CHAPTER 9

BALANCED FAULT

9.1 INTRODUCTION

Fault studies form an important part of power system analysis. The problem consists of determining bus voltages and line currents during various types of faults. Faults on power systems are divided into *three-phase balanced faults* and *unbalanced faults*. Different types of unbalanced faults are *single line-to-ground fault*, *line-to-line fault*, and *double line-to-ground fault*, which are dealt with in Chapter 10. The information gained from fault studies are used for proper relay setting and coordination. The three-phase balanced fault information is used to select and set phase relays, while the line-to-ground fault is used for ground relays. Fault studies are also used to obtain the rating of the protective switchgears.

The magnitude of the fault currents depends on the internal impedance of the generators plus the impedance of the intervening circuit. We have seen in Chapter 8 that the reactance of a generator under short circuit condition is not constant. For the purpose of fault studies, the generator behavior can be divided into three periods: the *subtransient period*, lasting only for the first few cycles; the *transient period*, covering a relatively longer time; and, finally, the *steady state period*. In this chapter, three-phase balanced faults are discussed. The bus impedance matrix by the *building algorithm* is formulated and is employed for the systematic computation of bus voltages and line currents during the fault. Two functions are developed for the formation of the bus impedance matrix. These function are **Zbus = zbuild(zdata)** and **Zbus = zbuildpi(linedata, gendata, yload)**. The latter one is compatible with power flow input/output files. A program named **symfault** is developed for systematic computation of three-phase balanced faults for a large interconnected power system.

9.2 BALANCED THREE-PHASE FAULT

This type of fault is defined as the simultaneous short circuit across all three phases. It occurs infrequently, but it is the most severe type of fault encountered. Because the network is balanced, it is solved on a per-phase basis. The other two phases carry identical currents except for the phase shift.

In Chapter 8 it was shown that the reactance of the synchronous generator under short-circuit conditions is a time-varying quantity, and for network analysis three reactances were defined. The subtransient reactance X''_d, for the first few cycles of the short circuit current, transient reactance X'_d, for the next (say) 30 cycles, and the synchronous reactance X_d, thereafter. Since the duration of the short circuit current depends on the time of operation of the protective system, it is not always easy to decide which reactance to use. Generally, the subtransient reactance is used for determining the interrupting capacity of the circuit breakers. In fault studies required for relay setting and coordination, transient reactance is used. Also, in typical transient stability studies, transient reactance is used.

A fault represents a structural network change equivalent with that caused by the addition of an impedance at the place of fault. If the fault impedance is zero, the fault is referred to as the *bolted fault* or the *solid fault*. The faulted network can be solved conveniently by the Thévenin's method. The procedure is demonstrated in the following example.

Example 9.1

The one-line diagram of a simple three-bus power system is shown in Figure 9.1. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common 100 MVA base, and for simplicity, resistances are neglected. The following assumptions are made.

(*i*) Shunt capacitances are neglected and the system is considered on no-load.

(*ii*) All generators are running at their rated voltage and rated frequency with their emfs in phase.

Determine the fault current, the bus voltages, and the line currents during the fault when a balanced three-phase fault with a fault impedance $Z_f = 0.16$ per unit occurs on

(a) Bus 3.
(b) Bus 2.
(c) Bus 1.

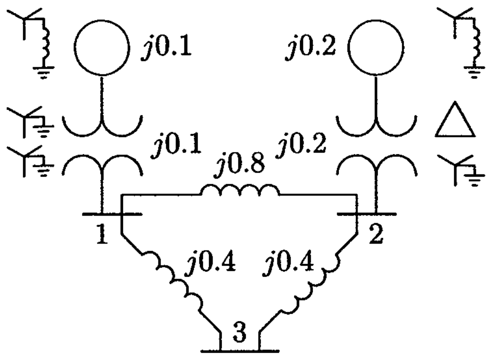

FIGURE 9.1
The impedance diagram of a simple power system.

The fault is simulated by switching on an impedance Z_f at bus 3 as shown in Figure 9.2(a). Thévenin's theorem states that the changes in the network voltage caused by the added branch (the fault impedance) shown in Figure 9.2(a) is equivalent to those caused by the added voltage $V_3(0)$ with all other sources short-circuited as shown in Figure 9.2(b).

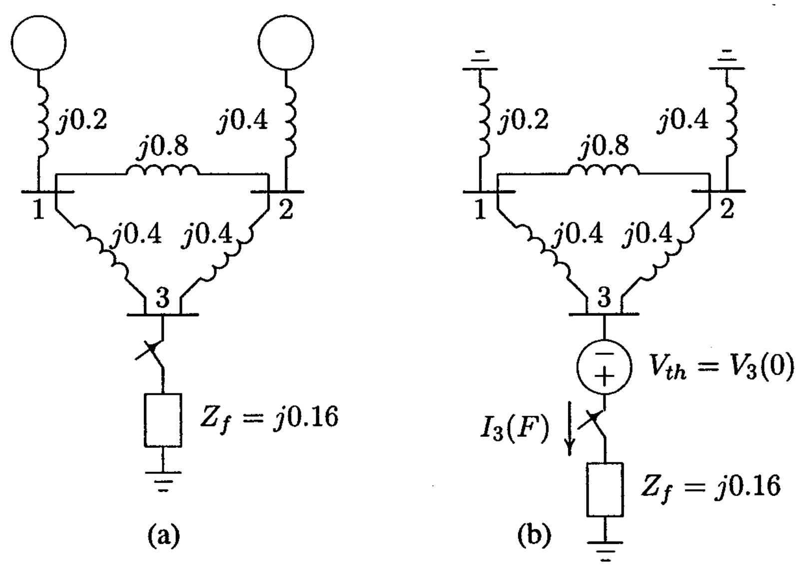

FIGURE 9.2
(a) The impedance network for fault at bus 3. (b) Thévenin's equivalent network.

(a) From 9.2(b), the fault current at bus 3 is

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f}$$

where $V_3(0)$ is the Thévenin's voltage or the prefault bus voltage. The prefault bus voltage can be obtained from the results of the power flow solution. In this example, since the loads are neglected and generator's emfs are assumed equal to the rated value, all the prefault bus voltages are equal to 1.0 per unit, i.e.,

$$V_1(0) = V_2(0) = V_3(0) = 1.0 \quad \text{pu}$$

Z_{33} is the Thévenin's impedance viewed from the faulted bus.

To find the Thévenin's impedance, we convert the Δ formed by buses 123 to an equivalent Y as shown in Figure 9.3(a).

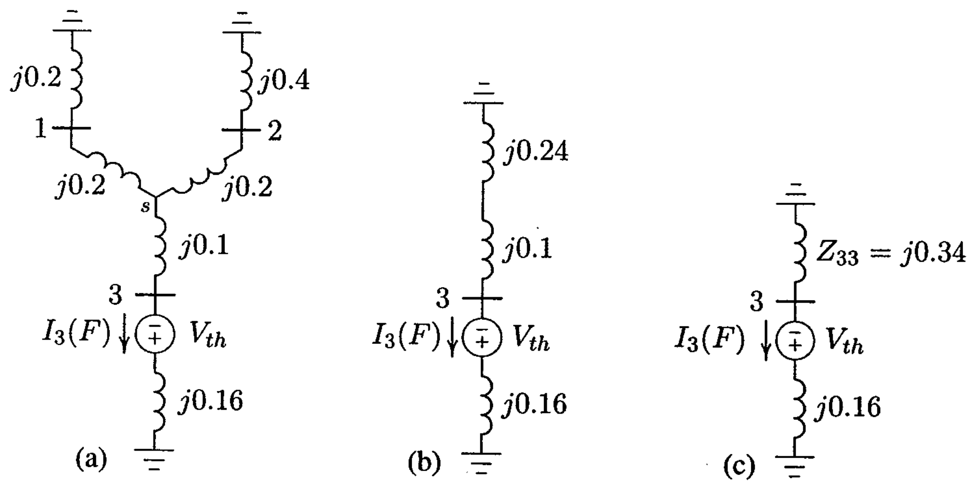

FIGURE 9.3
Reduction of Thévenin's equivalent network.

$$Z_{1s} = Z_{2s} = \frac{(j0.4)(j0.8)}{j1.6} = j0.2 \qquad Z_{3s} = \frac{(j0.4)(j0.4)}{j1.6} = j0.1$$

Combining the parallel branches, Thévenin's impedance is

$$\begin{aligned} Z_{33} &= \frac{(j0.4)(j0.6)}{j0.4 + j0.6} + j0.1 \\ &= j0.24 + j0.1 = j0.34 \end{aligned}$$

From Figure 9.3(c), the fault current is

$$I_3(F) = \frac{V_3(F)}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.16} = -j2.0 \quad \text{pu}$$

With reference to Figure 9.3(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.6}{j0.4 + j0.6} I_3(F) = -j1.2 \quad \text{pu}$$
$$I_{G2} = \frac{j0.4}{j0.4 + j0.6} I_3(F) = -j0.8 \quad \text{pu}$$

For the bus voltage changes from Figure 9.3(b), we get

$$\Delta V_1 = 0 - (j0.2)(-j1.2) \quad = -0.24 \quad \text{pu}$$
$$\Delta V_2 = 0 - (j0.4)(-j0.8) \quad = -0.32 \quad \text{pu}$$
$$\Delta V_3 = (j0.16)(-j2) - 1.0 = -0.68 \quad \text{pu}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, as shown in Figure 9.2(b), i.e.,

$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.24 = 0.76 \quad \text{pu}$$
$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.32 = 0.68 \quad \text{pu}$$
$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.68 = 0.32 \quad \text{pu}$$

The short circuit-currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.76 - 0.68}{j0.8} = -j0.1 \quad \text{pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.76 - 0.32}{j0.4} = -j1.1 \quad \text{pu}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.68 - 0.32}{j0.4} = -j0.9 \quad \text{pu}$$

(b) The fault with impedance Z_f at bus 2 is depicted in Figure 9.4(a), and its Thévenin's equivalent circuit is shown in Figure 9.4(b). To find the Thévenin's impedance, we combine the parallel branches in Figure 9.4(b). Also, combining parallel branches from ground to bus 2 in Figure 9.5(a), results in

$$Z_{22} = \frac{(j0.6)(j0.4)}{j0.6 + j0.4} = j0.24$$

From Figure 9.5(b), the fault current is

$$I_2(F) = \frac{V_2(0)}{Z_{22} + Z_f} = \frac{1.0}{j0.24 + j0.16} = -j2.5 \quad \text{pu}$$

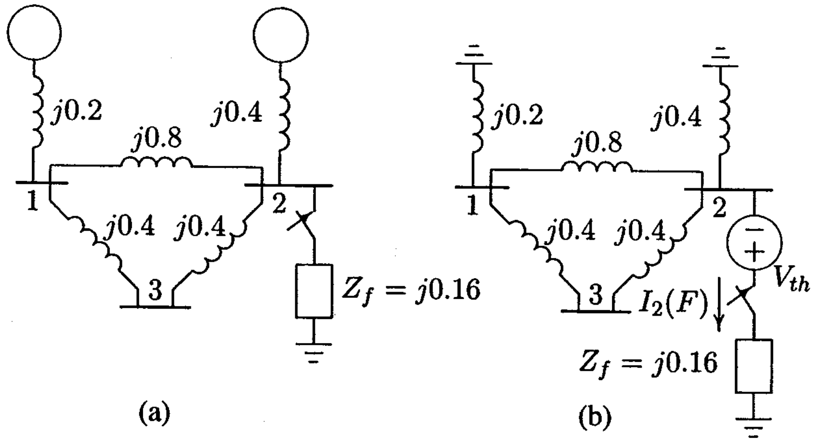

FIGURE 9.4
(a) The impedance network for fault at bus 2. (b) Thévenin's equivalent network.

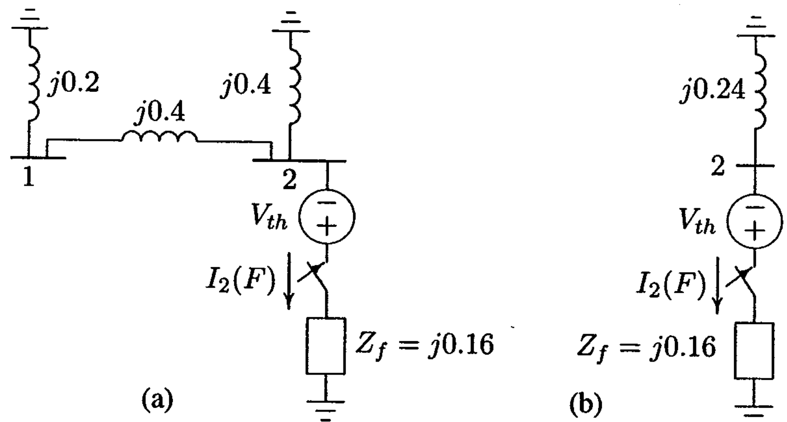

FIGURE 9.5
Reduction of Thévenin's equivalent network.

With reference to Figure 9.5(a), the current divisions between the generators are

$$I_{G1} = \frac{j0.4}{j0.4 + j0.6} I_2(F) = -j1.0 \quad \text{pu}$$

$$I_{G2} = \frac{j0.6}{j0.4 + j0.6} I_2(F) = -j1.5 \quad \text{pu}$$

For the bus voltage changes from Figure 9.4(a), we get

$$\Delta V_1 = 0 - (j0.2)(-j1.0) = -0.2 \quad \text{pu}$$

$$\Delta V_2 = 0 - (j0.4)(-j1.5) = -0.6 \quad \text{pu}$$

$$\Delta V_3 = -0.2 - (j0.4)(\frac{-j1.0}{2}) = -0.4 \quad \text{pu}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, as shown in Figure 9.4(b), i.e.,

$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.2 = 0.8 \quad \text{pu}$$
$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.6 = 0.4 \quad \text{pu}$$
$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.4 = 0.6 \quad \text{pu}$$

The short circuit-currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.8 - 0.4}{j0.8} = -j0.5 \quad \text{pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.8 - 0.6}{j0.4} = -j0.5 \quad \text{pu}$$
$$I_{32}(F) = \frac{V_3(F) - V_3(F)}{z_{32}} = \frac{0.6 - 0.4}{j0.4} = -j0.5 \quad \text{pu}$$

(c) The fault with impedance Z_f at bus 1 is depicted in Figure 9.6(a), and its Thévenin's equivalent circuit is shown in Figure 9.6(b).

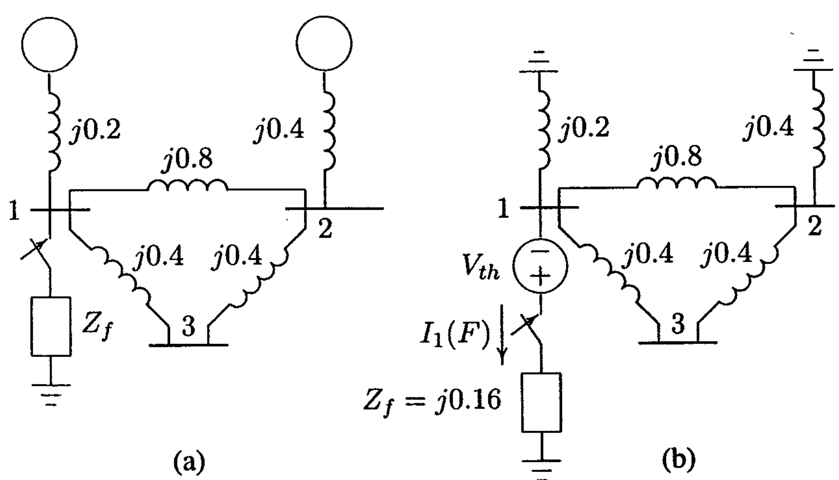

FIGURE 9.6
(a) The impedance network for fault at bus 1. (b) Thévenin's equivalent network.

To find the Thévenin's impedance, we combine the parallel branches in Figure 9.6(b). Also, combining parallel branches from ground to bus 1 in Figure 9.7(a),

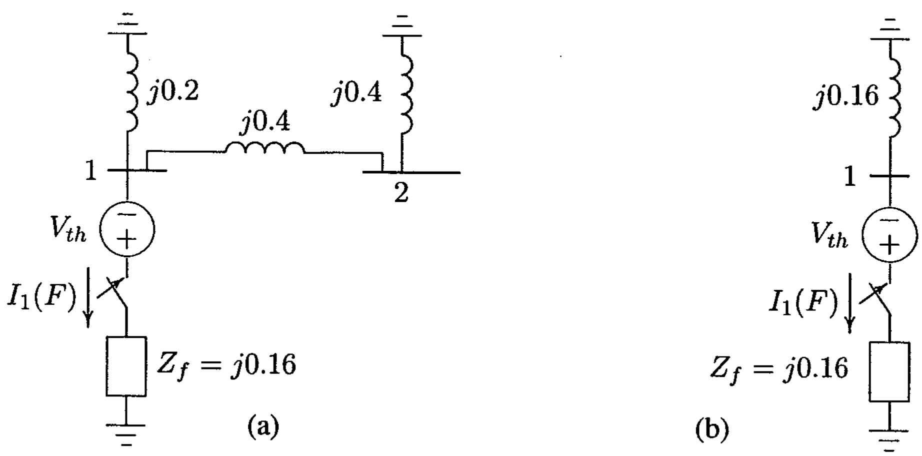

FIGURE 9.7
Reduction of Thévenin's equivalent network.

results in

$$Z_{11} = \frac{(j0.2)(j0.8)}{j0.2 + j0.8} = j0.16$$

From Figure 9.7(b), the fault current is

$$I_1(F) = \frac{V_1(0)}{Z_{11} + Z_f} = \frac{1.0}{j0.16 + j0.16} = -j3.125 \quad \text{pu}$$

With reference to Figure 9.7(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.8}{j0.2 + j0.8} I_2(F) = -j2.50 \quad \text{pu}$$
$$I_{G2} = \frac{j0.2}{j0.2 + j0.8} I_2(F) = -j0.625 \quad \text{pu}$$

For the bus voltage changes from Figure 9.6(b), we get

$$\Delta V_1 = 0 - (j0.2)(-j2.5) = -0.50 \quad \text{pu}$$
$$\Delta V_2 = 0 - (j0.4)(-j0.625) = -0.25 \quad \text{pu}$$
$$\Delta V_3 = -0.5 + (j0.4)(\frac{-j0.625}{2}) = -0.375 \quad \text{pu}$$

Bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, as shown in Figure 9.6(b), i.e.,

$$\begin{aligned} V_1(F) &= V_1(0) + \Delta V_1 = 1.0 - 0.50 = 0.50 \quad \text{pu} \\ V_2(F) &= V_2(0) + \Delta V_2 = 1.0 - 0.25 = 0.75 \quad \text{pu} \\ V_3(F) &= V_3(0) + \Delta V_3 = 1.0 - 0.375 = 0.625 \quad \text{pu} \end{aligned}$$

The short-circuit currents in the lines are

$$\begin{aligned} I_{21}(F) &= \frac{V_2(F) - V_1(F)}{z_{21}} = \frac{0.75 - 0.5}{j0.8} = -j0.3125 \quad \text{pu} \\ I_{31}(F) &= \frac{V_3(F) - V_1(F)}{z_{31}} = \frac{0.625 - 0.5}{j0.4} = -j0.3125 \quad \text{pu} \\ I_{23}(F) &= \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.75 - 0.625}{j0.4} = -j0.3125 \quad \text{pu} \end{aligned}$$

In the above example the load currents were neglected and all prefault bus voltages were assumed to be equal to 1.0 per unit. For more accurate calculation, the prefault bus voltages can be obtained from the power flow solution. As we have seen in Chapter 6, in a power system, loads are specified and the load currents are unknown. One way to include the effects of load currents in the fault analysis is to express the loads by a constant impedance evaluated at the prefault bus voltages. This is a very good approximation which results in linear nodal equations. The procedure is summarized in the following steps.

- The prefault bus voltages are obtained from the results of the power flow solution.
- In order to preserve the linearity feature of the network, loads are converted to constant admittances using the prefault bus voltages.
- The faulted network is reduced into a Thévenin's equivalent circuit as viewed from the faulted bus. Applying Thévenin's theorem, changes in the bus voltages are obtained.
- Bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages computed in the previous step.
- The currents during the fault in all branches of the network are then obtained.

9.3 SHORT-CIRCUIT CAPACITY (SCC)

The short-circuit capacity at a bus is a common measure of the strength of a bus. The short-circuit capacity or the short-circuit MVA at bus k is defined as the product of the magnitudes of the rated bus voltage and the fault current. The short-circuit MVA is used for determining the dimension of a bus bar, and the *interrupting* capacity of a circuit breaker. The interrupting capacity is only one of many ratings of a circuit breaker and should not be confused with the *momentary duty* of the breaker described in (8.63).

Based on the above definition, the short-circuit capacity or the short-circuit MVA at bus k is given by

$$SCC = \sqrt{3}\, V_{Lk} I_k(F) \times 10^{-3} \quad \text{MVA} \tag{9.1}$$

where the line-to-line voltage V_{Lk} is expressed in kilovolts and $I_k(F)$ is expressed in amperes. The symmetrical three-phase fault current in per unit is given by

$$I_k(F)_{pu} = \frac{V_k(0)}{X_{kk}} \tag{9.2}$$

where $V_k(0)$ is the per unit prefault bus voltage, and X_{kk} is the per unit reactance to the point of fault. System resistance is neglected and only the inductive reactance of the system is allowed for. This gives minimum system impedance and maximum fault current and a pessimistic answer. The base current is given by

$$I_B = \frac{S_B \times 10^3}{\sqrt{3}\, V_B} \tag{9.3}$$

where S_B is the base MVA and V_B is the line-to-line base voltage in kilovolts. Thus, the fault current in amperes is

$$\begin{aligned} I_k(F) &= I_k(F)_{pu} I_B \\ &= \frac{V_k(0)}{X_{kk}} \frac{S_B \times 10^3}{\sqrt{3}\, V_B} \end{aligned} \tag{9.4}$$

Substituting for $I_k(F)$ from (9.4) into (9.1) results in

$$SCC = \frac{V_k(0) S_B}{X_{kk}} \frac{V_L}{V_B} \tag{9.5}$$

If the base voltage is equal to the rated voltage, i.e., $V_L = V_B$

$$SCC = \frac{V_k(0) S_B}{X_{kk}} \tag{9.6}$$

The prefault bus voltage is usually assumed to be 1.0 per unit, and we therefore obtain from (9.6) the following approximate formula for the short-circuit capacity or the short-circuit MVA.

$$SCC = \frac{S_B}{X_{kk}} \text{ MVA} \tag{9.7}$$

9.4 SYSTEMATIC FAULT ANALYSIS USING BUS IMPEDANCE MATRIX

The network reduction used in the preceding example is not efficient and is not applicable to large networks. In this section a more general fault circuit analysis using nodal method is obtained. We see that by utilizing the elements of the bus impedance matrix, the fault current as well as the bus voltages during fault are readily and easily calculated.

Consider a typical bus of an n-bus power system network as shown in Figure 9.8. The system is assumed to be operating under balanced condition and a per phase circuit model is used. Each machine is represented by a constant voltage source behind proper reactances which may be X_d'', X_d', or X_d. Transmission lines are represented by their equivalent π model and all impedances are expressed in per unit on a common MVA base. A balanced three-phase fault is to be applied at bus k through a fault impedance Z_f. The prefault bus voltages are obtained from the power flow solution and are represented by the column vector

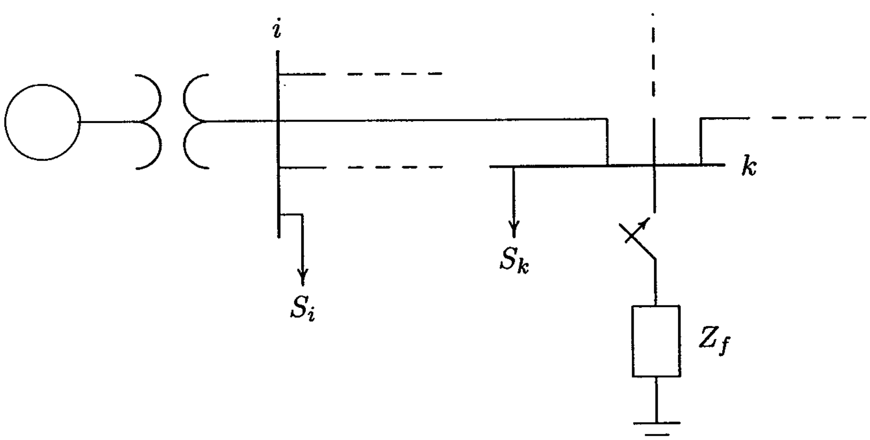

FIGURE 9.8
A typical bus of a power system.

$$\mathbf{V}_{bus}(0) = \begin{bmatrix} V_1(0) \\ \vdots \\ V_k(0) \\ \vdots \\ V_n(0) \end{bmatrix} \tag{9.8}$$

As already mentioned, short circuit currents are so much larger than the steady-state values that we may neglect the latter. However, a good approximation is to represent the bus load by a constant impedance evaluated at the prefault bus voltage, i.e.,

$$Z_{i\,L} = \frac{|V_i(0)|^2}{S_L^*} \tag{9.9}$$

The changes in the network voltage caused by the fault with impedance Z_f is equivalent to those caused by the added voltage $V_k(0)$ with all other sources short-circuited. Zeroing all voltage sources and representing all components and loads by their appropriate impedances, we obtain the Thévenin's circuit shown in Figure 9.9. The bus voltage changes caused by the fault in this circuit are represented by the column vector

$$\Delta\mathbf{V}_{bus} = \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix} \tag{9.10}$$

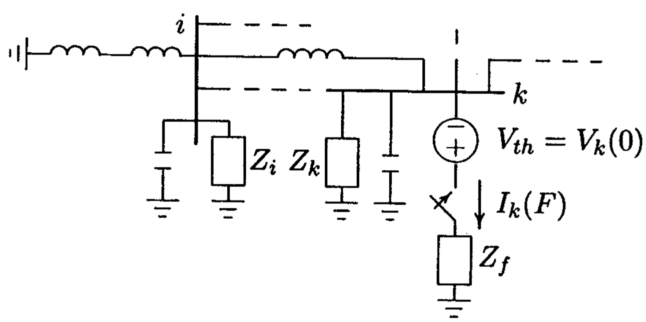

FIGURE 9.9
A typical bus of a power system.

From Thévenin's theorem bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages given by

$$\mathbf{V}_{bus}(F) = \mathbf{V}_{bus}(0) + \Delta\mathbf{V}_{bus} \tag{9.11}$$

In Section 6.2, we obtained the node-voltage equation for an n-bus network. The injected bus currents are expressed in terms of the bus voltages (with bus 0 as reference), i.e.,

$$\mathbf{I}_{bus} = \mathbf{Y}_{bus}\mathbf{V}_{bus} \tag{9.12}$$

where $\mathbf{I}_{bus}$ is the bus current vector entering the bus and $\mathbf{Y}_{bus}$ is the bus admittance matrix. The diagonal element of each bus is the sum of admittances connected to it, i.e.,

$$Y_{ii} = \sum_{j=0}^{m} y_{ij} \qquad j \neq i \tag{9.13}$$

The off-diagonal element is equal to the negative of the admittance between the buses, i.e.,

$$Y_{ij} = Y_{ji} = -y_{ij} \tag{9.14}$$

where y_{ij} (lower case) is the actual admittance of the line i-j. For more details refer to Section 6.2.

In the Thévenin's circuit of Figure 9.9, current entering every bus is zero except at the faulted bus. Since the current at faulted bus is leaving the bus, it is taken as a negative current entering bus k. Thus the nodal equation applied to the Thévenin's circuit in Figure 9.9 becomes

$$\begin{bmatrix} 0 \\ \vdots \\ -I_k(F) \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} y_{11} & \cdots & y_{1k} & \cdots & y_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{k1} & \cdots & y_{kk} & \cdots & y_{kn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{n1} & \cdots & y_{nk} & \cdots & y_{nn} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix} \tag{9.15}$$

or

$$\mathbf{I}_{bus}(F) = \mathbf{Y}_{bus}\Delta\mathbf{V}_{bus} \tag{9.16}$$

Solving for $\Delta\mathbf{V}_{bus}$, we have

$$\Delta\mathbf{V}_{bus} = \mathbf{Z}_{bus}\mathbf{I}_{bus}(F) \tag{9.17}$$

where $\mathbf{Z}_{bus} = \mathbf{Y}_{bus}^{-1}$ is known as the *bus impedance matrix*. Substituting (9.17) into (9.11), the bus voltage vector during the fault becomes

$$\mathbf{V}_{bus\,(F)} = \mathbf{V}_{bus}(0) + \mathbf{Z}_{bus}\mathbf{I}_{bus}(F) \tag{9.18}$$

Writing the above matrix equation in terms of its elements, we have

$$\begin{bmatrix} V_1(F) \\ \vdots \\ V_k(F) \\ \vdots \\ V_n(F) \end{bmatrix} = \begin{bmatrix} V_1(0) \\ \vdots \\ V_k(0) \\ \vdots \\ V_n(0) \end{bmatrix} + \begin{bmatrix} Z_{11} & \cdots & Z_{1k} & \cdots & Z_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{n1} & \cdots & Z_{nk} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ -I_k(F) \\ \vdots \\ 0 \end{bmatrix} \tag{9.19}$$

Since we have only one single nonzero element in the current vector, the kth equation in (9.19) becomes

$$V_k(F) = V_k(0) - Z_{kk}I_k(F) \tag{9.20}$$

Also from the Thévenin's circuit shown in Figure 9.9, we have

$$V_k(F) = Z_f I_k(F) \tag{9.21}$$

For bolted fault, $Z_f = 0$ and $V_k(F) = 0$. Substituting for $V_k(F)$ from (9.21) into (9.20) and solving for the fault current, we get

$$I_k(F) = \frac{V_k(0)}{Z_{kk} + Z_f} \tag{9.22}$$

Thus for a fault at bus k we need only the Z_{kk} element of the bus impedance matrix. This element is indeed the Thévenin's impedance as viewed from the faulted bus. Also, writing the ith equation in (9.19) in terms of its element, we have

$$V_i(F) = V_i(0) - Z_{ik}I_k(F) \tag{9.23}$$

Substituting for $I_k(F)$, bus voltage during the fault at bus i becomes

$$V_i(F) = V_i(0) - \frac{Z_{ik}}{Z_{kk} + Z_f}V_k(0) \tag{9.24}$$

With the knowledge of bus voltages during the fault, we can calculate the fault current in all the lines. For the line connecting buses i and j with impedance z_{ij}, the short circuit current in this line (defined positive in the direction $i \rightarrow j$) is

$$I_{ij}(F) = \frac{V_i(F) - V_j(F)}{z_{ij}} \tag{9.25}$$

We note that with the knowledge of the bus impedance matrix, the fault current and bus voltages during the fault are readily obtained for any faulted bus in the network. This method is very simple and practical. Thus, all fault calculations are formulated in the bus frame of reference using bus impedance matrix Z_{bus}.

One way to find $\mathbf{Z}_{bus}$ is to formulate Y_{bus} matrix for the system and then find its inverse. The matrix inversion for a large power system with a large number of buses is not feasible. A computationally attractive and efficient method for finding Z_{bus} matrix is "building" or "assembling" the impedance matrix by adding one network element at a time. In effect, this is an indirect matrix inversion of the bus admittance matrix. The algorithm for building the bus impedance matrix is described in the next section.

Example 9.2

A three-phase fault with a fault impedance $Z_f = j0.16$ per unit occurs at bus 3 in the network of Example 9.1. Using the bus impedance matrix method, compute the fault current, the bus voltages, and the line currents during the fault.

In this example the bus impedance matrix is obtained by finding the inverse of the bus admittance matrix. In the next section, we describe an efficient method of finding the bus impedance matrix by the method of building algorithm.

To find the bus admittance matrix, the Thévenin's circuit in Figure 9.2(b) is redrawn with impedances converted to admittances as shown in Figure 9.10. The ith diagonal element of the bus admittance matrix is the sum of all admittances connected to bus i, and the ijth off-diagonal element is the negative of the admittance between buses i and j. Referring to Figure 9.10, the bus admittance matrix by inspection is

$$\mathbf{Y}_{bus} = \begin{bmatrix} -j8.75 & j1.25 & j2.5 \\ j1.25 & -j6.25 & j2.5 \\ j2.5 & j2.5 & -j5.0 \end{bmatrix}$$

Using *MATLAB* inverse function **inv**, the bus impedance matrix is obtained

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.16 & j0.08 & j0.12 \\ j0.08 & j0.24 & j0.16 \\ j0.12 & j0.16 & j0.34 \end{bmatrix}$$

From (9.22), for a fault at bus 3 with fault impedance $Z_f = j0.16$ per unit, the fault current is

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.16} = -j2.0 \quad \text{pu}$$

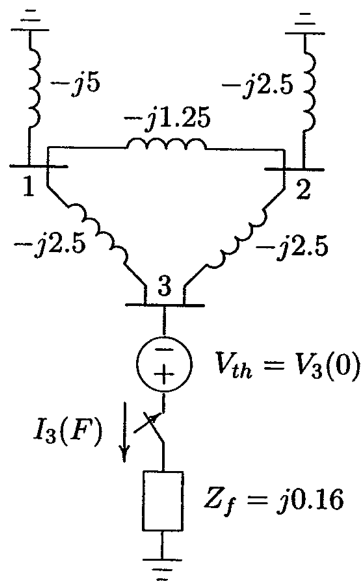

FIGURE 9.10
The admittance diagram for system of Figure 9.2 (b).

From (9.23), bus voltages during the fault are

$$\begin{aligned} V_1(F) &= V_1(0) - Z_{13}I_3(F) = 1.0 - (j0.12)(-j2.0) = 0.76 \quad \text{pu} \\ V_2(F) &= V_2(0) - Z_{23}I_3(F) = 1.0 - (j0.16)(-j2.0) = 0.68 \quad \text{pu} \\ V_3(F) &= V_3(0) - Z_{33}I_3(F) = 1.0 - (j0.34)(-j2.0) = 0.32 \quad \text{pu} \end{aligned}$$

From (9.25), the short circuit currents in the lines are

$$\begin{aligned} I_{12}(F) &= \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.76 - 0.68}{j0.8} = -j0.1 \quad \text{pu} \\ I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.76 - 0.32}{j0.4} = -j1.1 \quad \text{pu} \\ I_{23}(F) &= \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.68 - 0.32}{j0.4} = -j0.9 \quad \text{pu} \end{aligned}$$

The results are exactly the same as the values found in Example 9.1(a). The reader is encouraged to repeat the above calculations for fault at buses 2 and 1, and compare the results with those obtained from parts (b) and (c) in Example 9.1.

Note that the values of the diagonal elements in the bus impedance matrix are the same as the Thévenin's impedances found in Example 9.1, thus eliminating