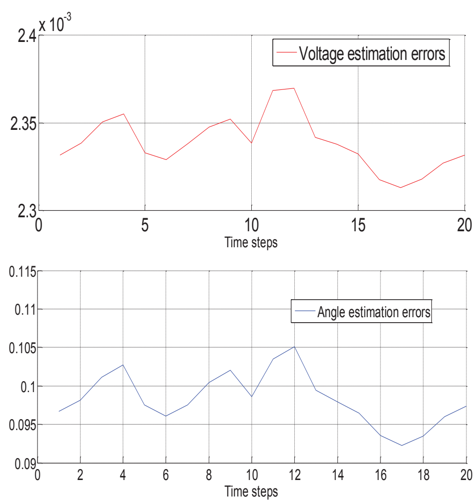

Fig.4 Bus 4 estimations using IEKF

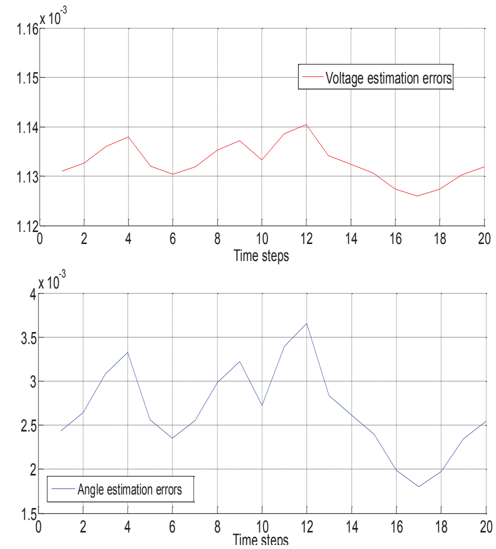

Fig.5 Bus 4 estimations using WLS

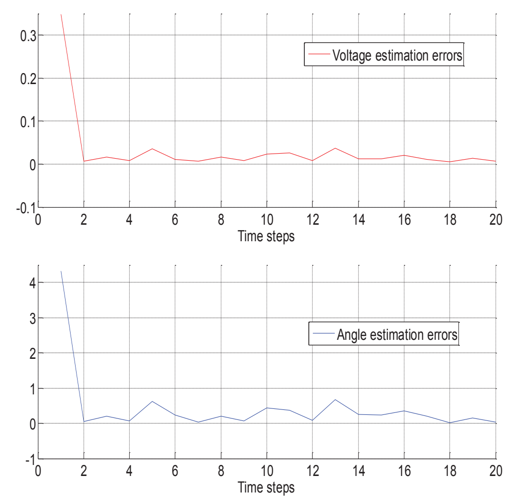

Fig.6 Bus 4 prediction estimations using IEKF

the grid in the time domain. Meanwhile, the Iterative Extended Kalman Filter (IEKF) was a dynamic estimator that can predict

the next state values based on past information. Despite both methods achieved reasonable estimation accuracy, IEKF incurred slightly higher errors than WLS based on the simulation results. Like WLS, largest normalized residual test (LRNT) was applied to the IEKF algorithm to enhance its bad data detection capability.

TABLE I. STATE ESTIMATIONS WITHOUT BAD DATA

Bus	Voltage (p.u)	Angle (degrees)
1	1.062	0.00
2	1.047	-5.21
3	1.012	-13.13
4	1.019	-10.80
5	1.021	-9.15
6	1.049	-14.76
7	1.039	-13.85
8	1.070	-13.85
9	1.031	-15.48
10	1.027	-15.64
11	1.034	-15.34
12	1.022	-15.47
13	1.008	-15.16
14	1.002	-16.46

TABLE II. STATE ESTIMATIONS WITH BAD DATA IN P_4

Bus	Voltage (p.u)	Angle (degrees)
1	1.055	0.00
2	1.041	-5.01
3	1.005	-12.83
4	1.015	-10.08
5	1.015	-8.79
6	1.043	-15.47
7	1.035	-13.86
8	1.067	-13.87
9	1.028	-15.87
10	1.022	-16.25
11	1.028	-16.13
12	1.014	-16.47
13	1.000	-16.07
14	0.995	-17.32

TABLE III. BAD DATA DETECTION IN P_4

	Max r^N
No Bad data	0.8682
With Bad data in P_4	9.6715

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