Estimation of Vehicle Steering Wheel Angle: An Unscented Kalman Smoother-Based Design

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***Abstract*—In recent years, there has been a growing interest in intelligent and adaptive vehicles. They have installed information and support systems, which may involve several applications and gadgets. A foremost reason on such an initiative could be for an adequate prevention of highway crashes and fatal accidents. Literature shows most of these accidents happen when the vehicle fail to estimate the sharp turns with steering angle. In this work, the focus is on an adequate estimation of the steering angle to minimize these accidents. Firstly, dynamics of vehicle have been derived. Secondly, an Unscented Kalman smoother (UKS) has been built to estimate the nonlinear parameters. Numerical evaluations were made on a vehicle model. Results ensure a better performance and accuracy of the proposed scheme.**

Keywords*—Automotive engineering, autonomous vehicles, electric vehicles (EVs), estimation, expected value, Kalman filter, smoother, unscented transformation, vehicle dynamics, wheels.

I. Introduction

The trend of renewable energy has introduced the concept of electric vehicles (EVs). The EVs have the Lithium-ion battery packs as their main source of energy. Since they are the only source of energy, the battery packs have hundreds of cells aligned in various structures, which are monitored to ensure accuracy and efficiency [1]–[3]. In order to facilitate these battery packs, the EV technology has been further enhanced by having intelligent and adaptive systems, which may provide a support function towards an efficient fuel consumption. Despite of having several gadgets and support functions, the number of road accidents are still a point of concern for the automobile industry [4].

These road accidents are usually due to the vehicle handling at sharp turns, which question the stability and maneuverability of the yaw angle and side-slip angle. Several works have been done for the control of yaw-angle [5], [6]. Yaw angle also plays a key role in handling of combination vehicles [5], [7]. It is also believed that the yaw angle and side-slip angle majorly contribute towards the stability of vehicle [5], [6]. Where the control of yaw angle is recommended for stability, some works have also recommended to estimate these two angles (yaw angle and side-slip angle) [7]–[11]. However, most of these works are based on the tire model, where the direct access to the steering wheel angle was not made using the in-vehicle sensor data. Note that the in-vehicle sensor data access is emphasized over here since in other direct measurements, the readings become meaningless. For example, many of the safety systems such as collision avoidance system, the lane keeping assistance system stop functioning normally if the information from steering wheel sensor is cut or being able to process due to a fault.

This paper proposes a vehicle steering angle estimation while considering the side-slip angle and steering wheel angle (yaw angle). It is shown that the side-slip angle is derived from the lateral motion of vehicle. And steering angle of vehicle is derived from the yaw motion of vehicle. An unscented smoother built on UKF has been proposed to estimate the steering dynamics. This is due to the unscented and non-linear property of the filter. Furthermore, a smoother is introduced here to smooth out any of the noises during the estimation process and accuracy of capturing all the variations could be enhanced. [12] proposed an estimation of steering angle using extended Kalman filter (EKF). The estimation was based on the direct measurements from in-vehicle sensor data. However, the non-linearity of the dynamics was compromised due to the Jacobian nature of EKF.

The write-up of this paper is made as follows: The proposed structure is for estimation of vehicle steering wheel angle is formulated in Section II. Implementation and evaluation of the scheme is made in Section III. Conclusions are made in Section IV.

II. Problem Formulation

The problem formulation of this paper follows follows the structure of the following framework.

A. Estimation of Vehicle Steering Wheel Angle

The focus of this framework is to have an accurate calculation of the steering wheel angle. This can be further explained by Fig. 1. A nonlinear state model of a formulated vehicle is defined in (1), An observation model based on an i-th inbuilt vehicle sensor is built on the state model in (2), sensor model (3), dynamics of electric ground vehicle are derived in: lateral motion motion (4), side-slip angle (5)-(10), yaw motion of the vehicle (11), and steering angle (12)-(14) respectively. A re-defined system model is expressed in (15). An unscented Kalman filter (UKF) is then built with steps of initialization, sigma points, time update, and measurement update in (16)-(28).

1) Nonlinear State Model: Consider a nonlinear system of a four-wheeled vehicle. It is represented by a Gaussian approximate state distribution considering: 1) lateral motion, 2) yaw motion, 3) tire side-slip angle, and 4) steering angle. The longitudinal motion is neglected here. The state model considering all variables can be represented as:

$$x(t+1) = f\Big(x(t),\mathcal{V}_y(t),\Omega_z(t),F_{yf}(t),F_{yr}(t)\Big) + G(t)w(t) \quad (1)$$

where $f(.)$ is the known nonlinear function representing the state transition model. $x_0(t) \in \mathbf{R}^{n\times 1}$ is the initial condition of the state of vehicle at time-instant t. $\mathcal{V}_y(t) \in \mathbf{R}^{n\times 1}$ is the

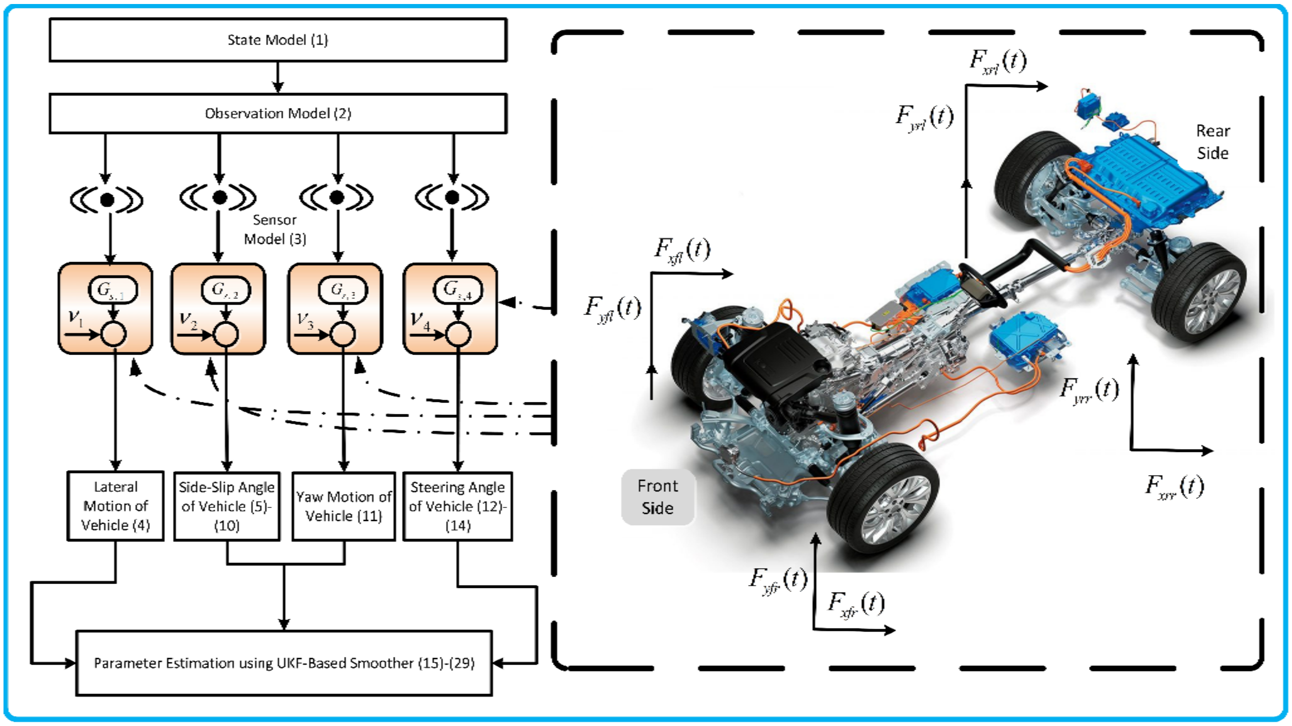

Fig. 1. Proposed framework of steering wheel angle estimation

variable for lateral motion. $\Omega_z(t) \in \mathbf{R}^{n\times 1}$ is the variable for yaw motion. $F_{yf}(t) \in \mathbf{R}^{n\times 1}$ is the force variable of tire for front lateral motion, and $F_{yr}(t) \in \mathbf{R}^{n\times 1}$ is the force variable of the tire for rear lateral motion. $G(t) \in \mathbf{R}^{n\times n}$ is the noise transition matrix, which can be defined as a probability vector whose elements are non-negative real numbers and sum to 1. $w(t) \in \mathbf{R}^{n\times 1}$ is the random process noise, t is the time instant, and T refers to the number of time instants.

2) Observation Model: Let the electric vehicle described in (1) be observed by the i-th in-vehicle sensor at time-instant t as:

$$y_i(t) = H_i(t)x(t) + \nu(t) \tag{2}$$

where $y_i(t) \in \mathbf{R}^{m\times 1}$ is the observation output of state of vehicle at the i-th in-vehicle sensor, m is the number of simultaneous observations for estimation made at time instant t, $H(t) \in \mathbf{R}^{m\times n}$ is the observation matrix of state, and $\nu(t) \in \mathbf{R}^{m\times 1}$ is the observation noise. Note that the noises in $w(t)$ and $\nu(t)$ in both the nonlinear state model and observation model have been assumed initially correlated zero-mean, and white Gaussian.

Once the observation model is extracted from the i-th in-vehicle sensor, the sensor model is defined for each vehicle dynamics.

3) Sensor Model: A sensor is modeled by a gain and an additive noise, as given below:

$$\Upsilon_{s,i}(t) = G_{si}\Upsilon^0_{s,i} + \nu_{s,i} \tag{3}$$

where $\Upsilon_{s,i}$, $\Upsilon^0_{s,i}$, and $\nu_{s,i}$ are the measured sensor output, true or fault-free output and additive sensor noise, respectively for an i-th sensor. Here i is equal to 1, 2, 3, or 4 for the lateral motion, side-slip angle, yaw motion and steering-wheel angle respectively. The gain G_{si} is such that $0 \leq G_{s,i} \leq 1$, with the degree of the fault ranging from no fault at all for $G_{s,i} = 1$ to a complete failure for $G_{si} = 0$.

Once the sensor model is defined, the dynamics of the vehicle along with motion equations are expressed.

4) Lateral Motion of Vehicle: Vehicle equations of motion can be expressed in detail in [13]. Note the longitudinal interaction of dynamics is neglected here. The lateral motion $V_y(t)$ can be represented as:

$$\begin{aligned}\hat{\mathcal{V}}_y(t) \quad &= \quad -\mathcal{V}_x(t)\Omega_z(t) + \frac{1}{M(t)}\Big[\big(F_{yfl}(t) + F_{yfr}(t)\big)\cos\sigma \\ &+ \quad F_{yrl}(t) + F_{yrr}(t)\Big]\end{aligned} \tag{4}$$

where $M(t)$ is the mass of vehicle, $F_{yfl}(t)$, $F_{yfr}(t)$, $F_{yrl}(t)$, and $F_{yrr}(t)$ are the lateral forces of front left tire, front right tire, rear left tire, and rear right tire respectively.

5) Side-Slip Angle of Vehicle: Let $F_y = \big(F_{yfl}(t) + F_{yfr}(t)\big)\cos\sigma + F_{yrl}(t) + F_{yrr}(t)$. This makes (4) as:

$$\hat{\mathcal{V}}_y(t) \quad = \quad -\mathcal{V}_x(t)\Omega_z(t) + \frac{1}{M(t)}F_y(t) \tag{5}$$

Taking $M(t)$ and $\mathcal{V}_x(t)$ common makes (5) as:

$$F_y(t) \quad = \quad M(t)\mathcal{V}_x(t)\Big(\frac{\hat{\mathcal{V}}_y(t)}{\mathcal{V}_x(t)} + \Omega_z(t)\Big) \tag{6}$$

In (6), let $\frac{1}{\mathcal{V}_x(t)} \approx 1$, $\hat{\mathcal{V}}_y(t) = \beta$, $\Omega_z(t) = \gamma$. This makes (6) as:

$$F_y(t) \quad = \quad M(t)\mathcal{V}_x(t)\Big(\hat{\beta} + \gamma\Big) \tag{7}$$

where $M(t)\mathcal{V}_x(t)\Big(\hat{\beta}+\gamma\Big) \approx F_{yf}(t)+F_{yr}(t)$. This makes (7) as:

$$F_y(t) = F_{yf}(t)+F_{yr}(t) \tag{8}$$

Since the lateral forces for front $F_{yf}(t)$ and rear $F_{yfr}(t)$ can be expressed in directions, (8) can be written as:

$$\begin{aligned} F_y(t) &= \Big(F_{yfl}(t)+F_{yfr}(t)\Big)\cos\delta \\ &+ \Big(F_{yrl}(t)+F_{yrr}(t)\Big)\sin\delta \end{aligned} \tag{9}$$

Let $\cos\delta = \alpha_f = \delta - \theta_{vf} = \delta - \beta - \frac{l_f\gamma}{\mathcal{V}_x}$. Also, let $\sin\delta = \alpha_r = -\theta_{vr} = -\beta + \frac{l_r\gamma}{\mathcal{V}_x}$. This gives the representation of $\hat{\beta}$ from (7)-(9) as:

$$\begin{aligned} \hat{\beta} &= -\frac{1}{M(t)\mathcal{V}_x(t)}\Big[\big(F_{yfl}(t)+F_{yfr}(t)\big)+\big(F_{yrl}(t) \\ &+ F_{yrr}(t)\big)\Big]\beta - \Big[1+\frac{1}{M(t)\mathcal{V}_x{}^2}\Big(\big(F_{yfl}(t)+F_{yfr}(t)\big) \\ &\times l_f(t)+\big(F_{yrl}(t)+F_{yrr}(t)\big)\times l_r(t)\Big)\Big]\gamma \\ &+ \Big[\frac{F_{yfl}(t)+F_{yfr}(t)}{M(t)\mathcal{V}_x(t)}\Big]\delta \end{aligned} \tag{10}$$

where $l_r(t)$ and $l_f(t)$ are the longitudinal distances from the center-of-gravity of vehicle to rear and front wheels respectively.

6) Yaw motion of Vehicle: The yaw motion of vehicle is shown as:

$$\begin{aligned} \hat{\Omega}_z(t) &= \frac{1}{I_z(t)}\Big[\Big(F_{yfl}(t)\sin\sigma - F_{yfr}(t)\sin\sigma\Big)l_s(t) - \Big(F_{yfr}(t) \\ &+ F_{yrr}(t)\Big)l_r(t) + \Big((F_{yfr}(t))\cos\sigma\Big)l_f(t)\Big] \end{aligned} \tag{11}$$

where $I_z(t)$ is the yaw inertia. $l_s(t)$ is the longitudinal distances from the center-of-gravity of vehicle to side.

7) Steering Angle of Vehicle: Let $M_z(t) = \Big[\Big(F_{yfl}(t)\sin\sigma - F_{yfr}(t)\sin\sigma\Big)l_s(t) - \Big(F_{yfr}(t) + F_{yrr}(t)\Big)l_r(t) + \Big((F_{yfr}(t))\cos\sigma\Big)l_f(t)\Big]$. This makes (11) as:

$$M_z(t) = I_z\hat{\Omega}_z(t) \tag{12}$$

where $I_z\hat{\Omega}_z(t) \approx \Big(F_{yfl}(t) + F_{yrf}(t)\Big)l_f(t) - \Big(F_{yrr}(t) + F_{yrl}(t)\Big)l_r(t)$. This makes (12) as:

$$\begin{aligned} M_z(t) &= \Big(F_{yfl}(t)+F_{yrf}(t)\Big)l_f(t) - \Big(F_{yrr}(t) \\ &+ F_{yrl}(t)\Big)l_r(t) \end{aligned} \tag{13}$$

Let γ be defined as the steering wheel angle, which can give representation of γ from (11)–(13).

$$\begin{aligned} \hat{\gamma} &= \frac{1}{I_z(t)}\Big[-\big(F_{yfl}(t)+F_{yfr}(t)\big)l_f(t)+\big(F_{yrl}(t) \\ &+ F_{yrr}(t)\big)l_r(t)\Big]\beta \end{aligned} \tag{14}$$

Thus the system model can be defined from vehicle dynamics in (10) and (14) as:

$$x(t) = [\hat{\beta} \quad \hat{\gamma}] \tag{15}$$

8) Parameter Estimation using UKF-Based Smoother (UKS): The purpose now is to individually estimate the nonlinear dynamics of measurements from each local sensor. This is achieved by UKS. Although UKF can perform well in calculating the nonlinear dynamics of the system and upgrading its covariance at each iteration. It is applied here to find $\hat{x}_{t|T}$. This will result in obtaining better optimal estimates, as compared to the estimates obtained when the final sub-optimal UKF estimate is extrapolated backwards in time.

The proposed UKS scheme is outlined as follows. It starts with an initial distribution of the latent variable, and the first observation $p_x(x_1|z_1) = p_x(z_1|x_1)p_x(x_1)$. Note that UKS assumed z_1 has a Gaussian distribution, where $p_x(z_1)$ is approximately $N(\mu_0, \sigma_0^2)$ with a mean μ_0 and a variance σ_0^2. While calculating the output, $\chi(t)$ was generated from sigma vectors transformation. For the forward recursion, the initial condition starts from $P_{0|-1} = var(z_0^T)$, and $x_{0|-1} = 0$. This shows the availability of $a-priori$ information at the previous instant of time. For $t = 1, ...T$, the vehicle state and covariance can be determined by the standard UKF [14] as. Let $\chi_i(t)$ be the computed state as:

$$\chi_i(t) = \Big[[\hat{\beta} \;\; \hat{\gamma}] \quad w(t)' \quad v'(t)\Big] \tag{16}$$

Let W be $T = 2n+1$ number of sigma-points. Here n is the dimension in which the state space model is defined. Sigma points are the sample points here which are chosen to represent a state distribution. It facilitates the filter to generate a non-linear initialization process. Each sigma-point is propagated to compute mean and covariance in the prediction step for the state model as follows:

$$\hat{x}_i(t+1) = \sum_{t=1}^{T} W(t)\chi_i(t|t+1) \tag{17}$$

$$\begin{aligned} \hat{P}_i(t+1) &= \sum_{t=1}^{T} W(t)[\hat{\chi}_i(t+1) - \hat{x}_i(t+1)] \\ &\quad [\hat{\chi}_i(t+1) - \hat{x}_i(t+1)]' \end{aligned} \tag{18}$$

where $P(t+1)$ represents the covariance of vehicle state. Once the state model is propagated, the sigma-points are propagated towards the observation model, followed by the respective mean and covariance as:

$$y_i(t+1) = H_i(t)\chi_i(t+1) + w_i(t) \tag{19}$$

$$\hat{y}_i(t+1) = \sum_{t=1}^{T} W(t)y_i(t+1) \tag{20}$$

$$\begin{aligned} \hat{P}_{i,\hat{y}\hat{y}}(t|t+1) &= \sum_{t=1}^{T} W(t)[y_i(t+1) - \hat{y}_i(t+1)] \\ &\quad [y_i(t+1) - \hat{y}_i(t+1)]' \end{aligned} \tag{21}$$

$$\begin{aligned} \hat{P}_{i,\hat{x}\hat{y}}(t|t+1) &= \sum_{t=1}^{T} W(t)[\hat{\chi}_i(t+1) - \hat{x}_i(t+1)] \\ &\quad [y_i(t+1) - \hat{y}_i(t+1)]' \end{aligned} \tag{22}$$

A gain $K(t)$ is introduced here, which represents as:

$$K(t+1) = P_{i,\hat{x}\hat{y}}(t+1)P_{i,\hat{y}\hat{y}}^{-1}(t+1) \tag{23}$$

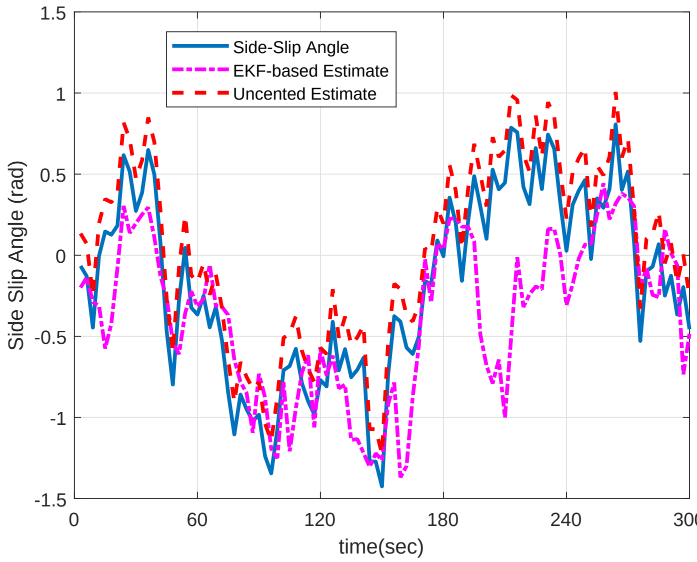

Fig. 2. Evaluation of side-slip angle and its estimate

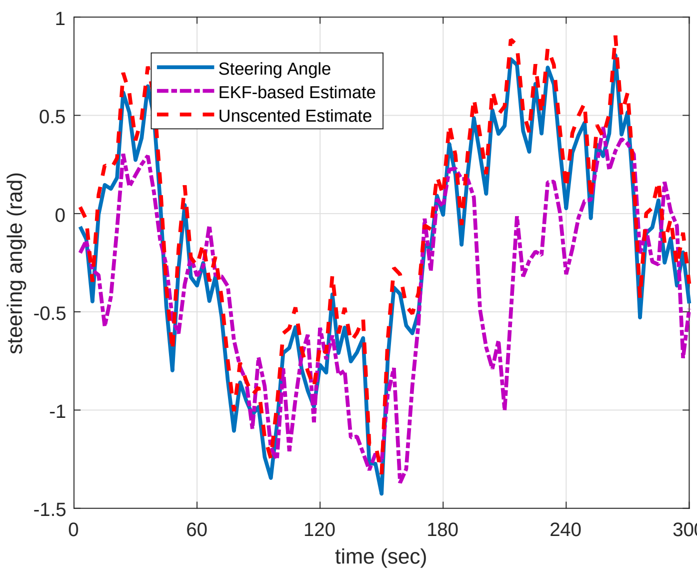

Fig. 3. Evaluation of steering angle and its estimate

This makes the updated state, covariance and posterior state covariance as:

$$\hat{x}_i(t|t) = \hat{x}_i(t+1|t)+K(t+1)\Big(y(t+1) - \hat{y}(t+1)\Big) \quad (24)$$

$$\hat{P}_i(t|t) = \hat{P}_i(t+1|t)–K_i(t+1)P_{i,y\hat{y}}(t|t+1) K_i^{'}(t+1) \quad (25)$$

$$\hat{P}_i(t+1|t) = F(t)\hat{P}_i(t|t)F^{'}(t)+Q(t) \quad (26)$$

The forward run, i.e. UKF, is calculated from (16) to (26). To implement the smoother run, i.e. the backward recursion, the sequence of T observations from UKF is required. This is based on the principle that the smoothed property of the latent variable is the probability at time instant t after a sequence of T observations, i.e. $p(x(t)|y_0^T)$. Note initial values of the state and $a-posteriori$ estimate covariance in (27)-(28) are final values of $\hat{x}(t)$ and $P(t|t)$ in the sequence calculated by UKF, respectively. This is when $t=T$. Moreover, assume that the function $f(.)$ can be simplified to $\frac{\partial f(x(t))}{\partial x(t)}|_{x(t)=x'(t)}$ in (1) and $f(.)$ is linearized around $x^{'}(t)$. Thus, $x(t|t-1) = F\hat{x}(t|t)$. For $t=T,\ T-1, ...,\ 0$, the smoothed error covariance and states are:

$$P_i^S(t|T) = F(t)P_i^S(t-1|T)F^{'}(t) + \Big(K(t)H_i(t) P_i^S(t-1|T) + K_i(t)R_i(e,t)\Big)\Big(K_i(t)H_i(t) P_i^S(t|T-1) + K_i(t)R_i(e,t)\Big)^{'} \quad (27)$$

$$\hat{x}_i(t|T) = \hat{x}_i(t|t-1) + P_i^S(t|T) \quad (28)$$

Meanwhile, the desired initial estimate is $\hat{x}_i(t|T)$, which estimates the state at t instants of time while the time sequence T is known.

One minor limitation is the computational complexity of the formulated UKS requires a considerable amount of storage and latency. This is due to the simultaneous execution of iteration for $T+1$ instances both in forward and backward run respectively. An optional standby option can be applied to reduce the time size T. To avoid latency, the filter can be run in the forward direction only, i.e. run (16)–(26) for the parameter estimation.

III. Numerical Results

The numerical results have been generated here. The derived vehicle dynamics of lateral motion of vehicle (4), side-slip angle (5)-(10), yaw motion of vehicle (11), and steering wheel angle of vehicle (12)-(14) are considered here for simulation. The following parameters are considered here: mass of vehicle M is 1410 kg, yaw moment of inertia Ω_z is 1800 kgm^2, longitudinal distances from center-of-gravity to side l_s, front l_f and rear l_r are: 1000 m, 1100 m and 1500 m respectively. Fig. 2 and 3 show the evaluation of side-slip angle and steering wheel angle respectively. A comparison has been made with EKF [12]. The EKF was unable to estimate the profiles in the beginning. This is due to its slow initialization procedure, which resulted in a slow time-tracking response. Likewise was case between $180-240$ s time window, where EKF lost the track. EKF may have suffered here due to its property of using predefined model for adequate approximation. This was not the case with UKS due to: 1) sigma-points based non-linear initialization of the filter, 2) unscented transformation, and 3) noise smoothing property of the smoother, thereby capturing all the minor details of the profile.

IV. Conclusions

Estimation of steering wheel angle of a four-wheeled independently driven (FWID) electric vehicle is analyzed here. Since side-slip angle plays a role in stability of vehicle steering angle, both angles have been firstly derived by the vehicle dynamics, and then being estimated using UKS. The UKS was able to capture the dynamics thoroughly. Future work would involve an adaptive steering wheel system, which considers a hardware-in-loop (HIL) real-time estimation and safe control of the vehicle steering.

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