Self-Charging System of Electric Vehicles: An Optimization Model with no Traffic Interactions

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***Abstract*—The battery packs are one of the primary sources of energy for the electric vehicles (EVs). In order to achieve the maximum power, the structural complexity of battery has to be raised with arrays of thousands of cells. However, this has also raised the cost of battery packs which is around half the cost of an EV. This has given the concept of inductive wireless charging (IWC), which could instantly charge the running EVs, while decreasing the size and thereby cost of the battery packs to a considerable extent. In this paper, the focus is to analyze an electric vehicle self-charging system (SCS), where such a system has been considered as a constrained optimization problem. The SCS contains a set of power transmitters which provide the facility of self-charging, where optimization is required during transition between different set-points of these power transmitters. This optimization will be achieved here by deriving an Augmented Lagrangian-based cost function. Moreover, the proposition is built on a condition where there are no traffic interactions. Performance evaluations under different constraints ensure accuracy of the proposed system.**

***Index Terms*—Battery packs, electric vehicle, optimization, wireless charging, wireless power transfer.**

ACRONYMS AND ABBREVIATIONS OF MATHEMATICAL FORMULATIONS

EV	Electric Vehicle
PEV	Plug-in electric vehicle
HEV	hybrid electric vehicle
PHEV	plug-in hybrid electric vehicle
IWC	Inductive Wireless Charging
P_{trans}	power transmission of the vehicle
F	modal matrix
$\mathcal{X}$	transmission transition variable
P_{inv}	power inverter
L	inductance of power
$\mathcal{E}$	energy capacity of battery
$\mathcal{E}_l$	lower limit of power
α	capacity parameter
E_{cap}	battery energy capacity
$\mathcal{E}_u$	upper limit of power
β	battery parameter
G	model matrix
w	random process noise
t	time-instant
T	number of time-instants
y	observation output
m	number of simultaneous observations
H	observation matrix of state
ν	observation noise
N	number of power transmitters installed
$\mathcal{C}_c$	charging consumption rate of battery
L_m	maximum length of a power transmission circuit

I. INTRODUCTION

Electric vehicles (EVs) are the emerging technology to handle the concept of zero-emission-based transportation sector. To compute the concept of lead-free environment, various types of EVs have been introduced such as: plug-in electric vehicle (PEV), hybrid electric vehicle (HEV), and plug-in hybrid electric vehicle (PHEV). The ideal EVs are solely relying for their energy source on lithium-ion battery packs. This is because the successful EV models have only electric motors for the active propulsion system, which also makes them more simpler and reliable [1].

However, the reliability of EVs from the perspective of battery pack is still a question mark, since the depletion process of battery is not guaranteed. Where EVs bring positive aspects of 1) being environmentally friendly, 2) relying on the concept of renewable, 3) cost effective and 4) energy efficient, they also bring the on-board challenges of 1) recharging the battery, 2) charging time, 3) driving range, and 4) health of the battery pack. The driving range depends on number of factors such as speed of vehicle, interaction of traffic, number and total weight of passengers on-board. All these factors contribute towards more load on the electric motors and propulsion system resulting in quick discharge of battery.

Though the health monitoring of battery pack [2–4] has facilitated to minimize the fluency of these factors, there is still an open forum of research to avoid dead-lock in the operation of EVs. The concept of inductive wireless charging (IWC) [5–7] of battery will effectively avoid this dead-lock. IWC is a system which consists a power supply, transmitter and receiver. These transmitter and receiver are in the form of primary and secondary coils respectively [8, 9]. The power is transmitted wirelessly via magnetic coupling between them. The receivers are always movable since they are installed in the EVs. The transmitters are replaced in the form of strips with a certain amount of gap between them. However, this gap is required to be optimized with the amount of transmitters installed on the wireless transmission track.

In this paper, IWC has been considered as a constrained optimization problem. The vehicle constraints to avoid complete depletion of the battery pack have been considered here. Since these constraints are involved in units of energy and length of the transmission circuit at the same time, a degeneracy factor could result in destabilizing the system with a situation of stranded EVs. The proposed solution considered here an Augmented Lagrangian-based optimization to guarantee a model with effective convergence towards vehicle charging in all situations.

The paper is written as follows: The proposed optimization model is formulated in Section II. In Section III the implementation and evaluation of the proposed scheme is made. Conclusions are drawn in Section IV.

II. PROBLEM FORMULATION

The problem is formulated based on the optimization model framework as shown in Fig. 1.

A. Optimization Framework

The optimization framework is expressed in Fig. 1. It elaborates the steps involved in the framework. 1) A state represen-

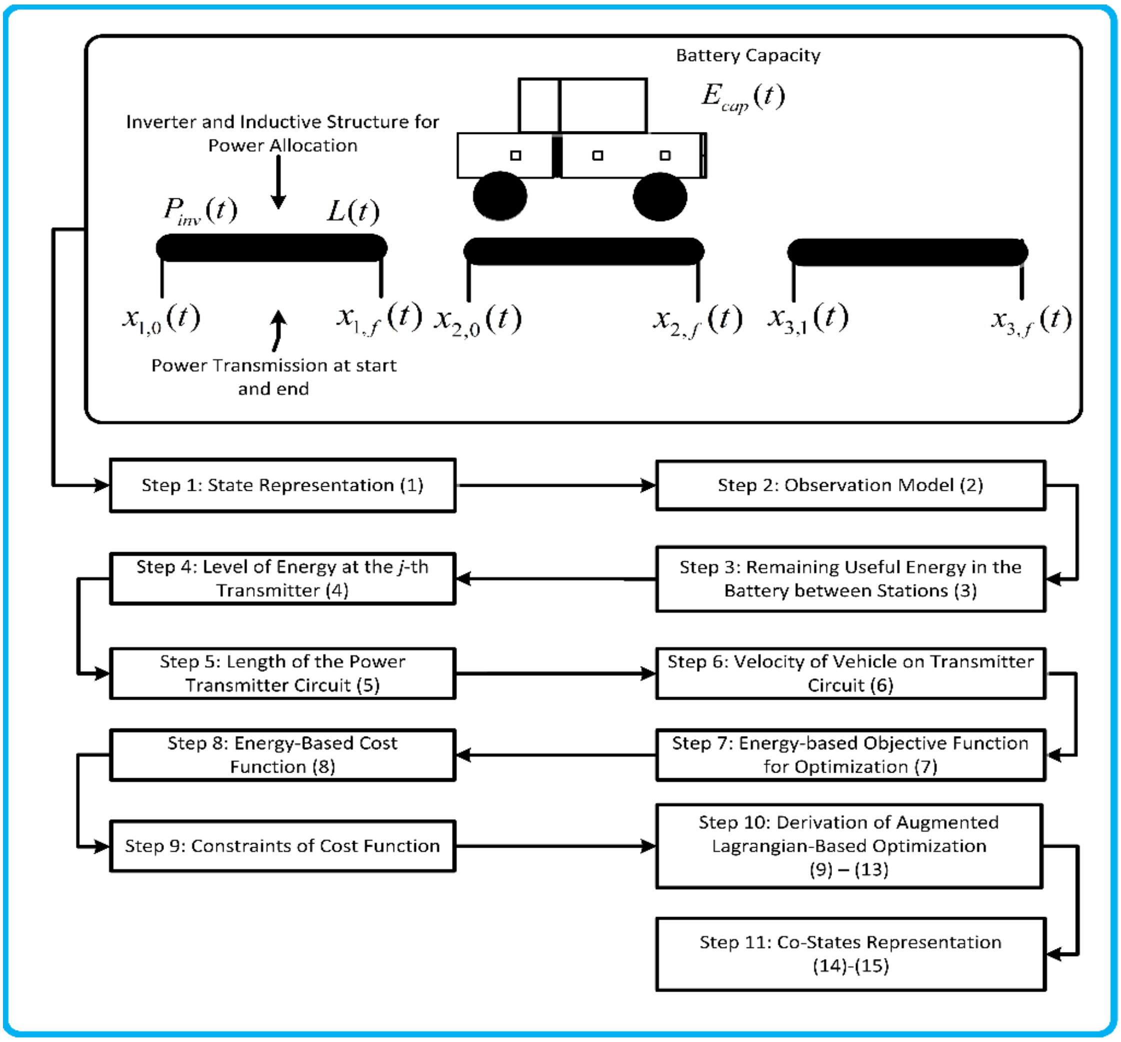

Fig. 1. Proposed framework of self-charging system (SCS)

tation for the vehicle charging system is represented, 2) An observation output is representing the simultaneous observations, 3) Remaining useful energy in the battery between transmitting stations is represented, 4) level of energy at the j-th transmitter is expressed in this step, 5) length of the power transmitter circuit is defined here, 6) velocity of vehicle on the transmitter circuit is calculated in this step, an energy-based objective function and cost function are expressed in (7)-(8) respectively. Constraints of the cost function are defined in (9). Derivation of Augmented Lagrangian with the cost function is made in (10), and finally, 11) co-state representation is in this step.

1) State Representation: Consider an electric vehicle self-charging system. The state representation of such a system can be expressed as:

$$P_{\text{trans}}(t+1)=F(t)P_{\text{trans}}(t)+\frac{\mathcal{X}_t}{2}\Big(P_{\text{inv}}(t)+L(t)\Big)P_{\text{var}}(t) + \mathcal{E}(t)P_{\text{var}}(t)+G(t)w(t) \quad (1)$$

where $P_{\text{trans}}(t) \in \mathbf{R}^{n\times 1}$ is the power transmission of the vehicle at time-instant t. The subscript 'trans' denotes the transmission. $F(t) \in \mathbf{R}^{n\times n}$ is a modal matrix of the state response. $\mathcal{X}(t) \in \mathbf{R}^{n\times n}$ is a transmission transition variable which shows the dynamics of power transmission at the start $x_0(t)$ and end $x_f(t)$ respectively. $P_{\text{inv}}(t)$ is the power inverter at time-instant t. $L(t) \in \mathbf{R}^{n}$ is the inductance of power. $\mathcal{E}(t) \in R^{n\times n}$ is the energy capacity of battery defined as $\mathcal{E}_l(t) < E(t) < \mathcal{E}_u(t)$, where $\mathcal{E}_l(t) = \alpha E_{\text{cap}}(t)$ is the lower limit of power, α is the capacity parameter, $E_{\text{cap}}(t)$ is the energy capacity of a battery. $\mathcal{E}_u(t) = \beta E_{\text{cap}}(t)$ is the upper limit of power, and β is the battery parameter. $G(t) \in \mathbf{R}^{n\times n}$ is the model matrix for process noise. $w(t) \in \mathbf{R}^{n\times 1}$ is the random process noise, t is the time instant, and T refers to the number of time instants.

2) Observation Model: Let the electric vehicle charging system described in (1) be observed at time-instant t as:

$$y(t) = H(t)x(t) + \nu(t) \quad (2)$$

In the observation model (2), $y(t) \in \mathbf{R}^{m\times 1}$ is the observation output of the vehicle charging system, m is the number of simultaneous observations for estimation made at time instant t, $H(t) \in \mathbf{R}^{m\times n}$ is the observation matrix of state, and $\nu(t) \in$

$\mathbf{R}^{m\times 1}$ is the observation noise.

Once the state model and the observation model are expressed, the states of the self-charging system will be defined. It is assumed here that there is no traffic interaction in the system. This allows to minimize the factors like velocity profile of vehicle, traffic congestion, traffic lights etc.

3) Remaining Useful Energy in the Battery between Stations: Let the self-charging vehicle system has N number of power transmitters installed. Let $x_{0,i}(t)$ and $x_{f,i}(t)$ are the starting and ending points of an i-th power transmitter respectively. Similarly, let t_0 be the starting time when the vehicle reaches an i-th power transmitter, and let t_f be the ending state of time, when the vehicle leaves an i-th power transmitter. The remaining useful energy in the battery between stations can be expressed as:

$$E_{\text{cap}}x_i(t_f) - \int_{x_i(t_f)}^{x_j(t_0)} \mathcal{C}_E dt - \mathcal{E}_L(t) = 0 \tag{3}$$

$E_{\text{cap}}x_i(t_f)$ is the energy capacity of the vehicle battery when it was at the t_f time of the i-th transmitter. $\int_{x_i(t_f)}^{x_j(t_0)} \mathcal{C}_E dt$ represents the energy consumption of the vehicle battery in the period from final time t_f at the i-th transmitter to the initial time t_0 of the j-th transmitter.

4) Level of Energy at the j-th Transmitter: This constraint defines the level of energy, which the battery should have at the final point of the j-th transmitter. It is expressed as:

$$\min \Big[E_{cap}x_i(t_f) - \int_{x_i(t_f)}^{x_j(t_f)} \mathcal{C}_E(t)dt + \mathcal{C}_c(t)(x_j(t_f) - x_j(t_0)), E_u(t)\Big] \tag{4}$$

where $\mathcal{C}_c(t)$ represents the charging consumption rate of the battery.

5) Length of the Power Transmitter Circuit: Length of the circuit having set of transmitters can be expressed as:

$$L_m - \Big(x_i(t_f) - x_i(t_0)\Big) \tag{5}$$

where L_m represents the maximum length of a power transmission circuit.

6) Velocity of vehicle on the Transmitter Circuit: The velocity projection of the vehicle on the transmitter circuit can be expressed as:

$$x_{0,f,i}(t) = \mathcal{V}_i(t_{0,f}) - \mathcal{V}_i \tag{6}$$

where $\mathcal{V}_i(t_{0,f})$ represents the velocity of vehicle at the i-th transmitter for all the time.

To avoid power variation in the vehicle charging system, the computation of individual dynamics is required. This is handled by developing a constrained optimization model for the system, which could optimize each constraint by considering it as a subproblem. This would contribute towards the original objective function of optimizing the SCS. In order to achieve that, an augmented Lagrangian-based solution is proposed here.

7) Energy-based Objective Function for Augmented Lagrangian-Based Optimization: The objective of optimal control is to bring the remaining useful energy in the battery between transmitting stations, level of energy at the transmitter, length of the power transmitter circuit, at an optimal point, while maintaining an adequate velocity of the vehicle. The energy-based objective function $f(E)$ is defined as:

$$\begin{aligned} f(E) = & \Big(E_{cap}(t) - \hat{E}_{cap}(t)\Big)^{'} \Big(E_{cap}(t) - \hat{E}_{cap}(t)\Big) \\ & + \Big(\mathcal{C}_E(t) - \hat{\mathcal{C}}_E(t)\Big)^{'} \Big(\mathcal{C}_E(t) - \hat{\mathcal{C}}_E(t)\Big) + \Big(L_m(t) \\ & - \hat{L}_m(t)\Big)^{'} \Big(L_m(t) - \hat{L}_m(t)\Big) + \Big(\mathcal{V}_i(t) - \hat{\mathcal{V}}_i(t)\Big)^{'} \\ & \Big(\mathcal{V}_i(t) - \hat{\mathcal{V}}_i(t)\Big) + \varDelta U(T)^{'} F(T) \end{aligned} \tag{7}$$

where $\varDelta U(T)$ is the variable representing sequence of inputs from the current observed states at time-instant t, such that $\varDelta U(T) = [\varDelta U(t), ...\varDelta U(t+T)]$. A symbol $'$ over a variable represents the transpose operator. $F(T)$ is a compatible vector.

This would now define the cost function for minimization.

8) Energy-Based Cost Function: The energy-based cost function can be represented as:

$$\begin{aligned} J_E = \min \varDelta U(T) = & \frac{1}{2}E_{cap}^2(t) + \frac{1}{2}\mathcal{C}_E^2(t) + \frac{1}{2}L_m^2(t) \\ & + \frac{1}{2}\mathcal{V}_i^2(t) \end{aligned} \tag{8}$$

9) Constraints of Cost Function: Constraints for this cost-function are defined. The first constraint represents the energy capacity of the battery as: $E_{cap}(t+1) = E_{cap}(t) + [E_{cap}(t)x_i(t_f) - \int_{x_i(t_f)}^{x_j(t_f)} \mathcal{C}_E(t)dt - \varepsilon_L(t)] \times \tau_s$. The second constraint represent the level of energy at j-th transmitter as: $\mathcal{C}_E(t+1) = \mathcal{C}_E(t) + \min[E_{cap}x_i(t_f) - \int_{x_i(t_f)}^{x_j(t_f)} \mathcal{C}_E(t)dt - \mathcal{C}_c(t)(x_j(t_f) - x_j(t_0)), E_u(t)\Big] \times \tau_s$. The third constraint represent velocity of vehicle at the i-th transmitter circuit as: $x_{0,f,i}(t+1) = x_{0,f,i}(t) + (\mathcal{V}_i(t_{0,f}) - \mathcal{V}_i) \times \tau_s$. The fourth constraint is about the length of the power transmitter circuit as: $L_m - \Big(x_i(t_f) - x_i(t_0)\Big)$.

10) Derivation of cost function with AL: The augmented Lagrangian-based optimization can be derived here. This will be achieved while considering J_E as the main function, $\varXi(t)$ as Lagrange multiplier, and $\varXi_r(t)$ as augmented Lagrange multiplier, such that $\varXi(t)$, $\varXi_r(t) \geq 0$ as:

$$\begin{aligned} J_E = \min \varDelta U(T) = & \frac{1}{2}E_{cap}^2(t) + \frac{1}{2}\mathcal{C}_E^2(t) + \frac{1}{2}L_m^2(t) + \frac{1}{2}\mathcal{V}_i^2(t) \\ & - \varXi_1(t) \times [E_{cap}(t+1) - E_{cap}(t) - [E_{cap}(t)x_i(t_f) \\ & + \int_{x_i(t_f)}^{x_j(t_f)} \mathcal{C}_E(t)dt + \varepsilon_L(t)] \times \tau_s] - \varXi_{1,r}(t) \times [E_{cap}(t+1) \\ & - E_{cap}(t) - [E_{cap}(t)x_i(t_f) + \int_{x_i(t_f)}^{x_j(t_f)} \mathcal{C}_E(t)dt + \varepsilon_L(t)] \\ & \times \tau_s]^2 - \varXi_2(t) \times \Big[\mathcal{C}_E(t+1) - \mathcal{C}_E(t) - \min[E_{cap}x_i(t_f) \\ & - \int_{x_i(t_f)}^{x_j(t_f)} \mathcal{C}_E(t)dt + \mathcal{C}_c(t)(x_j(t_f) - x_j(t_0)), E_u(t)] \times \tau_s\Big] \\ & - \varXi_{2,r}(t) \times \Big[\mathcal{C}_E(t+1) - \mathcal{C}_E(t) - \min[E_{cap}x_i(t_f) \\ & - \int_{x_i(t_f)}^{x_j(t_f)} \mathcal{C}_E(t)dt + \mathcal{C}_c(t)(x_j(t_f) - x_j(t_0)), E_u(t)] \times \tau_s\Big]^2 \\ & - \varXi_3(t) \times \Big[x_{0,f,i}(t+1) - x_{0,f,i}(t) - (\mathcal{V}_i(t_{0,f}) - \mathcal{V}_i) \times \tau_s\Big] \end{aligned}$$

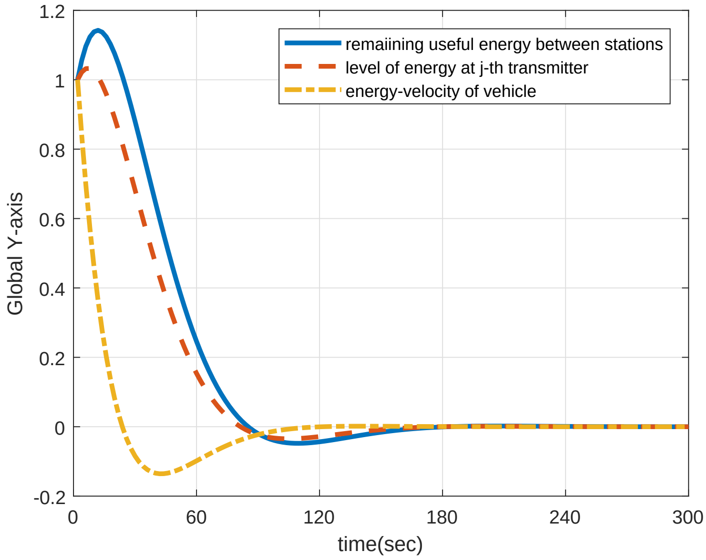

Fig. 2. Optimal control of energy parameters in self-charging system

$$- \varXi_{3,r}(t) \times \Big[x_{0,f,i}(t+1) - x_{0,f,i}(t) - (\mathcal{V}_i(t_{0,f}) - \mathcal{V}_i) \times \tau_s\Big]^2 - \varXi_4(t) \times \Big[[L_m - \Big(x_i(t_f) - x_i(t_0)\Big)] \times \tau_s\Big] - \varXi_{4,r}(t) \times \Big[[L_m - \Big(x_i(t_f) - x_i(t_0)\Big)] \times \tau_s\Big]^2 \tag{9}$$

Minimizing J_E with respect to $E_{cap}(t)$ gives:

$$\frac{\delta J_E}{\delta E_{cap}(t)} = E_{cap}(t) - \varXi_1(t) \times \tau_s + \varXi_1(t) x_i(t_f) \times \tau_s + 2\varXi_{1,r}(t) \times \tau_s + 2\varXi_{1,r}(t) E_{cap}(t) x_i^2(t_f) \times \tau_s - \varXi_2(t) E_{cap}(t) x_i(t_f) \times \tau_s + 2\varXi_{2,r}(t) E_{cap}(t) x_i^2(t_f) \times \tau_s \tag{10}$$

Minimizing J_E with respect to $\mathcal{C}_E(t)$ gives:

$$\frac{\delta J_E}{\delta \mathcal{C}_E(t)} = \mathcal{C}_E(t) - \varXi_1(t)\mathcal{C}_E(t) \times \tau_s - 2\varXi_{1,r}(t)\mathcal{C}_E(t) \times \tau_s + \varXi_2(t) \times \tau_s + \varXi_2(t)\mathcal{C}_E(t) \times \tau_s + 2\varXi_{2,r}(t) \times \tau_s + 2\varXi_{2,r}(t)\mathcal{C}_E(t) \times \tau_s \tag{11}$$

Minimizing J_E with respect to $\mathcal{V}_i(t)$ gives:

$$\frac{\delta J_E}{\delta \mathcal{V}_i(t)} = \mathcal{V}_i(t) + \varXi_3(t) \times \tau_s + 2\varXi_{3,r}(t)\mathcal{V}_i(t) \times \tau_s \tag{12}$$

Minimizing J_E with respect to L_m gives:

$$\frac{\delta J_E}{\delta L_m} = L_m - \varXi_4(t) \times \tau_s - 2\varXi_{4,r}(t) L_m \times \tau_s \tag{13}$$

11) Co-States Representation: The co-states $\varXi_k(t)$ are determined by backward integration of the adjunct state equation yielding:

$$\begin{bmatrix} \varXi_1 \\ \varXi_2 \\ \varXi_3 \\ \varXi_4 \end{bmatrix}_{t-1} = -2h(t)\frac{\delta E_d}{\delta x(t)} - F(t)^{'}\lambda(t) - h(t)[\sum_{i=1}^{N} \nabla_{x(t)} \psi_{\mu s}(\sigma(t)^i, s_i^{dl}(x(t)))] - h(t)[\sum_{j=1}^{N} \nabla_{x(t)} \psi_{\mu g}(\varrho(t)^j, g_j^D(x(t), \nu(t), h(t)))] \tag{14}$$

where,

$$\begin{aligned} & x(t+1) = f_{d_z}^D(x(t), \tau(t), h(t)), t = 0, ..., N-1 \\ & g_j^D(x(t), \nu(t), h(t)) \leq 0, \; j \in \{1, 2, ...j\} \\ & F_d^{'} = I_d \\ & E_d = \mathcal{V}_x(t), \mathcal{V}_y(t), \varOmega_z(t) \end{aligned} \tag{15}$$

III. Numerical Results

The numerical results have been generated here. The parameters of remaining useful energy in the battery between transmitting stations (3), level of energy at the j-th transmitter (4), length of power transmitter (5), velocity of vehicle on transmitter circuit (6), energy-based objective and cost functions (7)-(8), derivation of augmented Lagrangian (9)-(13), minimization with respect to co-states (14)-(15) are considered for the simulation. The parameters considered are as follows: E_{cap} is 7.67 kWh, $\mathcal{C}_c = 0.1 \; kWh/m$, $L_m = 50 \; m$. It can be seen in Fig. 2 that initially there was a sag in the profile of velocity of vehicle. This is due to the different scales of parameters to be optimized. However, all the three energy parameters were optimally controlled while achieving the steady-state value. This is due to the property of Augmented Lagrangian-based optimization and its co-states, which avoid the degeneracy to ensure convergence of all energy parameters.

IV. Conclusions

This paper considered the self-charging system (SCS) as a constrained optimization problem. Several dynamics of the SCS have been considered to achieve an adequate optimization. The proposed scheme was able to adequately provide optimization of the energy parameters of the system on the same scale. This was achieved by doing subproblem minimization of the parameters. Future work would involve to develop an optimization model which considers traffic interactions in the SCS. A robust SCS during heavy traffic situations would eventually contribute towards an improved zero-emission-based transportation sector.

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