Department of Electrical and Electronics Engineering

Higher Colleges of Technology

EEL 3003/ELE-3323 Electric Machines

LO 2 and LO3: Investigate the performance, design, operation of induction and synchronous machines

Class Instructor: Dr. Haris M. Khalid,
Faculty, Electrical and Electronics Engineering Department
Webpage: www.harismkhalid.com

كلّيات التقنية العليا
HIGHER COLLEGES OF TECHNOLOGY

CHAPTER

17

Introduction to Electric Machines

The objective of this chapter is to introduce the basic operation of rotating electric machines. The operation of the three major classes of electric machines—DC, synchronous, and induction—will first be described as intuitively as possible, building on the material presented in Chapter 16. The second part of the chapter will be devoted to a discussion of the applications and selection criteria for the different classes of machines.

The emphasis of this chapter will be on explaining the properties of each type of machine, with its advantages and disadvantages with regard to other types; and on classifying these machines in terms of their performance characteristics and preferred field of application. Chapter 18 will be devoted to a survey of special-purpose electric machines—many of which find common application in industry—such as stepper motors, brushless DC motors, switched reluctance motors, and single-phase induction motors. Selected examples and application notes will discuss some current issues of interest.

By the end of this chapter, you should be able to:

- Describe the principles of operation of DC and AC motors and generators.
- Interpret the nameplate data of an electric machine.
- Interpret the torque-speed characteristic of an electric machine.
- Specify the requirements of a machine given an application.

17.1 ROTATING ELECTRIC MACHINES

The range of sizes and power ratings and the different physical features of rotating machines are such that the task of explaining the operation of rotating machines in a single chapter may appear formidable at first. Some features of rotating machines, however, are common to all such devices. This introductory section is aimed at explaining the common properties of all rotating electric machines. We begin our discussion with reference to Figure 17.1, in which a hypothetical rotating machine is depicted in a cross-sectional view. In the figure, a box with a cross inscribed in it indicates current flowing into the page, while a dot represents current out of the plane of the page.

In Figure 17.1, we identify a **stator,** of cylindrical shape, and a **rotor,** which, as the name indicates, rotates inside the stator, separated from the latter by means of an air gap. The rotor and stator each consist of a magnetic core, some electrical insulation, and the windings necessary to establish a magnetic flux (unless this is created by a permanent magnet). The rotor is mounted on a bearing-supported shaft, which can be connected to *mechanical loads* (if the machine is a motor) or to a *prime mover* (if the machine is a generator) by means of belts, pulleys, chains, or other mechanical couplings. The windings carry the electric currents that generate the magnetic fields and flow to the electrical loads, and also provide the closed loops in which voltages will be induced (by virtue of Faraday's law, as discussed in the previous chapter).

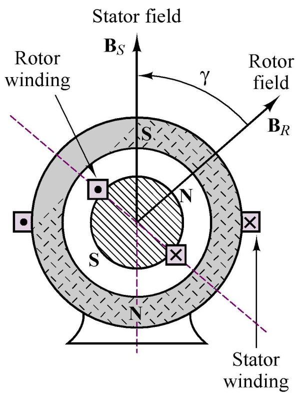


Figure 17.1 A rotating electric machine

Basic Classification of Electric Machines

An immediate distinction can be made between different types of windings characterized by the nature of the current they carry. If the current serves the sole purpose of providing a magnetic field and is independent of the load, it is called a *magnetizing,* or excitation, current, and the winding is termed a **field winding.** Field currents are nearly always DC and are of relatively low power, since their only purpose is to magnetize the core (recall the important role of high-permeability cores in generating large magnetic fluxes from relatively small currents). On the other hand, if the winding carries only the load current, it is called an **armature.** In DC and AC synchronous machines, separate windings exist to carry field and armature currents. In the induction motor, the magnetizing and load currents flow in the same winding, called the *input winding,* or *primary;* the output winding is then called the *secondary*. As we shall see, this terminology, which is reminiscent of transformers, is particularly appropriate for induction motors, which bear a significant analogy to the operation of the transformers studied in Chapters 7 and 16. Table 17.1 characterizes the principal machines in terms of their field and armature configuration.

It is also useful to classify electric machines in terms of their energy-conversion characteristics. A machine acts as a **generator** if it converts mechanical energy from a prime mover—e.g., an internal combustion engine—to electrical form. Examples of generators are the large machines used in power-generating plants, or the common automotive alternator. A machine is classified as a **motor** if it converts electrical energy to mechanical form. The latter class of machines is probably of more direct interest to you, because of its widespread application in engineering practice. Electric motors are used to provide forces and torques to generate motion in countless industrial applications. Machine tools, robots, punches, presses, mills, and propulsion systems for electric vehicles are but a few examples of the application of electric machines in engineering.

Table 17.1 Configurations of the three types of electric machines

Machine type	Winding	Winding type	Location	Current
DC	Input and output	Armature	Rotor	AC (winding) DC (at brushes)
	Magnetizing	Field	Stator	DC
Synchronous	Input and output	Armature	Stator	AC
	Magnetizing	Field	Rotor	DC
Induction	Input	Primary	Stator	AC
	Output	Secondary	Rotor	AC

Note that in Figure 17.1 we have explicitly shown the direction of two magnetic fields: that of the rotor, $\mathbf{B}_R$, and that of the stator, $\mathbf{B}_S$. Although these fields are generated by different means in different machines (e.g., permanent magnets, AC currents, DC currents), the presence of these fields is what causes a rotating machine to turn and enables the generation of electric power. In particular, we see that in Figure 17.1 the north pole of the rotor field will seek to align itself with the south pole of the stator field. It is this magnetic attraction force that permits the generation of torque in an electric motor; conversely, a generator exploits the laws of electromagnetic induction to convert a changing magnetic field to an electric current.

To simplify the discussion in later sections, we shall presently introduce some basic concepts that apply to all rotating electric machines. Referring to Figure 17.2, we note that for all machines the force on a wire is given by the expression

$$\mathbf{f} = i_w \mathbf{l} \times \mathbf{B} \tag{17.1}$$

where i_w is the current in the wire, $\mathbf{l}$ is a vector along the direction of the wire, and $\times$ denotes the cross product of two vectors. Then the torque for a multiturn coil

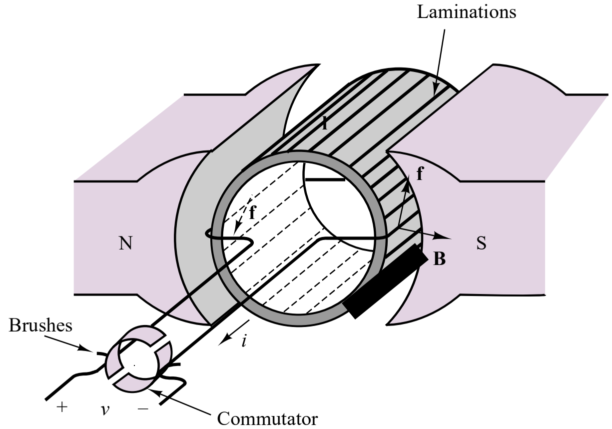


Figure 17.2 Stator and rotor fields and the force acting on a rotating machine

becomes

$$T = KBi_w \sin\alpha \tag{17.2}$$

where

B = magnetic flux density caused by the stator field

K = constant depending on coil geometry

α = angle between **B** and the normal to the plane of the coil

In the hypothetical machine of Figure 17.2, there are two magnetic fields: one generated within the stator, the other within the rotor windings. Either (but not both) of these fields could be generated either by a current or by a permanent magnet. Thus, we could replace the permanent-magnet stator of Figure 17.2 with a suitably arranged winding to generate a stator field in the same direction. If the stator were made of a toroidal coil of radius R (see Chapter 16), then the magnetic field of the stator would generate a flux density B, where

$$B = \mu H = \mu \frac{Ni}{2\pi R} \tag{17.3}$$

and where N is the number of turns and i is the coil current. The direction of the torque is always the direction determined by the rotor and stator fields as they seek to align to each other (i.e., counterclockwise in the diagram of Figure 17.1).

It is important to note that Figure 17.2 is merely a general indication of the major features and characteristics of rotating machines. A variety of configurations exist, depending on whether each of the fields is generated by a current in a coil or by a permanent magnet, and on whether the load and magnetizing currents are direct or alternating. The type of excitation (AC or DC) provided to the windings permits a first classification of electric machines (see Table 17.1). According to this classification, one can define the following types of machines:

- *Direct-current machines:* DC current in both stator and rotor
- *Synchronous machines:* AC current in one winding, DC in the other
- *Induction machines:* AC current in both

In most industrial applications, the induction machine is the preferred choice, because of the simplicity of its construction. However, the analysis of the performance of an induction machine is rather complex. On the other hand, DC machines are quite complex in their construction but can be analyzed relatively simply with the analytical tools we have already acquired. Therefore, the progression of this chapter will be as follows. We start with a section that discusses the physical construction of DC machines, both motors and generators. Then we continue with a discussion of synchronous machines, in which one of the currents is now alternating, since these can easily be understood as an extension of DC machines. Finally, we consider the case where both rotor and stator currents are alternating, and analyze the induction machine.

Performance Characteristics of Electric Machines

As already stated earlier in this chapter, electric machines are **energy-conversion devices,** and we are therefore interested in their energy-conversion **efficiency.** Typical applications of electric machines as motors or generators must take into consideration the energy losses associated with these devices. Figure 17.3(a) and (b) represent the various loss mechanisms you must consider in analyzing the efficiency of an electric machine for the case of direct-current machines. It is important for you to keep in mind this conceptual flow of energy when analyzing electric machines. The sources of loss in a rotating machine can be separated into three fundamental groups: electrical (I^2R) losses, core losses, and mechanical losses.

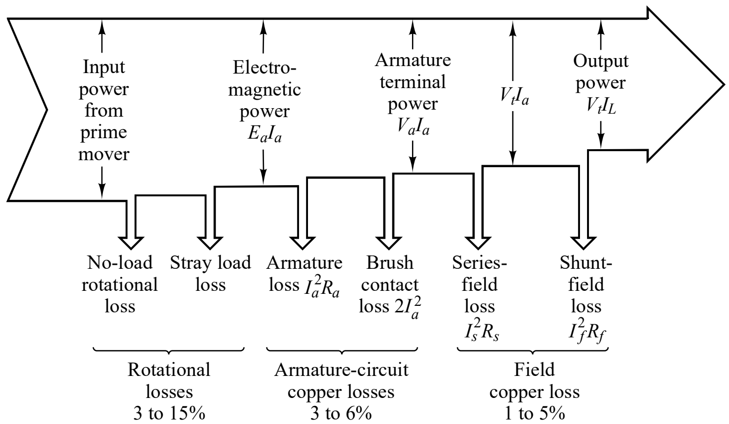


Figure 17.3(a) Generator losses, direct current

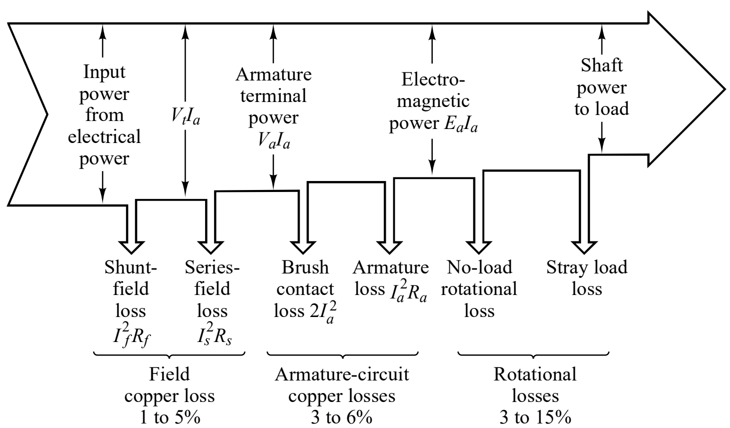


Figure 17.3(b) Motor losses, direct current

I^2R losses are usually computed on the basis of the DC resistance of the windings at 75°C; in practice, these losses vary with operating conditions. The difference between the nominal and actual I^2R loss is usually lumped under the category of *stray-load loss*. In direct-current machines, it is also necessary to account for the *brush contact loss* associated with slip rings and commutators.

Mechanical losses are due to *friction* (mostly in the bearings) and *windage,* that is, the air drag force that opposes the motion of the rotor. In addition, if external devices (e.g., blowers) are required to circulate air through the machine for cooling purposes, the energy expended by these devices is also included in the mechanical losses.

Open-circuit core losses consist of *hysteresis* and *eddy current* losses, with only the excitation winding energized (see Chapter 16 for a discussion of hysteresis and eddy currents). Often these losses are summed with friction and windage losses to give rise to the *no-load rotational loss*. The latter quantity is useful if one simply wishes to compute efficiency. Since open-circuit core losses do not account for the changes in flux density caused by the presence of load currents, an additional magnetic loss is incurred that is not accounted for in this term. *Stray-load losses* are used to lump the effects of nonideal current distribution in the windings and of the additional core losses just mentioned. Stray-load losses are difficult to determine exactly and are often assumed to be equal to 1.0 percent of the output power for DC machines; these losses can be determined by experiment in synchronous and induction machines.

The performance of an electric machine can be quantified in a number of ways. In the case of an electric motor, it is usually portrayed in the form of a graphical **torque-speed characteristic** and **efficiency map.** The torque-speed characteristic of a motor describes how the torque supplied by the machine varies as a function of the speed of rotation of the motor for steady speeds. As we shall see in later sections, the torque-speed curves vary in shape with the type of motor (DC, induction, synchronous) and are very useful in determining the performance of the motor when connected to a mechanical load. Figure 17.4(a) depicts the torque-speed curve of a hypothetical motor. Figure 17.4(b) depicts a typical efficiency map for a DC machine. It is quite likely that in most engineering applications, the engineer is required to make a decision regarding the performance characteristics of the motor best suited to a specified task. In this context, the torque-speed curve of a machine is a very useful piece of information.

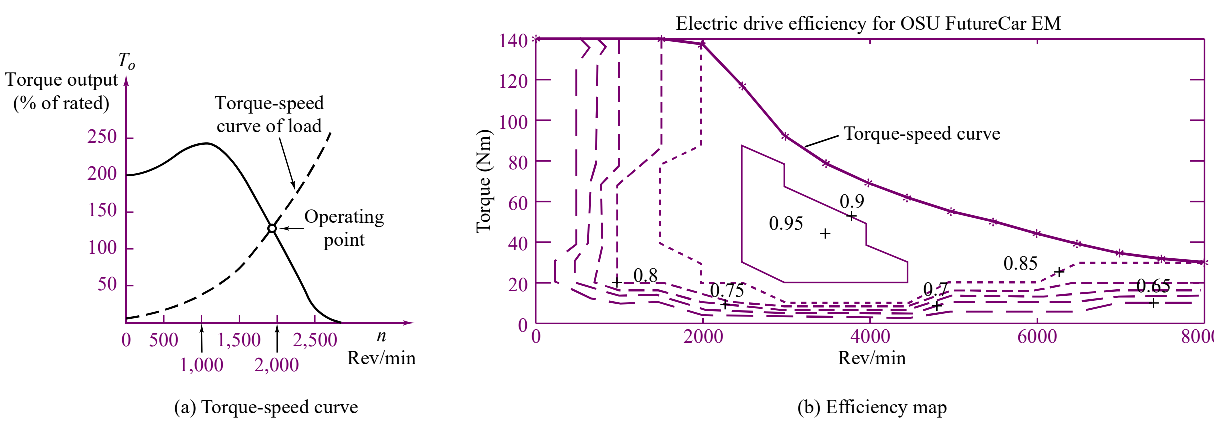


Figure 17.4 Torque-speed and efficiency curves for an electric motor

The first feature we note of the torque-speed characteristic is that it bears a strong resemblance to the i-v characteristics used in earlier chapters to represent the behavior of electrical sources. It should be clear that, according to this torque-speed curve, the motor is not an ideal source of torque (if it were, the curve would appear as a horizontal line across the speed range). One can readily see, for example, that the hypothetical motor represented by the curves of Figure 17.4(a) would produce maximum torque in the range of speeds between approximately 800 and 1,400 rev/min. What determines the actual speed of the motor (and therefore its output torque and power) is the torque-speed characteristic of the load connected to it, much as a resistive load determines the current drawn from a voltage source. In the figure, we display the torque-speed curve of a load, represented by the dashed line; the operating point of the motor-load pair is determined by the intersection of the two curves.

Another important observation pertains to the fact that the motor of Figure 17.4(a) produces a nonzero torque at zero speed. This fact implies that as soon as electric power is connected to the motor, the latter is capable of supplying a certain amount of torque; this zero-speed torque is called the **starting torque.** If the load the motor is connected to requires less than the starting torque the motor can provide, then the motor can accelerate the load, until the motor speed and torque settle to a stable value, at the operating point. The motor-load pair of Figure 17.4(a) would behave in the manner just described. However, there may well be circumstances in which a motor might not be able to provide a sufficient starting torque to overcome the static load torque that opposes its motion. Thus, we see that a torque-speed characteristic can offer valuable insight into the operation of a motor. As we proceed to discuss each type of machine in greater detail, we shall devote some time to the discussion of its torque-speed curve.

FIND IT ON THE WEB

The efficiency of an electric machine is also an important design and performance characteristic. The **1995 Department of Energy Energy Policy Act,** also known as EPACT, has required electric motor manufacturers to guarantee a minimum efficiency. The efficiency of an electric motor is usually described using a contour plot of the efficiency value (a number between 0 and 1) in the torque-speed plane. This representation permits a determination of the motor efficiency as a function of its performance and operating conditions. Figure 17.4(b) depicts the efficiency map of an electric drive used in a hybrid-electric vehicle—a 20-kW permanent magnet AC (or brushless DC) machine. We shall discuss this type of machine in Chapter 18. Note that the peak efficiency can be as high as 0.95 (95 percent), but that the efficiency decreases significantly away from the optimum point (around 3500 rev/min and 45 N-m), to values as low as 0.65.

The most common means of conveying information regarding electric machines is the *nameplate*. Typical information conveyed by the nameplate is:

1. Type of device (e.g., DC motor, alternator)
2. Manufacturer
3. Rated voltage and frequency
4. Rated current and volt-amperes
5. Rated speed and horsepower

The **rated voltage** is the terminal voltage for which the machine was designed, and which will provide the desired magnetic flux. Operation at higher voltages will increase magnetic core losses, because of excessive core saturation. The **rated current** and **rated volt-amperes** are an indication of the typical current and power levels at the terminal that will not cause undue overheating due to copper losses (I^2R losses) in the windings. These ratings are not absolutely precise, but they give an indication of the range of excitations for which the motor will perform without overheating. Peak power operation in a motor may exceed rated torque, power, or currents by a substantial factor (up to as much as 6 or 7 times the rated value); however, continuous operation of the motor above the rated performance will cause the machine to overheat, and eventually to sustain damage. Thus, it is important to consider both peak and continuous power requirements when selecting a motor for a specific application. An analogous discussion is valid for the speed rating: While an electric machine may operate above rated speed for limited periods of time, the large centrifugal forces generated at high rotational speeds will eventually cause undesirable mechanical stresses, especially in the rotor windings, leading eventually even to self-destruction.

Another important feature of electric machines is the **regulation** of the machine speed or voltage, depending on whether it is used as a motor or as a generator, respectively. Regulation is the ability to maintain speed or voltage constant in the face of load variations. The ability to closely regulate speed in a motor or voltage in a generator is an important feature of electric machines; regulation is often improved by means of feedback control mechanisms, some of which will be briefly introduced in this chapter. We shall take the following definitions as being adequate for the intended purpose of this chapter:

$$\text{Speed regulation} = \frac{\text{Speed at no load} - \text{Speed at rated load}}{\text{Speed at rated load}} \tag{17.4}$$

$$\text{Voltage regulation} = \frac{\text{Voltage at no load} - \text{Voltage at rated load}}{\text{Voltage at rated load}} \tag{17.5}$$

Please note that the rated value is usually taken to be the nameplate value, and that the meaning of *load* changes depending on whether the machine is a motor, in which case the load is mechanical, or a generator, in which case the load is electrical.

EXAMPLE 17.1 Regulation

Problem

Find the percent speed regulation of a shunt DC motor.

Solution

Known Quantities: No-load speed, speed at rated load.

Find: Percent speed regulation, SR%.

Schematics, Diagrams, Circuits, and Given Data:

n_{nl} = no-load speed = 1,800 rev/min

n_{nr} = rated-load speed = 1,760 rev/min

Analysis:

$$\text{SR}\% = \frac{n_{\text{nl}} - n_{\text{rl}}}{n_{\text{rl}}} \times 100 = \frac{1{,}800 - 1{,}760}{1{,}800} \times 100 = 2.27\%$$

Comments: Speed regulation is an intrinsic property of a motor; however, external speed controls can be used to regulate the speed of a motor to any (physically achievable) desired value. Some motor control concepts are discussed later in this chapter.

EXAMPLE 17.2 Nameplate Data

Problem

Discuss the nameplate data, shown below, of a typical induction motor.

Solution

Known Quantities: Nameplate data.

Find: Motor characteristics.

Schematics, Diagrams, Circuits, and Given Data: The nameplate appears below.

MODEL	19308 J-X		
TYPE	CJ4B	FRAME	324TS
VOLTS	230/460	°C AMB.	40
		INS. CL.	B
FRT. BRG	210SF	EXT. BRG	312SF
SERV FACT	1.0	OPER INSTR	C-517
PHASE | 3	Hz | 60	CODE | G	WDGS | 1
H.P.	40		
R.P.M.	3,565		
AMPS	106/53		
NEMA NOM.	EFF		
NOM. P.F.			
DUTY	CONT.	NEMA DESIGN	B

Analysis: The nameplate of a typical induction motor is shown in the table above. The model number (sometimes abbreviated as MOD) uniquely identifies the motor to the manufacturer. It may be a style number, a model number, an identification number, or an instruction sheet reference number.

The term *frame* (sometimes abbreviated as FR) refers principally to the physical size of the machine, as well as to certain construction features.

Ambient temperature (abbreviated as AMB, or MAX. AMB) refers to the maximum ambient temperature in which the motor is capable of operating. Operation of the motor in a higher ambient temperature may result in shortened motor life and reduced torque.

Insulation class (abbreviated as INS. CL.) refers to the type of insulation used in the motor. Most often used are class A (105°C) and class B (130°C).

The duty (DUTY), or time rating, denotes the length of time the motor is expected to be able to carry the rated load under usual service conditions. "CONT." means that the machine can be operated continuously.

The "CODE" letter sets the limits of starting kVA per horsepower for the machine. There are 19 levels, denoted by the letters A through V, excluding I, O, and Q.

Service factor (abbreviated as SERV FACT) is a term defined by NEMA (the National Electrical Manufacturers Association) as follows: "The service factor of a general-purpose alternating-current motor is a multiplier which, when applied to the rated horsepower, indicates a permissible horsepower loading which may be carried under the conditions specified for the service factor."

The voltage figure given on the nameplate refers to the voltage of the supply circuit to which the motor should be connected. Sometimes two voltages are given, for example, 230/460. In this case, the machine is intended for use on either a 230-V or a 460-V circuit. Special instructions will be provided for connecting the motor for each of the voltages.

The term "BRG" indicates the nature of the bearings supporting the motor shaft.

EXAMPLE 17.3 Torque-Speed Curves

Problem

Discuss the significance of the torque-speed curve of an electric motor.

Solution

A variable-torque variable-speed motor has a torque output that varies directly with speed; hence, the horsepower output varies directly with the speed. Motors with this characteristic are commonly used with fans, blowers, and centrifugal pumps. Figure 17.5 shows typical torque-speed curves for this type of motor. Superimposed on the motor torque-speed curve is the torque-speed curve for a typical fan where the input power to the fan varies as the cube of the fan speed. Point A is the desired operating point, which could be determined graphically by plotting the load line and the motor torque-speed curve on the same graph, as illustrated in Figure 17.5.

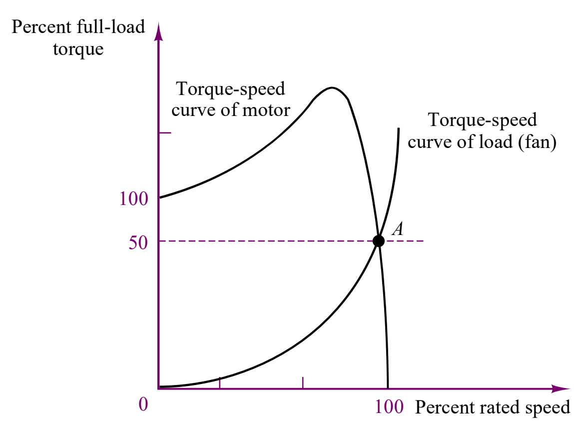


Figure 17.5

Basic Operation of All Rotating Machines

We have already seen in Chapter 16 how the magnetic field in electromechanical devices provides a form of coupling between electrical and mechanical systems. Intuitively, one can identify two aspects of this coupling, both of which play a role in the operation of electric machines:

1. Magnetic attraction and repulsion forces generate mechanical torque.
2. The magnetic field can induce a voltage in the machine windings (coils) by virtue of Faraday's law.

Thus, we may think of the operation of an electric machine in terms of either a motor or a generator, depending on whether the input power is electrical and mechanical power is produced (motor action), or the input power is mechanical and the output power is electrical (generator action). Figure 17.6 illustrates the two cases graphically.

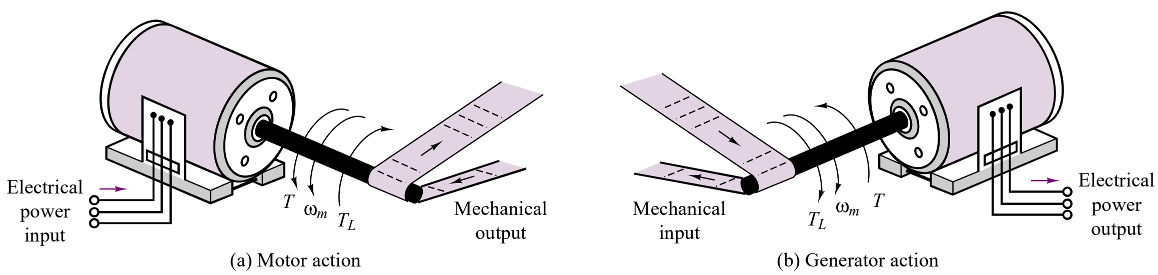


Figure 17.6 Generator and motor action in an electric machine

The coupling magnetic field performs a dual role, which may be explained as follows. When a current i flows through conductors placed in a magnetic field, a force is produced on each conductor, according to equation 17.1. If these conductors are attached to a cylindrical structure, a torque is generated, and if the structure is free to rotate, then it will rotate at an angular velocity ω_m. As the conductors rotate, however, they move through a magnetic field and cut through flux lines, thus generating an electromotive force in opposition to the excitation. This emf is also called "counter" emf, as it opposes the source of the current i. If, on the other hand, the rotating element of the machine is driven by a prime mover (for example, an internal combustion engine), then an emf is generated across the coil that is rotating in the magnetic field (the armature). If a load is connected to the armature, a current i will flow to the load, and this current flow will in turn cause a reaction torque on the armature that opposes the torque imposed by the prime mover.

You see, then, that for energy conversion to take place, two elements are required:

1. A coupling field, **B**, usually generated in the field winding.
2. An armature winding that supports the load current, i, and the emf, e.

Magnetic Poles in Electric Machines

Before discussing the actual construction of a rotating machine, we should spend a few paragraphs to illustrate the significance of **magnetic poles** in an electric machine. In an electric machine, torque is developed as a consequence of magnetic forces of attraction and repulsion between magnetic poles on the stator and on the rotor; these poles produce a torque that accelerates the rotor and a reaction torque on the stator. Naturally, we would like a construction such that the torque generated as a consequence of the magnetic forces is continuous and in a constant direction. This can be accomplished if the number of rotor poles is equal to the number of stator poles. It is also important to observe that the number of poles must be even, since there have to be equal numbers of north and south poles.

The motion and associated electromagnetic torque of an electric machine are the result of two magnetic fields that are trying to align with each other so that the south pole of one field attracts the north pole of the other. Figure 17.7 illustrates this action by analogy with two permanent magnets, one of which is allowed to rotate about its center of mass.

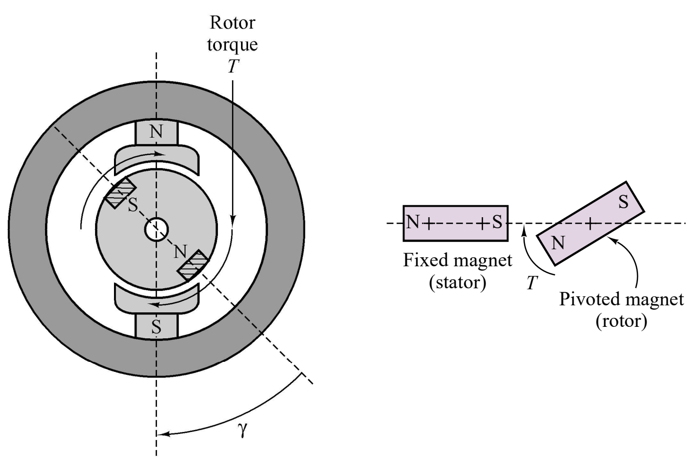


Figure 17.7 Alignment action of poles

Figure 17.8 depicts a two-pole machine in which the stator poles are constructed in such a way as to project closer to the rotor than to the stator structure. This type of construction is rather common, and poles constructed in this fashion are called **salient poles.** Note that the rotor could also be constructed to have salient poles.

To understand magnetic polarity, we need to consider the direction of the magnetic field in a coil carrying current. Figure 17.9 shows how the *right-hand rule* can be employed to determine the direction of the magnetic flux. If one were to grasp the coil with the right hand, with the fingers curling in the direction of current flow, then the thumb would be pointing in the direction of the magnetic flux. Magnetic flux is by convention viewed as entering the south pole and exiting from the north pole. Thus, to determine whether a magnetic pole is north or south, we must consider the direction of the flux. Figure 17.10 shows a cross section of a coil wound around a pair of salient rotor poles. In this case, one can readily identify the direction of the magnetic flux and therefore the magnetic polarity of the poles by applying the right-hand rule, as illustrated in the figure.

Often, however, the coil windings are not arranged as simply as in the case of salient poles. In many machines, the windings are embedded in slots cut into the stator or rotor, so that the situation is similar to that of the stator depicted in

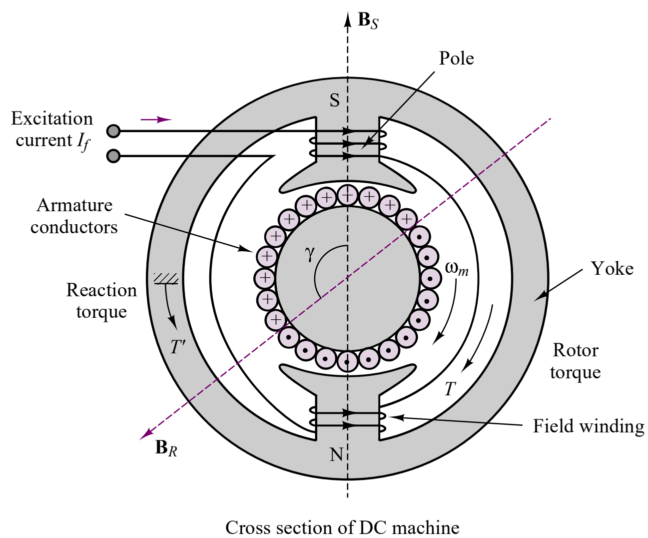


Figure 17.8 A two-pole machine with salient stator poles

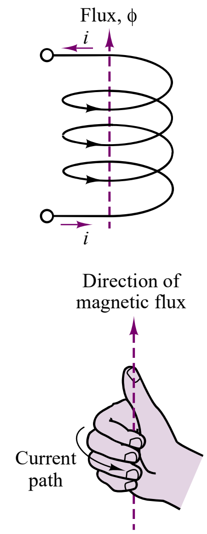


Figure 17.9 Right-hand rule

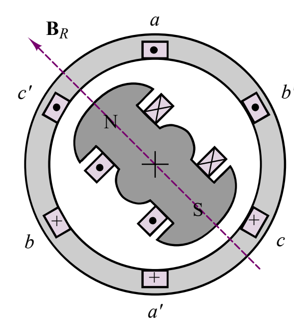


Figure 17.10 Magnetic field in a salient rotor winding

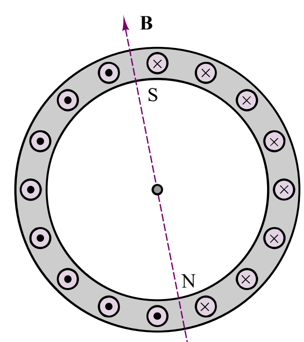


Figure 17.11 Magnetic field of stator

Figure 17.11. This figure is a cross section in which the wire connections between "crosses" and "dots" have been cut away. In Figure 17.11, the dashed line indicates the axis of the stator flux according to the right-hand rule, indicating that the slotted stator in effect behaves like a pole pair. The north and south poles indicated in the figure are a consequence of the fact that the flux exits the bottom part of the structure (thus, the north pole indicated in the figure) and enters the top half of the structure (thus, the south pole). In particular, if you consider that the windings are arranged so that the current entering the right-hand side of the stator (to the right of the dashed line) flows through the back end of the stator and then flows outward from the left-hand side of the stator slots (left of the dashed line), you can visualize the windings in the slots as behaving in a manner similar to the coils of Figure 17.10, where the flux axis of Figure 17.11 corresponds to the flux axis of each of the coils of Figure 17.10. The actual circuit that permits current flow is completed by the front and back ends of the stator, where the wires are connected according to the pattern a-a', b-b', c-c', as depicted in the figure.

Another important consideration that facilitates understanding the operation of electric machines pertains to the use of AC currents. It should be apparent by now that if the current flowing into the slotted stator is alternating, the direction of the flux will also alternate, so that in effect the two poles will reverse polarity every time the current reverses direction, that is, every half-cycle of the sinusoidal current. Further—since the magnetic flux is approximately proportional to the current in the coil—as the amplitude of the current oscillates in a sinusoidal fashion, so will the flux density in the structure. Thus, *the magnetic field developed in the stator changes both spatially and in time.*

This property is typical of AC machines, where a *rotating magnetic field* is established by energizing the coil with an alternating current. As we shall see in the next section, the principles underlying the operation of DC and AC machines are quite different: in a direct-current machine, there is no rotating field, but a mechanical switching arrangement (the *commutator*) makes it possible for the rotor and stator magnetic fields to always align at right angles to each other.

The accompanying CD-ROM includes 2-D "movies" of the most common types of electric machines. You might wish to explore these animations to better understand the basic concepts described in this section.

Check Your Understanding

17.1 The percent speed regulation of a motor is 10 percent. If the full-load speed is 50π rad/s, find (a) the no-load speed in rad/s, and (b) the no-load speed in rev/min.

17.2 The percent voltage regulation for a 250-V generator is 10 percent. Find the no-load voltage of the generator.

17.3 The nameplate of a three-phase induction motor indicates the following values:

H.P. = 10	Volt = 220 V
R.P.M. = 1,750	Service factor = 1.15
Temperature rise = 60°C	Amp = 30A

Find the rated torque, rated volt-amperes, and maximum continuous output power.

17.4 A motor having the characteristics shown in Figure 17.4 is to drive a load; the load has a linear torque-speed curve and requires 150 percent of rated torque at 1,500 rev/min. Find the operating point for this motor-load pair.

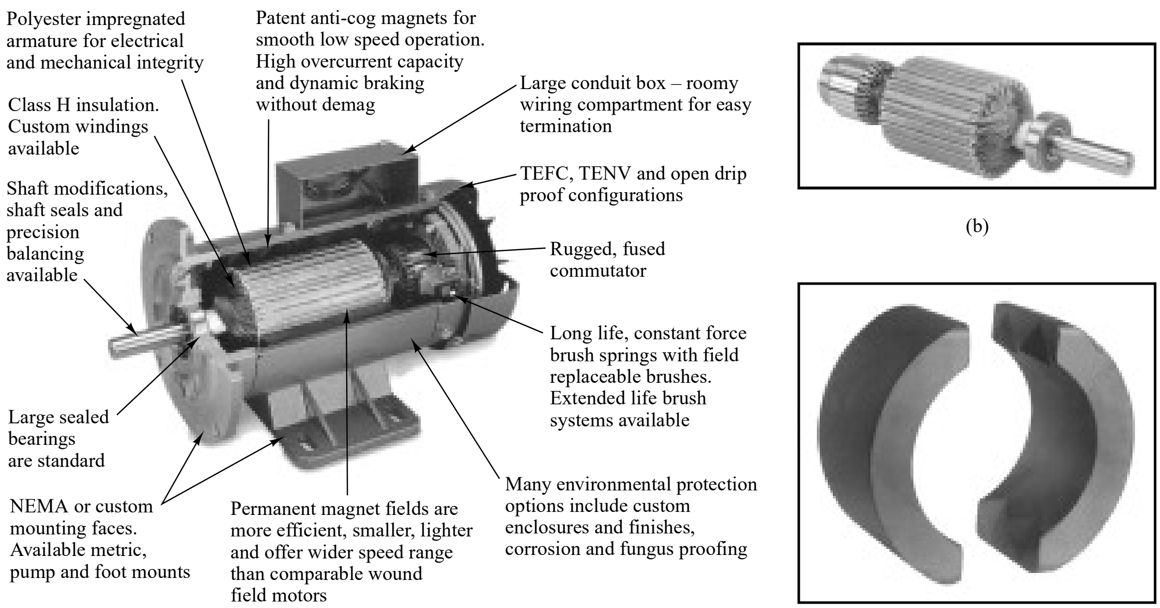
17.2 DIRECT-CURRENT MACHINES

As explained in the introductory section, direct-current (DC) machines are easier to analyze than their AC counterparts, although their actual construction is made rather complex by the need to have a commutator, which reverses the direction of currents and fluxes to produce a net torque. The objective of this section is to describe the major construction features and the operation of direct-current machines, as well as to develop simple circuit models that are useful in analyzing the performance of this class of machines.

Physical Structure of DC Machines

A representative DC machine was depicted in Figure 17.8, with the magnetic poles clearly identified, for both the stator and the rotor. Figure 17.12 is a photograph of the same type of machine. Note the salient pole construction of the stator and the slotted rotor. As previously stated, the torque developed by the machine is a consequence of the magnetic forces between stator and rotor poles. This torque is maximum when the angle γ between the rotor and stator poles is 90°. Also, as you can see from the figure, in a DC machine the armature is usually on the rotor, and the field winding is on the stator.

To keep this torque angle constant as the rotor spins on its shaft, a mechanical switch, called a **commutator,** is configured so the current distribution in the rotor winding remains constant and therefore the rotor poles are consistently at 90° with respect to the fixed stator poles. In a DC machine, the magnetizing current is DC, so that there is no spatial alternation of the stator poles due to time-varying currents. To understand the operation of the commutator, consider the simplified diagram of Figure 17.13. In the figure, the brushes are fixed, and the rotor revolves at an angular velocity ω_m; the instantaneous position of the rotor is given by the expression $\theta = \omega_m t - \gamma$.



(a) (c)

Figure 17.12 (a) DC machine; (b) rotor; (c) permanent magnet stator

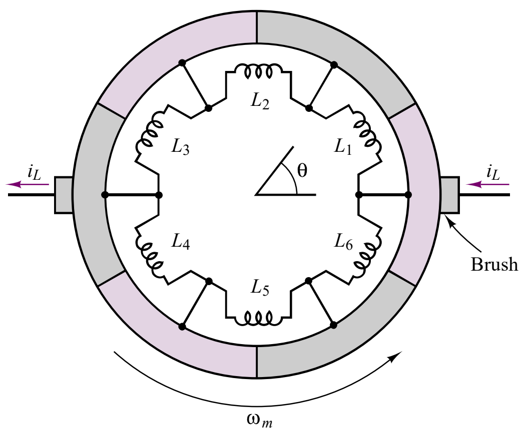


Figure 17.13 Rotor winding and commutator

The commutator is fixed to the rotor and is made up in this example of six segments that are made of electrically conducting material but are insulated from each other. Further, the rotor windings are configured so that they form six coils, connected to the commutator segments as shown in Figure 17.13.

As the commutator rotates counterclockwise, the rotor magnetic field rotates with it up to $\theta = 30^\circ$. At that point, the direction of the current changes in coils L_3 and L_6 as the brushes make contact with the next segment. Now the direction of the magnetic field is -30°. As the commutator continues to rotate, the direction of the rotor field will again change from -30° to $+30^\circ$, and it will switch again when the brushes switch to the next pair of segments. In this machine, then, the torque angle, γ, is not always 90°, but can vary by as much as ±30°; the actual torque produced by the machine would fluctuate by as much as ±14 percent, since the torque is proportional to $\sin\gamma$. As the number of segments increases, the torque fluctuation produced by the commutation is greatly reduced. In a practical machine, for example, one might have as many as 60 segments, and the variation of γ from 90° would be only ±3°, with a torque fluctuation of less than 1 percent. Thus, the DC machine can produce a nearly constant torque (as a motor) or voltage (as a generator).

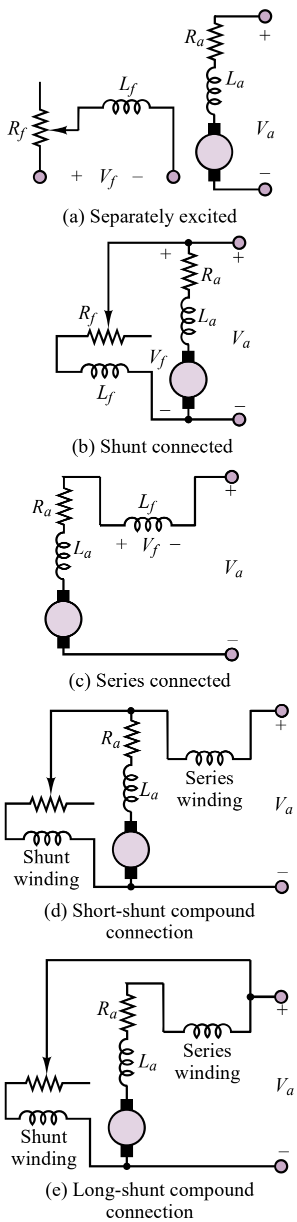


Figure 17.14

Configuration of DC Machines

In DC machines, the field excitation that provides the magnetizing current is occasionally provided by an external source, in which case the machine is said to be **separately excited** (Figure 17.14(a)). More often, the field excitation is derived from the armature voltage and the machine is said to be **self-excited.** The latter configuration does not require the use of a separate source for the field excitation and is therefore frequently preferred. If a machine is in the separately excited configuration, an additional source, V_f, is required. In the self-excited case, one method used to provide the field excitation is to connect the field in parallel with the armature; since the field winding typically has significantly higher resistance than the armature circuit (remember that it is the armature that carries the load current), this will not draw excessive current from the armature. Further, a series resistor can be added to the field circuit to provide the means for adjusting the field current independent of the armature voltage. This configuration is called a **shunt-connected** machine and is depicted in Figure 17.14(b). Another method for self-exciting a DC machine consists of connecting the field in series with the armature, leading to the **series-connected** machine, depicted in Figure 17.14(c); in this case, the field winding will support the entire armature current, and thus the field coil must have low resistance (and therefore relatively few turns). This configuration is rarely used for generators, since the generated voltage and the load voltage must always differ by the voltage drop across the field coil, which varies with the load current. Thus, a series generator would have poor (large) regulation. However, series-connected motors are commonly used in certain applications, as will be discussed in a later section.

The third type of DC machine is the **compound-connected** machine, which consists of a combination of the shunt and series configurations. Figures 17.14(d) and (e) show the two types of connections, called the **short shunt** and the **long shunt,** respectively. Each of these configurations may be connected so that the series part of the field adds to the shunt part (**cumulative compounding**) or so that it subtracts (**differential compounding**).

DC Machine Models

As stated earlier, it is relatively easy to develop a simple model of a DC machine, which is well suited to performance analysis, without the need to resort to the details of the construction of the machine itself. This section will illustrate the development of such models in two steps. First, steady-state models relating field and armature currents and voltages to speed and torque are introduced; second, the differential equations describing the dynamic behavior of DC machines are derived.

When a field excitation is established, a magnetic flux, ϕ, is generated by the field current, I_f. From equation 17.2, we know that the torque acting on the rotor is proportional to the product of the magnetic field and the current in the load-carrying wire; the latter current is the armature current, I_a (i_w, in equation 16.2). Assuming that, by virtue of the commutator, the torque angle, γ, is kept very close to 90°, and therefore $\sin\gamma = 1$, we obtain the following expression for the torque (in units of N-m) in a DC machine:

$$T = k_T \phi I_a \qquad \text{for } \gamma = 90^\circ \tag{17.6}$$

You may recall that this is simply a consequence of the Bli law of Chapter 16. The mechanical power generated (or absorbed) is equal to the product of the machine torque and the mechanical speed of rotation, ω_m (in rad/s), and is therefore given by

$$P_m = \omega_m T = \omega_m k_T \phi I_a \tag{17.7}$$

Recall now that the rotation of the armature conductors in the field generated by the field excitation causes a **back emf,** E_b, in a direction that opposes the rotation of the armature. According to the Blu law (see Chapter 16), then, this back emf is given by the expression

$$E_b = k_a \phi \omega_m \tag{17.8}$$

where k_a is called the **armature constant** and is related to the geometry and magnetic properties of the structure. The voltage E_b represents a countervoltage (opposing the DC excitation) in the case of a motor, and the generated voltage in the case of a generator. Thus, the electric power dissipated (or generated) by the machine is given by the product of the back emf and the armature current:

$$P_e = E_b I_a \tag{17.9}$$

The constants k_T and k_a in equations 17.6 and 17.8 are related to geometry factors, such as the dimension of the rotor and the number of turns in the armature winding; and to properties of materials, such as the permeability of the magnetic materials. Note that in the ideal energy-conversion case, $P_m = P_e$, and therefore $k_a = k_T$. We shall in general assume such ideal conversion of electrical to mechanical energy (or vice versa) and will therefore treat the two constants as being identical: $k_a = k_T$. The constant k_a is given by

$$k_a = \frac{pN}{2\pi M} \tag{17.10}$$

where

p = number of magnetic poles

N = number of conductors per coil

M = number of parallel paths in armature winding

An important observation concerning the units of angular speed must be made at this point. The equality (under the no-loss assumption) between the constants k_a and k_T in equations 17.6 and 17.8 results from the choice of consistent units, namely, volts and amperes for the electrical quantities, and newton-meters and radians per second for the mechanical quantities. You should be aware that it is fairly common practice to refer to the speed of rotation of an electric machine in units of revolutions per minute (rev/min).[1] In this book, we shall uniformly use the symbol n to denote angular speed in rev/min; the following relationship should be committed to memory:

$$n \text{ (rev/min)} = \frac{60}{2\pi}\omega_m \text{ (rad/s)} \tag{17.11}$$

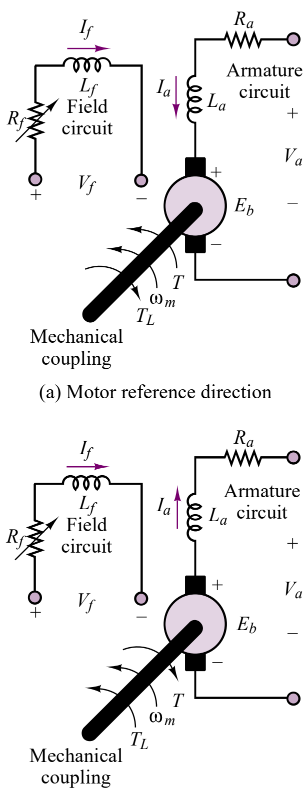
If the speed is expressed in rev/min, the armature constant changes as follows:

$$E_b = k_a'\phi n \tag{17.12}$$

where

$$k_a' = \frac{pN}{60M} \tag{17.13}$$

Having introduced the basic equations relating torque, speed, voltages, and currents in electric machines, we may now consider the interaction of these quantities in a DC machine at steady state, that is, operating at constant speed and field excitation. Figure 17.15 depicts the electrical circuit model of a separately excited DC machine, illustrating both motor and generator action. It is very important to note the reference direction of armature current flow, and of the developed torque, in order to make a distinction between the two modes of operation. The field excitation is shown as a voltage, V_f, generating the field current, I_f, that flows through a variable resistor, R_f, and through the field coil, L_f. The variable resistor permits adjustment of the field excitation. The armature circuit, on the other hand, consists of a voltage source representing the back emf, E_b, the armature resistance, R_a, and the armature voltage, V_a. This model is appropriate both for motor and for generator action. When $V_a < E_b$, the machine acts as a generator (I_a flows out of the machine). When $V_a > E_b$, the machine acts as a motor (I_a flows into the machine). Thus, according to the circuit model of Figure 17.15, the operation of a DC machine at steady state (i.e., with the inductors in the circuit replaced by short circuits) is described by the following equations:



(b) Generator reference direction

Figure 17.15 Electrical circuit model of a separately excited DC machine

$$-I_f + \frac{V_f}{R_f} = 0 \quad \text{and} \quad V_a - R_a I_a - E_b = 0 \quad \text{(motor action)}$$
$$-I_f + \frac{V_f}{R_f} = 0 \quad \text{and} \quad V_a + R_a I_a - E_b = 0 \quad \text{(generator action)} \tag{17.14}$$

Equation pair 17.14 together with equations 17.6 and 17.8 may be used to determine the steady-state operating condition of a DC machine.

The circuit model of Figure 17.15 permits the derivation of a simple set of differential equations that describe the *dynamic* analysis of a DC machine. The dynamic equations describing the behavior of a separately excited DC machine are as follows:

$$V_a(t) - I_a(t)R_a - L_a\frac{dI_a(t)}{dt} - E_b(t) = 0 \quad \text{(armature circuit)} \tag{17.15a}$$

$$V_f(t) - I_f(t)R_f - L_f\frac{dI_f(t)}{dt} = 0 \quad \text{(field circuit)} \tag{17.15b}$$

[1] Note that the abbreviation RPM, although certainly familiar to the reader, is not a standard unit, and its use should be discouraged.

These equations can be related to the operation of the machine in the presence of a load. If we assume that the motor is rigidly connected to an inertial load with moment of inertia J and that the friction losses in the load are represented by a viscous friction coefficient, b, then the torque developed by the machine (in the motor mode of operation) can be written as follows:

$$T(t) = T_L + b\omega_m(t) + J\frac{d\omega_m(t)}{dt} \quad \textbf{(17.16)}$$

where T_L is the load torque. T_L is typically either constant or some function of speed, ω_m, in a motor. In the case of a generator, the load torque is replaced by the torque supplied by a prime mover, and the machine torque, $T(t)$, opposes the motion of the prime mover, as shown in Figure 17.15. Since the machine torque is related to the armature and field currents by equation 17.6, equations 17.16 and 17.17 are coupled to each other; this coupling may be expressed as follows:

$$T(t) = k_a\phi I_a(t) \quad \textbf{(17.17)}$$

or

$$k_a\phi I_a(t) = T_L + b\omega_m(t) + J\frac{d\omega_m(t)}{dt} \quad \textbf{(17.18)}$$

The dynamic equations described in this section apply to any DC machine. In the case of a *separately excited* machine, a further simplification is possible, since the flux is established by virtue of a separate field excitation, and therefore

$$\phi = \frac{N_f}{\mathcal{R}}I_f = k_f I_f \quad \textbf{(17.19)}$$

where N_f is the number of turns in the field coil, $\mathcal{R}$ is the reluctance of the structure, and I_f is the field current.

17.3 DIRECT-CURRENT GENERATORS

To analyze the performance of a DC generator, it would be useful to obtain an open-circuit characteristic capable of predicting the voltage generated when the machine is driven at a constant speed ω_m by a prime mover. The common arrangement is to drive the machine at rated speed by means of a prime mover (or an electric motor). Then, with no load connected to the armature terminals, the armature voltage is recorded as the field current is increased from zero to some value sufficient to produce an armature voltage greater than the rated voltage. Since the load terminals are open-circuited, $I_a = 0$ and $E_b = V_a$; and since $k_a\phi = E_b/\omega_m$, the magnetization curve makes it possible to determine the value of $k_a\phi$ corresponding to a given field current, I_f, for the rated speed.

Figure 17.16 depicts a typical magnetization curve. Note that the armature voltage is nonzero even when no field current is present. This phenomenon is due to the *residual magnetization* of the iron core. The dashed lines in Figure 17.16 are called **field resistance curves** and are a plot of the voltage that appears across the field winding plus rheostat (variable resistor; see Figure 17.15) versus the field current, for various values of field winding plus rheostat resistance. Thus, the slope of the line is equal to the total field circuit resistance, R_f.

The operation of a DC generator may be readily understood with reference to the magnetization curve of Figure 17.16. As soon as the armature is connected

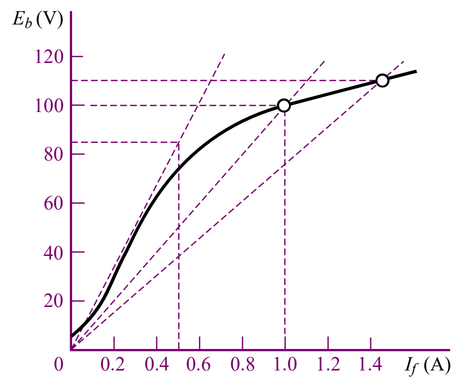


Figure 17.16 DC machine magnetization curve

across the shunt circuit consisting of the field winding and the rheostat, a current will flow through the winding, and this will in turn act to increase the emf across the armature. This **buildup** process continues until the two curves meet, that is, until the current flowing through the field winding is exactly that required to induce the emf. By changing the rheostat setting, the operating point at the intersection of the two curves can be displaced, as shown in Figure 17.16, and the generator can therefore be made to supply different voltages. The following examples illustrate the operation of the separately excited DC generator.

EXAMPLE 17.4 Separately Excited DC Generator

Problem

A separately excited DC generator is characterized by the magnetization curve of Figure 17.16.

1. If the prime mover is driving the generator at 800 rev/min, what is the no-load terminal voltage, V_a?
2. If a 1-Ω load is connected to the generator, what is the generated voltage?

Solution

Known Quantities: Generator magnetization curve and ratings.

Find: Terminal voltage with no load and 1-Ω load.

Schematics, Diagrams, Circuits, and Given Data:
Generator ratings: 100 V, 100 A, 1,000 rev/min.
Circuit parameters: $R_a = 0.14\ \Omega$; $V_f = 100$ V; $R_f = 100\ \Omega$.

Analysis:

1. The field current in the machine is

$$I_f = \frac{V_f}{R_f} = \frac{100\ \text{V}}{100\ \Omega} = 1\ \text{A}$$

From the magnetization curve, it can be seen that this field current will produce

100 V at a speed of 1,000 rev/min. Since this generator is actually running at 800 rev/min, the induced emf may be found by assuming a linear relationship between speed and emf. This approximation is reasonable, provided that the departure from the nominal operating condition is small. Let n_0 and E_{b0} be the nominal speed and emf, respectively (i.e., 1,000 rev/min and 100 V); then,

$$\frac{E_b}{E_{b0}} = \frac{n}{n_0}$$

and therefore

$$E_b = \frac{n}{n_0} E_{b0} = \frac{800 \text{ rev/min}}{1{,}000 \text{ rev/min}} \times 100 \text{ V} = 80 \text{ V}$$

The open-circuit (output) terminal voltage of the generator is equal to the emf from the circuit model of Figure 17.15; therefore:

$$V_a = E_b = 80 \text{ V}$$

2. When a load resistance is connected to the circuit (the practical situation), the terminal (or load) voltage is no longer equal to E_b, since there will be a voltage drop across the armature winding resistance. The armature (or load) current may be determined from the expression

$$I_a = I_L = \frac{E_b}{R_a + R_L} = \frac{80 \text{ V}}{(0.14 + 1)\Omega} = 70.2 \text{ A}$$

where $R_L = 1\ \Omega$ is the load resistance. The terminal (load) voltage is therefore given by

$$V_L = I_L R_L = 70.2 \times 1 = 70.2 \text{ V}$$

EXAMPLE 17.5 Separately Excited DC Generator

Problem

Determine the following quantities for a separately excited DC:

1. Induced voltage
2. Machine constant
3. Torque developed at rated conditions

Solution

Known Quantities: Generator ratings and machine parameters.

Find: E_b, k_a, T.

Schematics, Diagrams, Circuits, and Given Data:
Generator ratings: 1,000 kW; 2,000 V; 3,600 rev/min
Circuit parameters: $R_a = 0.1\ \Omega$; flux per pole $= \phi = 0.5$ Wb

Analysis:

1. The armature current may be found by observing that the rated power is equal to the product of the terminal (load) voltage and the armature (load) current; thus,

$$I_a = \frac{P_{\text{rated}}}{V_L} = \frac{1{,}000 \times 10^3}{2{,}000} = 500 \text{ A}$$

The generated voltage is equal to the sum of the terminal voltage and the voltage drop across the armature resistance (see Figure 16.14):

$$E_b = V_a + I_a R_a = 2{,}000 + 500 \times 0.1 = 2{,}050 \text{ V}$$

2. The speed of rotation of the machine in units of rad/s is

$$\omega_m = \frac{2\pi n}{60} = \frac{2\pi \times 3{,}600 \text{ rev/min}}{60 \text{ s/min}} = 377 \text{ rad/s}$$

Thus, the machine constant is found to be

$$k_a = \frac{E_b}{\phi\omega_m} = \frac{2{,}050 \text{ V}}{0.5 \text{ Wb} \times 377 \text{ rad/s}} = 10.876 \frac{\text{V-s}}{\text{Wb-rad}}$$

3. The torque developed is found from equation 16.6:

$$T = k_a\phi I_a = 10.875 \text{ V-s/Wb-rad} \times 0.5 \text{ Wb} \times 500 \text{ A} = 2{,}718.9 \text{ N-m}$$

Comments: In many practical cases, it is not actually necessary to know the armature constant and the flux separately, but it is sufficient to know the value of the product $k_a\phi$. For example, suppose that the armature resistance of a DC machine is known and that, given a known field excitation, the armature current, load voltage, and speed of the machine can be measured. Then, the product $k_a\phi$ may be determined from equation 16.20, as follows:

$$k_a\phi = \frac{E_b}{\omega_m} = \frac{V_L + I_a(R_a + R_S)}{\omega_m}$$

where V_L, I_a, and ω_m are measured quantities for given operating conditions.

Since the compound-connected generator contains both a shunt and a series field winding, it is the most general configuration, and the most useful for developing a circuit model that is as general as possible. In the following discussion, we shall consider the so-called short-shunt, compound-connected generator, in which the flux produced by the series winding adds to that of the shunt winding. Figure 17.17 depicts the equivalent circuit for the compound generator; circuit models for the shunt generator and for the rarely used series generator can be obtained by removing the shunt or series field winding element, respectively. In the circuit of Figure 17.17, the generator armature has been replaced by a voltage source corresponding to the induced emf and a series resistance, R_a, corresponding to the resistance of the armature windings. The equations describing the DC generator at steady state (i.e., with the inductors acting as short circuits) are:

DC Generator Steady-State Equations

$$E_b = k_a\phi\omega_m \text{ V} \quad \textbf{(17.20)}$$

$$T = \frac{P}{\omega_m} = \frac{E_b I_a}{\omega_m} = k_a\phi I_a \text{ N-m} \quad \textbf{(17.21)}$$

$$V_L = E_b - I_a R_a - I_S R_S \quad \textbf{(17.22)}$$

$$I_a = I_S + I_f \quad \textbf{(17.23)}$$

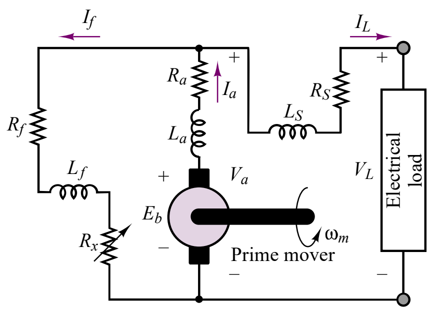


Figure 17.17 Compound generator circuit model

Note that in the circuit of Figure 17.17, the load and armature voltages are not equal, in general, because of the presence of a series field winding, represented by the resistor R_S and by the inductor L_S where the subscript "S" stands for "series." The expression for the armature emf is dependent on the air-gap flux, ϕ, to which the series and shunt windings in the compound generator both contribute, according to the expression

$$\phi = \phi_{\text{sh}} \pm \phi_S = \phi_{\text{sh}} \pm k_S I_a \tag{17.24}$$

Check Your Understanding

17.5 A 24-coil, 2-pole DC generator has 16 turns per coil in its armature winding. The field excitation is 0.05 Wb per pole, and the armature angular velocity is 180 rad/s. Find the machine constant and the total induced voltage.

17.6 A 1,000-kW, 1,000-V, 2,400-rev/min separately excited DC generator has an armature circuit resistance of 0.04 Ω. The flux per pole is 0.4 Wb. Find: (a) the induced voltage; (b) the machine constant; and (c) the torque developed at the rated conditions.

17.7 A 100-kW, 250-V shunt generator has a field circuit resistance of 50 Ω and an armature circuit resistance of 0.05 Ω. Find: (a) the full-load line current flowing to the load; (b) the field current; (c) the armature current; and (d) the full-load generator voltage.

17.4 DIRECT-CURRENT MOTORS

DC motors are widely used in applications requiring accurate speed control—for example, in servo systems. Having developed a circuit model and analysis methods for the DC generator, we can extend these results to DC motors, since these are in effect DC generators with the roles of input and output reversed. Once again, we shall analyze the motor by means of both its magnetization curve and a circuit model. It will be useful to begin our discussion by referring to the schematic diagram of a cumulatively compounded motor, as shown in Figure 17.18. The choice of the compound-connected motor is the most convenient, since its model can be used to represent either a series or a shunt motor with minor modifications.

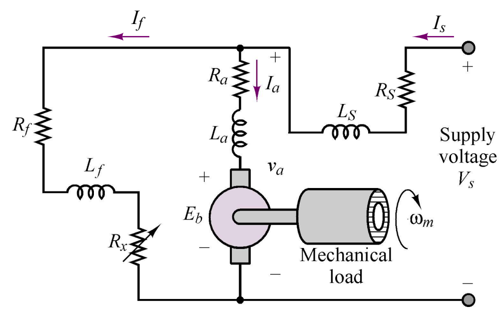


Figure 17.18 Equivalent circuit of a cumulatively compounded motor

The equations that govern the behavior of the DC motor follow and are similar to those used for the generator. Note that the only differences between these equations and those that describe the DC generator appear in the last two equations in the group, where the source voltage is equal to the *sum* of the emf and the voltage drop across the series field resistance and armature resistance, and where the source current now equals the *sum* of the field shunt and armature series currents.

DC Motor Steady-State Equations

$$E_b = k_a \phi \omega_m \tag{17.25}$$

$$T = k_a \phi I_a \tag{17.26}$$

$$V_s = E_b + I_a R_a + I_s R_S \tag{17.27}$$

$$I_s = I_f + I_a \tag{17.28}$$

Note that in these equations we have replaced the symbols V_L and I_L, used in the generator circuit model to represent the generator load current and voltage, with the symbols V_s and I_s, indicating the presence of an external source.

Speed-Torque and Dynamic Characteristics of DC Motors

The Shunt Motor

In a shunt motor (similar to the configuration of Figure 17.18, but with the series field short-circuited), the armature current is found by dividing the net voltage across the armature circuit (source voltage minus back emf) by the armature resistance:

$$I_a = \frac{V_s - k_a \phi \omega_m}{R_a} \tag{17.29}$$

An expression for the armature current may also be obtained from equation 16.26, as follows:

$$I_a = \frac{T}{k_a \phi} \tag{17.30}$$

It is then possible to relate the torque requirements to the speed of the motor by substituting equation 17.29 in equation 17.30:

$$\frac{T}{k_a \phi} = \frac{V_s - k_a \phi \omega_m}{R_a} \tag{17.31}$$

Equation 17.31 describes the steady-state torque-speed characteristic of the shunt motor. To understand this performance equation, we observe that if V_s, k_a, ϕ, and R_a are fixed in equation 17.31 (the flux is essentially constant in the shunt motor for a fixed V_s), then the speed of the motor is directly related to the armature current. Now consider the case where the load applied to the motor is suddenly increased, causing the speed of the motor to drop. As the speed decreases, the armature current increases, according to equation 17.29. The excess armature current causes the motor to develop additional torque, according to equation 17.30, until a new equilibrium is reached between the higher armature current and developed torque and the lower speed of rotation. The equilibrium point is dictated by the balance of mechanical and electrical power, in accordance with the relation

$$E_b I_a = T \omega_m \tag{17.32}$$

Thus, the shunt DC motor will adjust to variations in load by changing its speed to preserve this power balance. The torque-speed curves for the shunt motor may be obtained by rewriting the equation relating the speed to the armature current:

$$\omega_m = \frac{V_s - I_a R_a}{k_a \phi} = \frac{V_s}{k_a \phi} - \frac{R_a T}{(k_a \phi)^2} \tag{17.33}$$

To interpret equation 17.33, one can start by considering the motor operating at rated speed and torque. As the load torque is reduced, the armature current will also decrease, causing the speed to increase in accordance with equation 17.33. The increase in speed depends on the extent of the voltage drop across the armature resistance, $I_a R_a$. The change in speed will be on the same order of magnitude as this drop; it typically takes values around 10 percent. This corresponds to a relatively good speed regulation, which is an attractive feature of the shunt DC motor (recall the discussion of regulation in Section 17.1). Normalized torque and speed vs. power curves for the shunt motor are shown in Figure 17.19. Note that, over a reasonably broad range of powers, up to rated value, the curve is relatively flat, indicating that the DC shunt motor acts as a reasonably constant-speed motor.

The dynamic behavior of the shunt motor is described by equations 17.15 through 17.18, with the additional relation

$$I_a(t) = I_s(t) - I_f(t) \tag{17.34}$$

Compound Motors

It is interesting to compare the performance of the shunt motor with that of the compound-connected motor; the comparison is easily made if we recall that a series field resistance appears in series with the armature resistance and that the

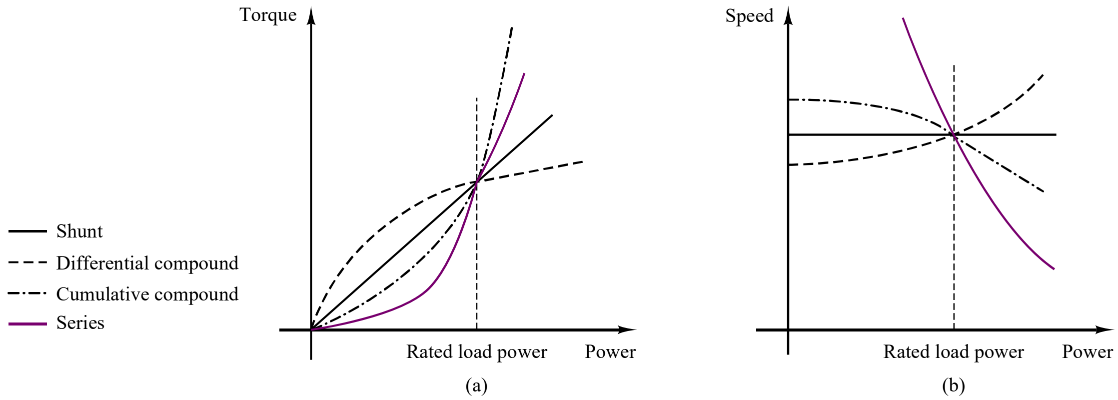


Figure 17.19 DC motor operating characteristics

flux is due to the contributions of both series and shunt fields. Thus, the speed equation becomes

$$\omega_m = \frac{V_s - I_a(R_a + R_S)}{k_a(\phi_{\mathrm{sh}} \pm \phi_S)} \tag{17.35}$$

where

$+$ in the denominator is for a cumulatively compounded motor.

$-$ in the denominator is for a differentially compounded motor.

ϕ_{sh} is the flux set up by the shunt field winding, assuming that it is constant.

ϕ_S is the flux set up by the series field winding, $\phi_S = k_S I_a$.

For the cumulatively compound motor, two effects are apparent: the flux is increased by the presence of a series component, ϕ_S; and the voltage drop due to I_a in the numerator term is increased by an amount proportional to the resistance of the series field winding, R_S. As a consequence, when the load to the motor is reduced, the numerator increases more dramatically than in the case of the shunt motor, because of the corresponding decrease in armature current, while at the same time the series flux decreases. Each of these effects causes the speed to increase; therefore, it stands to reason that the speed regulation of the compound-connected motor is poorer than that of the shunt motor. Normalized torque and speed vs. power curves for the compound motor (both differential and cumulative connections) are shown in Figure 17.19.

The differential equation describing the behavior of a compound motor differs from that for the shunt motor in having additional terms due to the series field component:

$$\begin{aligned} V_s &= E_b(t) + I_a(t)R_a + L_a\frac{dI_a(t)}{dt} + I_s(t)R_S + L_S\frac{dI_s(t)}{dt} \\ &= V_a(t) + I_s(t)R_S + L_S\frac{dI_s(t)}{dt} \end{aligned} \tag{17.36}$$

The differential equation for the field circuit can be written as

$$V_a = I_f(t)(R_f + R_x) + L_f\frac{dI_f(t)}{dt} \tag{17.37}$$

We also have the following basic relations:

$$I_a(t) = I_s(t) - I_f(t) \tag{17.38}$$

and

$$E_b(t) = k_a I_a(t)\omega_m(t) \qquad \text{and} \qquad T(t) = k_a \phi I_a(t) \tag{17.39}$$

Series Motors

The series motor [see Figure 17.14(c)] behaves somewhat differently from the shunt and compound motors because the flux is established solely by virtue of the series current flowing through the armature. It is relatively simple to derive an expression for the emf and torque equations for the series motor if we approximate the relationship between flux and armature current by assuming that the motor operates in the linear region of its magnetization curve. Then we can write

$$\phi = k_S I_a \tag{17.40}$$

and the emf and torque equations become

$$E_b = k_a \omega_m \phi = k_a \omega_m k_S I_a \tag{17.41}$$

$$T = k_a \phi I_a = k_a k_S I_a^2 \tag{17.42}$$

The circuit equation for the series motor becomes

$$V_s = E_b + I_a(R_a + R_S) = (k_a \omega_m k_S + R_T) I_a \tag{17.43}$$

where R_a is the armature resistance, R_S is the series field winding resistance, and R_T is the total series resistance. From equation 17.43, we can solve for I_a and substitute in the torque expression (equation 17.42) to obtain the following torque-speed relationship:

$$T = k_a k_S \frac{V_s^2}{(k_a \omega_m k_S + R_T)^2} \tag{17.44}$$

which indicates the inverse squared relationship between torque and speed in the series motor. This expression describes a behavior that can, under certain conditions, become unstable. Since the speed increases when the load torque is reduced, one can readily see that if one were to disconnect the load altogether, the speed would tend to increase to dangerous values. To prevent excessive speeds, series motors are always mechanically coupled to the load. This feature is not necessarily a drawback, though, because series motors can develop very high torque at low speeds, and therefore can serve very well for traction-type loads (e.g., conveyor belts or vehicle propulsion systems). Torque and speed vs. power curves for the series motor are also shown in Figure 17.19.

The differential equation for the armature circuit of the motor can be given as

$$\begin{aligned} V_s &= I_a(t)(R_a + R_S) + L_a \frac{dI_a(t)}{dt} + L_S \frac{dI_a(t)}{dt} + E_b \\ &= I_a(t)(R_a + R_S) + L_a \frac{dI_a(t)}{dt} + L_S \frac{dI_a(t)}{dt} + k_a k_S I_a \omega_m \end{aligned} \tag{17.45}$$

Permanent-Magnet DC Motors

Permanent-magnet (PM) DC motors have become increasingly common in applications requiring relatively low torques and efficient use of space. The construction of PM direct-current motors differs from that of the motors considered thus far in that the magnetic field of the stator is produced by suitably located poles made of magnetic materials. Thus, the basic principle of operation, including the idea of commutation, is unchanged with respect to the wound-stator DC motor. What changes is that there is no need to provide a field excitation, whether separately or by means of the self-excitation techniques discussed in the preceding sections. Therefore, the PM motor is intrinsically simpler than its wound-stator counterpart.

The equations that describe the operation of the PM motor follow. The torque produced is related to the armature current by a torque constant, k_{PM}, which is determined by the geometry of the motor:

$$T = k_{TPM} I_a \tag{17.46}$$

As in the conventional DC motor, the rotation of the rotor produces the usual counter or back emf, E_b, which is linearly related to speed by a voltage constant, k_{aPM}:

$$E_b = k_{aPM}\omega_m \tag{17.47}$$

The equivalent circuit of the PM motor is particularly simple, since we need not model the effects of a field winding. Figure 17.20 shows the circuit model and the torque-speed curve of a PM motor.

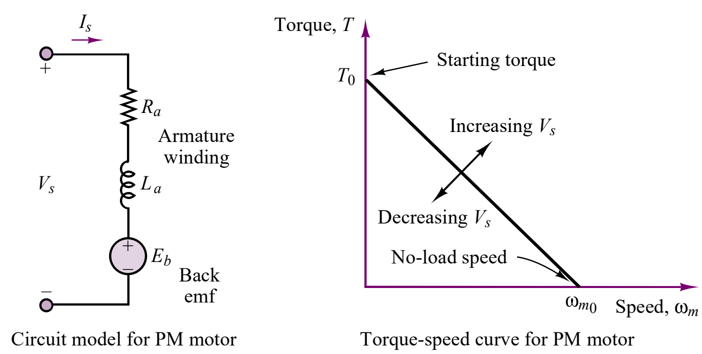


Figure 17.20 Circuit model and torque-speed curve of PM motor

We can use the circuit model of Figure 17.20 to predict the torque-speed curve shown in the same figure, as follows. From the circuit model, for a constant speed (and therefore constant current), we may consider the inductor a short circuit and write the equation

$$\begin{aligned} V_s &= I_a R_a + E_b = I_a R_a + k_{aPM}\omega_m \\ &= \frac{T}{k_{TPM}} R_a + k_{aPM}\omega_m \end{aligned} \tag{17.48}$$

thus obtaining the equations relating speed and torque:

$$\omega_m = \frac{V_s}{k_{aPM}} - \frac{T R_a}{k_{aPM} k_{TPM}} \tag{17.49}$$

and

$$T = \frac{V_s}{R_a} k_{TPM} - \frac{\omega_m}{R_a} k_{aPM} k_{TPM} \tag{17.50}$$

From these equations, one can extract the stall torque, T_0, that is, the zero-speed torque:

$$T_0 = \frac{V_s}{R_a} k_{TPM} \tag{17.51}$$

and the no-load speed, ω_{m0}:

$$\omega_{m0} = \frac{V_s}{k_{aPM}} \tag{17.52}$$

Under dynamic conditions, assuming an inertia plus viscous friction load, the torque produced by the motor can be expressed as

$$T = k_{TPM} I_a(t) = T_{\text{load}}(t) + d\omega_m(t) + J \frac{d\omega_m(t)}{dt} \tag{17.53}$$

The differential equation for the armature circuit of the motor is therefore given by:

$$\begin{aligned} V_s &= I_a(t) R_a + L_a \frac{dI_a(t)}{dt} + E_b \\ &= I_a(t) R_a + L_a \frac{dI_a(t)}{dt} + k_{aPM} \omega_m(t) \end{aligned} \tag{17.54}$$

The fact that the air-gap flux is constant in a permanent-magnet DC motor makes its characteristics somewhat different from those of the wound DC motor. A direct comparison of PM and wound-field DC motors reveals the following advantages and disadvantages of each configuration.

Comparison of Wound-Field and PM DC Motors

1. PM motors are smaller and lighter than wound motors for a given power rating. Further, their efficiency is greater because there are no field winding losses.
2. An additional advantage of PM motors is their essentially linear speed-torque characteristic, which makes analysis (and control) much easier. Reversal of rotation is also accomplished easily, by reversing the polarity of the source.
3. A major disadvantage of PM motors is that they can become demagnetized by exposure to excessive magnetic fields, application of excessive voltage, or operation at excessively high or low temperatures.
4. A less obvious drawback of PM motors is that their performance is subject to greater variability from motor to motor than is the case for wound motors, because of variations in the magnetic materials.

In summary, four basic types of **DC motors** are commonly used. Their principal operating characteristics are summarized as follows, and their general torque and speed versus power characteristics are depicted in Figure 17.19, assuming motors with identical voltage, power, and speed ratings.

FIND IT ON THE WEB

Shunt wound motor: Field connected in parallel with the armature. With constant armature voltage and field excitation, the motor has good speed regulation (flat speed-torque characteristic).

Compound wound motor: Field winding has both series and shunt components. This motor offers better starting torque than the shunt motor, but worse speed regulation.

Series wound motor: The field winding is in series with the armature. The motor has very high starting torque and poor speed regulation. It is useful for low-speed, high-torque applications.

Permanent-magnet motor: Field windings are replaced by permanent magnets. The motor has adequate starting torque, with speed regulation somewhat worse than that of the compound wound motor.

EXAMPLE 17.6 DC Shunt Motor Analysis

Problem

Find the speed and torque generated by a four-pole DC shunt motor.

Solution

Known Quantities: Motor ratings; circuit and magnetic parameters.

Find: ω_m, T.

Schematics, Diagrams, Circuits, and Given Data:
Motor ratings: 3 hp; 240 V; 120 rev/min.
Circuit and magnetic parameters: $I_S = 30$ A; $I_f = 1.4$ A; $R_a = 0.6\ \Omega$; $\phi = 20$ mWb; $N = 1{,}000$; $M = 4$ (see equation 17.10).

Analysis: We convert the power to SI units:

$$P_{\text{RATED}} = 3 \text{ hp} \times 746 \frac{\text{W}}{\text{hp}} = 2.238 \text{ W}$$

Next, we compute the armature current as the difference between source and field current (equation 17.34):

$$I_a = I_s - I_f = 30 - 1.4 = 28.6 \text{ A}$$

The no-load armature voltage, E_b, is given by:

$$E_b = V_s - I_a R_a = 240 - 28.6 \times 0.6 = 222.84 \text{ V}$$

and equation 17.10 can be used to determine the armature constant:

$$k_a = \frac{pN}{2\pi M} = \frac{4 \times 1000}{2\pi \times 4} = 159.15 \frac{\text{V-s}}{\text{Wb-rad}}$$

Knowing the motor constant, we can calculate the speed, after equation 17.25:

$$\omega_m = \frac{E_a}{k_a \phi} = \frac{222.84 \text{ V}}{159.15 \frac{\text{V-s}}{\text{Wb-rad}} \times 0.002 \text{ Wb}} = 70 \frac{\text{rad}}{\text{s}}$$

Finally, the torque developed by the motor can be found as the ratio of the power to the angular velocity:

$$T = \frac{P}{\omega_m} = \frac{2{,}238 \text{ W}}{70\dfrac{\text{rad}}{\text{s}}} = 32 \text{ N-m}$$

EXAMPLE 17.7 DC Shunt Motor Analysis

Problem

Determine the following quantities for the DC shunt motor, connected as shown in the circuit Figure 17.21:

1. Field current required for full-load operation
2. No-load speed
3. Plot the speed torque curve of the machine in the range from no-load torque to rated torque
4. Power output at rated torque.

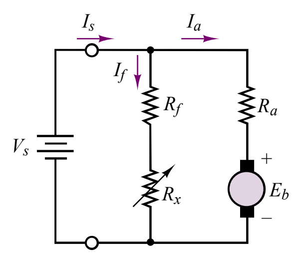


Figure 17.21 Shunt motor configuration

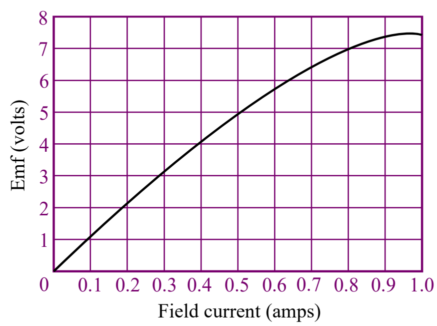


Figure 17.22 Magnetization curve for a small DC motor

Solution

Known Quantities: Magnetization curve, rated current, rated speed, circuit parameters.

Find: I_f; $n_{\text{no-load}}$; T-n curve, P_{rated}.

Schematics, Diagrams, Circuits, and Given Data: Figure 17.22 (magnetization curve)
Motor ratings: 8 A, 120 rev/min
Circuit parameters: $R_a = 0.2\ \Omega$; $V_s = 7.2$ V; $N =$ number of coil turns in winding $= 200$

Analysis:

1. To find the field current, we must find the generated emf since R_f is not known. Writing KVL around the armature circuit, we obtain

$$V_s = E_b + I_a R_a$$

$$E_b = V_s - I_a R_a = 7.2 - 8(0.2) = 5.6 \text{ V}$$

Having found the back emf, we can find the field current from the magnetization curve. At $E_b = 5.6$ V, we find that the field current and field resistance are

$$I_f = 0.6 \text{ A} \qquad \text{and} \qquad R_f = \frac{7.2}{0.6} = 12\ \Omega$$

2. To obtain the no-load speed, we use the equations

$$E_b = k_a \phi \frac{2\pi n}{60} \qquad T = k_a \phi I_a$$

leading to

$$V_s = I_a R_a + E_b = I_a R_a + k_a \phi \frac{2\pi}{60} n$$

or

$$n = \frac{V_s - I_a R_a}{k_a \phi (2\pi/60)}$$

At no load, and assuming no mechanical losses, the torque is zero, and we see that the current I_a must also be zero in the torque equation ($T = k_a \phi I_a$). Thus, the motor speed at no load is given by

$$n_{\text{no-load}} = \frac{V_s}{k_a \phi (2\pi/60)}$$

We can obtain an expression for $k_a \phi$ knowing that, at full load,

$$E_b = 5.6 \text{ V} = k_a \phi \frac{2\pi n}{60}$$

so that, for constant field excitation,

$$k_a \phi = E_b \left(\frac{60}{2\pi n} \right) = 5.6 \left(\frac{60}{2\pi (120)} \right) = 0.44563 \frac{\text{V} \cdot \text{s}}{\text{rad}}$$

Finally, we may solve for the no-load speed, in rev/min:

$$n_{\text{no-load}} = \frac{V_s}{k_a \phi (2\pi/60)} = \frac{7.2}{(0.44563)(2\pi/60)} = 154.3 \text{ rev/min}$$

3. The torque at rated speed and load may be found as follows:

$$T_{\text{rated load}} = k_a \phi I_a = (0.44563)(8) = 3.565 \text{ N-m}$$

Now we have the two points necessary to construct the torque-speed curve for this motor, which is shown in Figure 17.23.

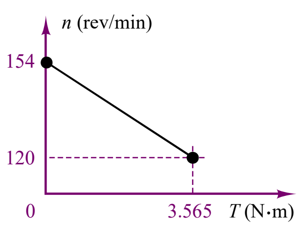


Figure 17.23

4. The power is related to the torque by the frequency of the shaft:

$$P_{\text{rated}} = T \omega_m = (3.565) \left(\frac{120}{60} \right) 2\pi = 44.8 \text{ W}$$

or, equivalently,

$$P = 44.8 \text{ W} \times \frac{1}{746} \frac{\text{hp}}{\text{W}} = 0.06 \text{ hp}$$

EXAMPLE 17.8 DC Series Motor Analysis

Problem

Determine the torque developed by a DC series motor when the current supplied to the motor is 60 A.

Solution

Known Quantities: Motor ratings; operating conditions.

Find: T_{60}, torque delivered at 60-A series current.

Schematics, Diagrams, Circuits, and Given Data: Motor ratings: 10 hp; 115 V; full load speed = 1,800 rev/min
Operating conditions: motor draws 40 A

Assumptions: The motor operates in the linear region of the magnetization curve.

Analysis: Within the linear region of operation, the flux per pole is directly proportional to the current in the field winding. That is,

$$\phi = k_S I_a$$

The full-load speed is

$$n = 1{,}800 \text{ rev/min}$$

or

$$\omega_m = \frac{2\pi n}{60} = 60\pi \text{ rad/s}$$

Rated output power is

$$P_{\text{rated}} = 10 \text{ hp} \times 746 \text{ W/hp} = 7{,}460 \text{ W}$$

and full-load torque is

$$T_{40\text{A}} = \frac{P_{\text{rated}}}{\omega_m} = \frac{7{,}460}{60\pi} = 39.58 \text{ N-m}$$

Thus, the machine constant may be computed from the torque equation for the series motor:

$$T = k_a k_S I_a^2 = K I_a^2$$

At full load,

$$K = k_a k_S = \frac{39.58 \text{ N-m}}{40^2 \text{ A}^2} = 0.0247 \frac{\text{N-m}}{\text{A}^2}$$

and we can compute the torque developed for a 60-A supply current to be

$$T_{60\text{A}} = K I_a^2 = 0.0247 \times 60^2 = 88.92 \text{ N-m}$$

EXAMPLE 17.9 Dynamic Response of PM DC Motor

Problem

Develop a set of differential equations and a transfer function describing the dynamic response of the motor angular velocity of a PM DC motor connected to a mechanical load.

Solution

Known Quantities: PM DC motor circuit model; mechanical load model.

Find: Differential equations and transfer functions of electromechanical system.

Analysis: The dynamic response of the electromechanical system can be determined by applying KVL to the electrical circuit (Figure 17.20), and Newton's second law to the mechanical system. These equations will be coupled to one another, as you shall see, because of the nature of the motor back emf and torque equations.

Applying KVL and equation 17.47 to the electrical circuit we obtain:

$$V_L(t) - R_a I_a(t) - L_a \frac{dI_a(t)}{dt} - E_b(t) = 0$$

or

$$L_a \frac{dI_a(t)}{dt} + R_a I_a(t) + K_{a\text{PM}}\omega_m(t) = V_L(t)$$

Applying Newton's second law and equation 17.46 to the load inertia, we obtain:

$$J\frac{d\omega(t)}{dt} = T(t) - T_{\text{load}}(t) - b\omega$$

or

$$-K_{T\,\text{PM}} I_a(t) + J\frac{d\omega(t)}{dt} + b\omega(t) = T_{\text{load}}(t)$$

These two differential equations are coupled because the first depends on ω_m and the second on I_a. Thus, they need to be solved simultaneously.

To derive the transfer function, we Laplace-transform the two equations to obtain:

$$(sL_a + R_a)I_a(s) + K_{a\text{PM}}\Omega(s) = V_L(s)$$
$$-K_{T\,\text{PM}} I_a(s) + (sJ + b)\Omega(s) = T_{\text{load}}(s)$$

We can write the above equations in matrix form and resort to Cramer's rule to solve for $\Omega_m(s)$ as a function of $V_L(s)$ and $T_{\text{load}}(s)$.

$$\begin{bmatrix} (sL_a + R_a) & K_{a\text{PM}} \\ -K_{T\,\text{PM}} & (sL + b) \end{bmatrix} \begin{bmatrix} I_a(s) \\ \Omega_m(s) \end{bmatrix} = \begin{bmatrix} V_L(s) \\ T_{\text{load}}(s) \end{bmatrix}$$

with solution

$$\Omega_m(s) = \frac{\det\begin{bmatrix} (sL_a + R_a) & V_L(s) \\ K_{T\,\text{PM}} & T_{\text{load}}(s) \end{bmatrix}}{\det\begin{bmatrix} (sL_a + R_a) & K_{a\,\text{PM}} \\ -K_{T\,\text{PM}} & (sJ + b) \end{bmatrix}}$$

or

$$\Omega_m(s) = \frac{(sL_a + R_a)}{(sL_a + R_a)(sJ + b) + K_{a\,\text{PM}}K_{T\,\text{PM}}} T_{\text{load}}(s) + \frac{K_{T\,\text{PM}}}{(sL_a + R_a)(sJ + b) + K_{a\,\text{PM}}K_{T\,\text{PM}}} V_L(s)$$

Comments: Note that the dynamic response of the motor angular velocity depends on both the input voltage and on the load torque. This problem is explored further in the homework problems.

DC Drives and DC Motor Speed Control

FIND IT ON THE WEB

The advances made in power semiconductors have made it possible to realize low-cost **speed control systems for DC motors.** The basic operation of *controlled rectifier* and *chopper* drives for DC motors was described in Chapter 11. In the present section we describe some of the considerations that are behind the choice of a specific drive type, and of some of the loads that are likely to be encountered.

Constant-torque loads are quite common, and are characterized by a need for constant torque over the entire speed range. This need is usually due to friction; the load will demand increasing horsepower at higher speeds, since power is the product of speed and torque. Thus, the power required will increase linearly with speed. This type of loading is characteristic of conveyors, extruders, and surface winders.

Another type of load is one that requires *constant horsepower* over the speed range of the motor. Since torque is inversely proportional to speed with constant horsepower, this type of load will require higher torque at low speeds. Examples of constant-horsepower loads are machine tool spindles (e.g., lathes). This type of application requires very high starting torques.

Variable-torque loads are also common. In this case, the load torque is related to the speed in some fashion, either linearly or geometrically. For some loads, for example, torque is proportional to the speed (and thus horsepower is proportional to speed squared); examples of loads of this type are positive displacement pumps. More common than the linear relationship is the squared-speed dependence of inertial loads such as centrifugal pumps, some fans, and all loads in which a flywheel is used for energy storage.

To select the appropriate motor and adjustable speed drive for a given application, we need to examine how each method for speed adjustment operates on a DC motor. Armature voltage control serves to smoothly adjust speed from 0 to 100 percent of the nameplate rated value (i.e., base speed), provided that the field excitation is also equal to the rated value. Within this range, it is possible to fully control motor speed for a constant-torque load, thus providing a linear increase in horsepower, as shown in Figure 17.24. Field weakening allows for increases in speed of up to several times the base speed; however, field control changes the characteristics of the DC motor from constant torque to constant horsepower, and therefore the torque output drops with speed, as shown in Figure 17.24. Operation above base speed requires special provision for field control, in addition to the circuitry required for armature voltage control, and is therefore more complex and costly.

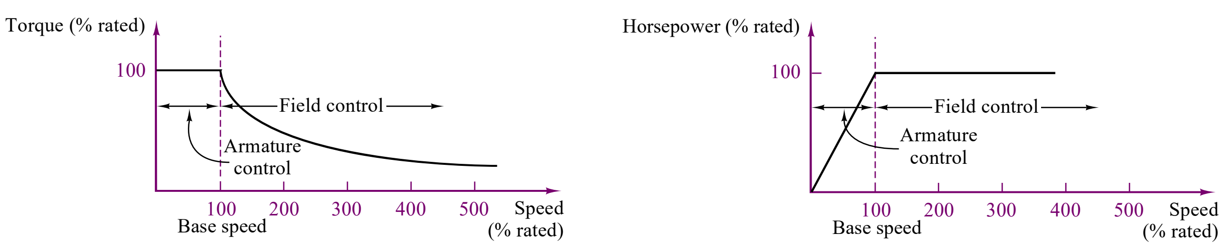


Figure 17.24 Speed control in DC motors

Check Your Understanding

17.8 A series motor draws a current of 25 A and develops a torque of 100 N-m Find: (a) the torque when the current rises to 30 A if the field is unsaturated; and (b) the torque when the current rises to 30 A and the increase in current produces a 10 percent increase in flux.

17.9 A 200-V DC shunt motor draws 10 A at 1,800 rev/min. The armature circuit resistance is 0.15 Ω and the field winding resistance is 350 Ω. What is the torque developed by the motor?

17.10 Describe the cause-and-effect behavior of the speed control method of changing armature voltage for a shunt DC motor.

17.5 AC MACHINES

From the previous sections, it should be apparent that it is possible to obtain a wide range of performance characteristics from DC machines, as both motors and generators. A logical question at this point should be, Would it not be more convenient in some cases to take advantage of the single- or multiphase AC power that is available virtually everywhere than to expend energy and use additional hardware to rectify and regulate the DC supplies required by direct-current motors? The answer to this very obvious question is certainly a resounding yes. In fact, the AC induction motor is the workhorse of many industrial applications, and synchronous generators are used almost exclusively for the generation of electric power worldwide. Thus, it is appropriate to devote a significant portion of this chapter to the study of AC machines, and of induction motors in particular. The objective of this section is to explain the basic operation of both synchronous and induction machines, and to outline their performance characteristics. In doing so, we shall also point out the relative advantages and disadvantages of these machines in comparison with direct-current machines. The motor "movies" included in the CD-ROM may help you visualize the operation of AC machines.

Rotating Magnetic Fields

As mentioned in Section 17.1, the fundamental principle of operation of AC machines is the generation of a rotating magnetic field, which causes the rotor to turn at a speed that depends on the speed of rotation of the magnetic field. We shall now explain how a rotating magnetic field can be generated in the stator and air gap of an AC machine by means of AC currents.

Consider the stator shown in Figure 17.25, which supports windings a-a', b-b' and c-c'. The coils are geometrically spaced 120° apart, and a three-phase voltage is applied to the coils. As you may recall from the discussion of AC power in Chapter 7, the currents generated by a three-phase source are also spaced by 120°, as illustrated in Figure 17.26. The phase voltages referenced the neutral

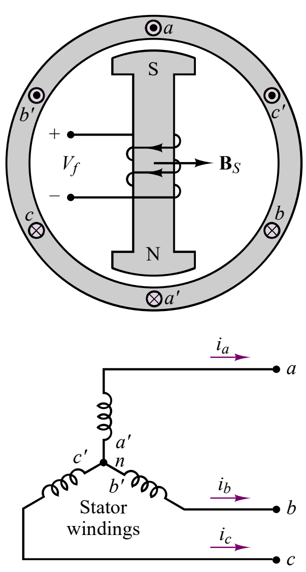


Figure 17.25 Two-pole three-phase stator

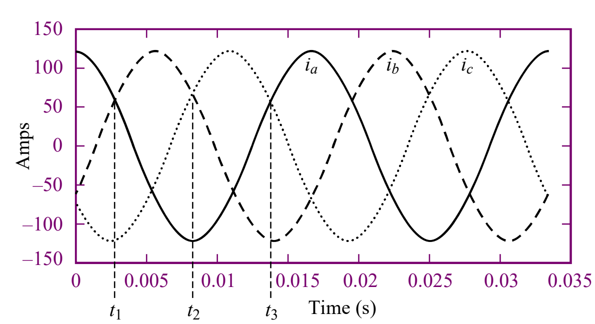


Figure 17.26 Three-phase stator winding currents

terminal, would then be given by the expressions

$$v_a = A\cos(\omega_e t)$$
$$v_b = A\cos\left(\omega_e t - \frac{2\pi}{3}\right)$$
$$v_c = A\cos\left(\omega_e t + \frac{2\pi}{3}\right)$$

where ω_e is the frequency of the AC supply, or line frequency. The coils in each winding are arranged in such a way that the flux distribution generated by any one winding is approximately sinusoidal. Such a flux distribution may be obtained by appropriately arranging groups of coils for each winding over the stator surface. Since the coils are spaced 120° apart, the flux distribution resulting from the sum of the contributions of the three windings is the sum of the fluxes due to the separate windings, as shown in Figure 17.27. Thus, the flux in a three-phase machine rotates in space according to the vector diagram of Figure 17.28, and is constant in amplitude. A stationary observer on the machine's stator would see a sinusoidally varying flux distribution as shown in Figure 17.27.

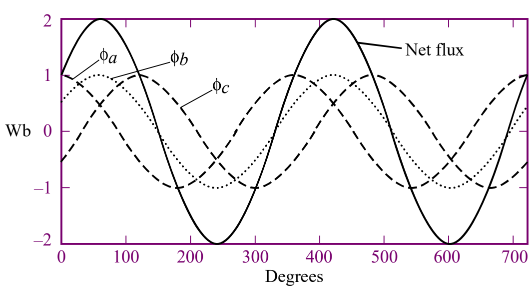


Figure 17.27 Flux distribution in a three-phase stator winding as a function of angle of rotation

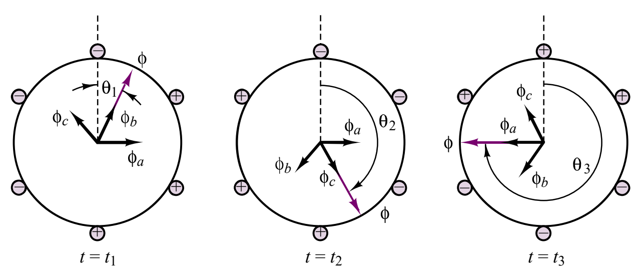


Figure 17.28 Rotating flux in a three-phase machine

Since the resultant flux of Figure 17.27 is generated by the currents of Figure 17.26, the speed of rotation of the flux must be related to the frequency of the sinusoidal phase currents. In the case of the stator of Figure 17.25, the number of magnetic poles resulting from the winding configuration is two; however, it is also possible to configure the windings so that they have more poles. For example, Figure 17.29 depicts a simplified view of a four-pole stator.

In general, the speed of the rotating magnetic field is determined by the frequency of the excitation current, f, and by the number of poles present in the stator, p, according to the equation

$$n_s = \frac{120 f}{p}\ \text{rev/min}$$

or **(17.55)**

$$\omega_s = \frac{2\pi n_S}{60} = \frac{2\pi \times 2f}{p}$$

where n_s (or ω_s) is usually called the **synchronous speed.**

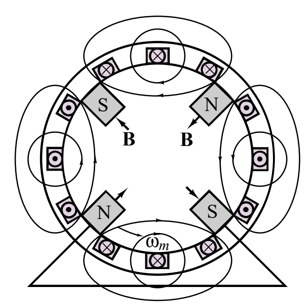


Figure 17.29 Four-pole stator

Now, the structure of the windings in the preceding discussion is the same whether the AC machine is a motor or a generator; the distinction between the two depends on the direction of power flow. In a generator, the electromagnetic torque is a reaction torque that opposes rotation of the machine; this is the torque against which the prime mover does work. In a motor, on the other hand, the rotational (motional) voltage generated in the armature opposes the applied voltage; this voltage is the counter (or back) emf. Thus, the description of the rotating magnetic field given thus far applies to both motor and generator action in AC machines.

As described a few paragraphs earlier, the stator magnetic field rotates in an AC machine, and therefore the rotor cannot "catch up" with the stator field and is in constant pursuit of it. The speed of rotation of the rotor will therefore depend on the number of magnetic poles present in the stator and in the rotor. The magnitude of the torque produced in the machine is a function of the angle γ between the stator and rotor magnetic fields; precise expressions for this torque depend on how the magnetic fields are generated and will be given separately for the two cases of synchronous and induction machines. What is common to all rotating machines is that the number of stator and rotor poles must be identical if any torque is to be generated. Further, the number of poles must be even, since for each north pole there must be a corresponding south pole.

One important desired feature in an electric machine is an ability to generate a constant electromagnetic torque. With a constant-torque machine, one can avoid torque pulsations that could lead to undesired mechanical vibration in the motor itself and in other mechanical components attached to the motor (e.g., mechanical loads, such as spindles or belt drives). A constant torque may not always be achieved, although it will be shown that it is possible to accomplish this goal when the excitation currents are multiphase. A general rule of thumb, in this respect, is that it is desirable, insofar as possible, to produce a constant flux per pole.

17.6 THE ALTERNATOR (SYNCHRONOUS GENERATOR)

One of the most common AC machines is the **synchronous generator,** or **alternator.** In this machine, the field winding is on the rotor, and the connection is made by means of brushes, in an arrangement similar to that of the DC machines studied earlier. The rotor field is obtained by means of a DC current provided to the rotor winding, or by permanent magnets. The rotor is then connected to a mechanical source of power and rotates at a speed that we will consider constant to simplify the analysis.

Figure 17.30 depicts a two-pole three-phase synchronous machine. Figure 17.31 depicts a four-pole three-phase alternator, in which the rotor poles are generated by means of a wound salient pole configuration and the stator poles are the result of windings embedded in the stator according to the simplified arrangement shown in the figure, where each of the pairs a/a', b/b', and so on, contributes to the generation of the magnetic poles, as follows. The group a/a', b/b', c/c' produces a sinusoidally distributed flux (see Figure 17.27) corresponding to one of the pole pairs, while the group $-a/-a'$, $-b/-b'$, $-c/-c'$ contributes the other pole pair. The connections of the coils making up the windings are also shown in Figure 17.31. Note that the coils form a wye connection (see Chapter 7). The resulting flux distribution is such that the flux completes two sinusoidal cycles around the circumference of the air gap. Note also that each arm of the

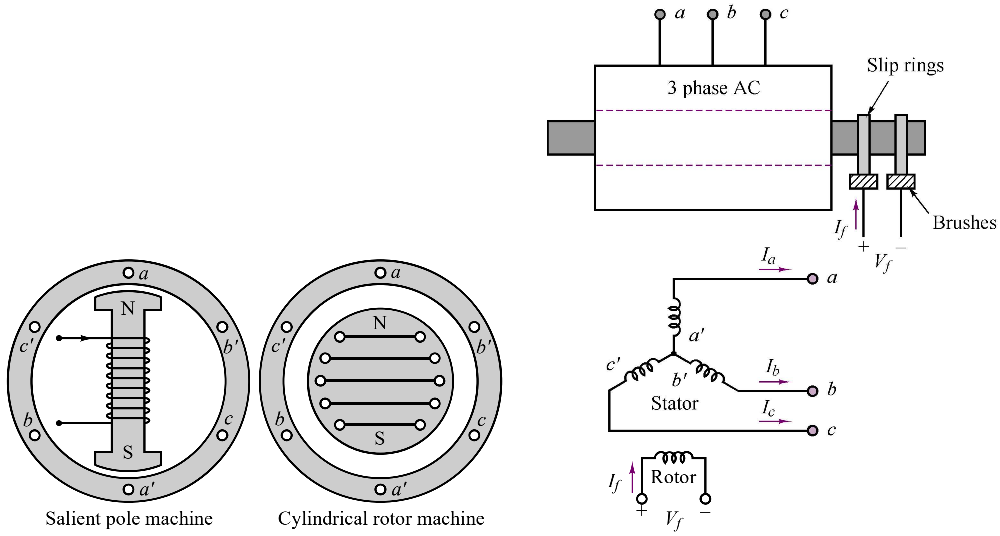


Figure 17.30 Two-pole synchronous machine

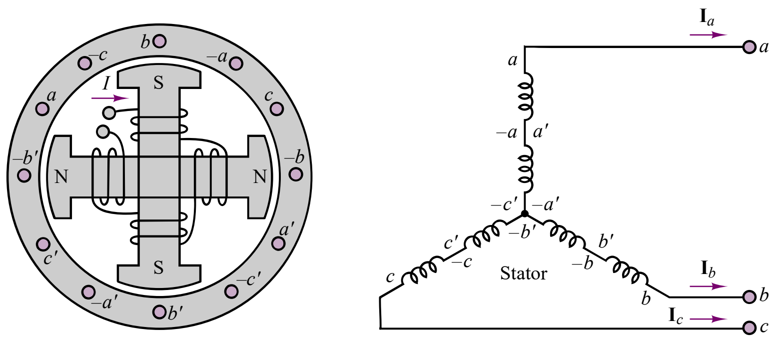


Figure 17.31 Four-pole three-phase alternator

three-phase wye connection has been divided into two coils, wound in different locations, according to the schematic stator diagram of Figure 17.31. One could then envision analogous configurations with greater numbers of poles, obtained in the same fashion, that is, by dividing each arm of a wye connection into more windings.

The arrangement shown in Figure 17.31 requires that a further distinction be made between mechanical degrees, θ_m, and electrical degrees, θ_e. In the four-pole alternator, the flux will see two complete cycles during one rotation of the rotor, and therefore the voltage that is generated in the coils will also oscillate at twice the frequency of rotation. In general, the electrical degrees (or radians) are related to the mechanical degrees by the expression

$$\theta_e = \frac{p}{2}\theta_m \tag{17.56}$$

where p is the number of poles. In effect, the voltage across a coil of the machine goes through one cycle every time a pair of poles moves past the coil. Thus, the frequency of the voltage generated by a synchronous generator is

$$f = \frac{p}{2}\frac{n}{60} \text{ Hz} \qquad \textbf{(17.57)}$$

where n is the mechanical speed in rev/min. Alternatively, if the speed is expressed in rad/s, we have

$$\omega_e = \frac{p}{2}\omega_m \qquad \textbf{(17.58)}$$

where ω_m is the mechanical speed of rotation in rad/s. The number of poles employed in a synchronous generator is then determined by two factors: the frequency desired of the generated voltage (e.g., 60 Hz, if the generator is used to produce AC power), and the speed of rotation of the prime mover. In the latter respect, there is a significant difference, for example, between the speed of rotation of a steam turbine generator and that of a hydroelectric generator, the former being much greater.

A common application of the alternator is in automotive battery-charging systems—in which, however, the generated AC voltage is rectified to provide the DC current required for charging the battery. Figure 17.32 depicts an automotive alternator.

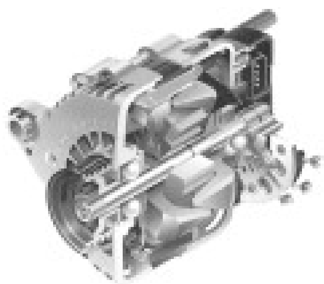


Figure 17.32 Automotive alternator (Courtesy: Delphi Automotive Systems)

17.7 THE SYNCHRONOUS MOTOR

Synchronous motors are virtually identical to synchronous generators with regard to their construction, except for an additional winding for helping start the motor and minimizing motor speed over- and undershoots. The principle of operation is, of course, the opposite: an AC excitation provided to the armature generates a magnetic field in the air gap between stator and rotor, resulting in a mechanical torque. To generate the rotor magnetic field, some DC current must be provided to the field windings; this is often accomplished by means of an **exciter,** which consists of a small DC generator propelled by the motor itself, and therefore mechanically connected to it. It was mentioned earlier that to obtain a constant torque in an electric motor, it is necessary to keep the rotor and stator magnetic fields constant relative to each other. This means that the electromagnetically rotating field in the stator and the mechanically rotating rotor field should be aligned at all times. The only condition for which this can occur is if both fields are rotating at the synchronous speed, $n_s = 120f/p$. Thus, synchronous motors are by their very nature constant-speed motors.

For a non–salient pole (cylindrical-rotor) synchronous machine, the torque can be written in terms of the AC stator current, $i_S(t)$, and of the DC rotor current, I_f:

$$T = k i_S(t) I_f \sin(\gamma) \qquad \textbf{(17.59)}$$

where γ is the angle between the stator and rotor fields (see Figure 17.7). Let the angular speed of rotation be

$$\omega_m = \frac{d\theta_m}{dt} \text{ rad/s} \qquad \textbf{(17.60)}$$

where $\omega_m = 2\pi n/60$, and let ω_e be the electrical frequency of $i_S(t)$, where $i_S(t) = \sqrt{2} I_S \sin(\omega_e t)$. Then the torque may be expressed as follows:

$$T = k\sqrt{2} I_S \sin(\omega_e t) I_f \sin(\gamma) \qquad \textbf{(17.61)}$$

where k is a machine constant, I_S is the rms value of the stator current, and I_f is the DC rotor current. Now, the rotor angle γ can be expressed as a function of time by

$$\gamma = \gamma_0 + \omega_m t \tag{17.62}$$

where γ_0 is the angular position of the rotor at $t = 0$; the torque expression then becomes

$$\begin{aligned} T &= k\sqrt{2}I_S I_f \sin(\omega_e t)\sin(\omega_m t + \gamma_0) \\ &= k\frac{\sqrt{2}}{2}I_S I_f \cos[(\omega_m - \omega_e)t - \gamma_0] - \cos[(\omega_m + \omega_e)t + \gamma_0] \end{aligned} \tag{17.63}$$

It is a straightforward matter to show that the average value of this torque, $\langle T \rangle$, is different from zero only if $\omega_m = \pm\omega_e$, that is, only if the motor is turning at the synchronous speed. The resulting average torque is then given by

$$\langle T \rangle = k\sqrt{2}I_S I_f \cos(\gamma_0) \tag{17.64}$$

Note that equation 17.63 corresponds to the sum of an average torque plus a fluctuating component at twice the original electrical (or mechanical) frequency. The fluctuating component results because, in the foregoing derivation, a single-phase current was assumed. The use of multiphase currents reduces the torque fluctuation to zero and permits the generation of a constant torque.

A per-phase circuit model describing the synchronous motor is shown in Figure 17.33, where the rotor circuit is represented by a field winding equivalent resistance and inductance, R_f and L_f, respectively, and the stator circuit is represented by equivalent stator winding inductance and resistance, L_S and R_S, respectively, and by the induced emf, E_b. From the exact equivalent circuit as given in Figure 17.33, we have

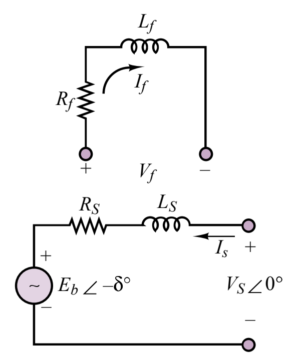


Figure 17.33 Per-phase circuit model

$$V_S = E_b + I_S(R_S + jX_S) \tag{17.65}$$

where X_S is known as the synchronous reactance and includes magnetizing reactance.

The motor power is

$$P_{\text{out}} = \omega_S T = |V_S||I_S|\cos(\theta) \tag{17.66}$$

for each phase, where T is the developed torque and θ is the angle between V_S and I_S.

When the phase winding resistance R_S is neglected, the circuit model of a synchronous machine can be redrawn as shown in Figure 17.34. The input power (per phase) is equal to the output power in this circuit, since no power is dissipated in the circuit; that is:

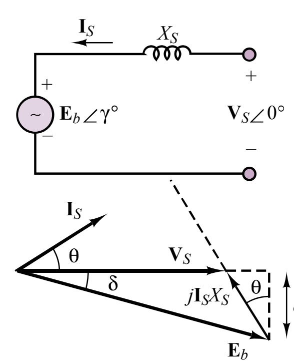


Figure 17.34

$$P_\phi = P_{\text{in}} = P_{\text{out}} = |\mathbf{V}_S||\mathbf{I}_S|\cos(\theta) \tag{17.67}$$

Also by inspection of Figure 17.34, we have

$$d = |\mathbf{E}_b|\sin(\delta) = |\mathbf{I}_S|X_S\cos(\theta) \tag{17.68}$$

Then

$$|\mathbf{E}_b||\mathbf{V}_S|\sin(\delta) = |\mathbf{V}_S||\mathbf{I}_S|X_S\cos(\theta) = X_S P_\phi \tag{17.69}$$

The total power of a three-phase synchronous machine is then given by

$$P = (3)\frac{|\mathbf{V}_S||\mathbf{E}_b|}{X_S}\sin(\delta) \tag{17.70}$$

Because of the dependence of the power upon the angle δ, this angle has come to be called the **power angle.** If δ is zero, the synchronous machine cannot develop useful power. The developed power has its maximum value at δ equal to 90°. If we assume that $|\mathbf{E}_b|$ and $|\mathbf{V}_S|$ are constant, we can draw the curve shown in Figure 17.35, relating the power and power angle in a synchronous machine.

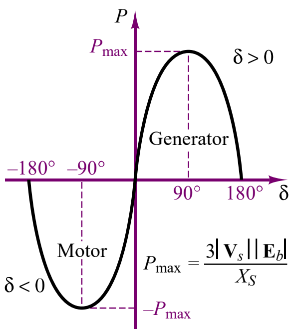


Figure 17.35 Power versus power angle for a synchronous machine

A synchronous generator is usually operated at a power angle varying from 15° to 25°. For synchronous motors and small loads, δ is close to 0°, and the motor torque is just sufficient to overcome its own windage and friction losses; as the load increases, the rotor field falls further out of phase with the stator field (although the two are still rotating at the same speed), until δ reaches a maximum at 90°. If the load torque exceeds the maximum torque, which is produced for $\delta = 90°$, the motor is forced to slow down below synchronous speed. This condition is undesirable, and provisions are usually made to shut the motor down automatically whenever synchronism is lost. The maximum torque is called the **pull-out torque** and is an important measure of the performance of the synchronous motor.

Accounting for each of the phases, the total torque is given by

$$T = \frac{m}{\omega_s}|\mathbf{V}_S||\mathbf{I}_S|\cos(\theta) \tag{17.71}$$

where m is the number of phases. From Figure 17.34, we have $E_b\sin(\delta) = X_S I_S\cos(\theta)$. Therefore, for a three-phase machine, the developed torque is

$$T = \frac{P}{\omega_s} = \frac{3}{\omega_s}\frac{|\mathbf{V}_S||\mathbf{E}_b|}{X_S}\sin(\delta) \qquad \text{N-m} \tag{17.72}$$

Typically, analysis of multiphase motors is performed on a per-phase basis, as illustrated in the examples that follow.

EXAMPLE 17.10 Synchronous Motor Analysis

Problem

Find the kVA rating, the induced voltage and the power angle of the rotor for a fully loaded synchronous motor.

Solution

Known Quantities: Motor ratings; motor synchronous impedance.

Find: S; $\mathbf{E}_b$; δ.

Schematics, Diagrams, Circuits, and Given Data: Motor ratings: 460 V; 3 ϕ; pf = 0.707 lagging; full-load stator current: 12.5 A. $Z_S = 1 + j12\ \Omega$.

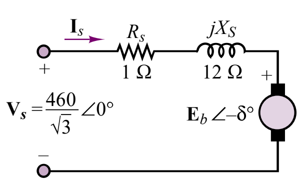


Figure 17.36

Assumptions: Use per-phase analysis.

Analysis: The circuit model for the motor is shown in Figure 17.36. The per-phase current in the wye-connected stator winding is

$$I_S = |\mathbf{I}_S| = 12.5 \text{ A}$$

The per-phase voltage is

$$V_S = |\mathbf{V}_S| = \frac{460 \text{ V}}{\sqrt{3}} = 265.58 \text{ V}$$

The kVA rating of the motor is expressed in terms of the apparent power, S (see Chapter 7):

$$S = 3V_S I_S = 3 \times 265.58 \text{ V} \times 12.5 \text{ A} = 9{,}959 \text{ W}$$

From the equivalent circuit, we have

$$\begin{aligned}\mathbf{E}_b &= \mathbf{V}_S - \mathbf{I}_S(R_S + jX_S) \\ &= 265.58 - (12.5\angle -45^\circ \text{ A}) \times (1 + j12\ \Omega) = 179.31\angle -32.83^\circ \text{ V}\end{aligned}$$

The induced line voltage is defined to be

$$V_{\text{line}} = \sqrt{3}E_b = \sqrt{3} \times 179.31 \text{ V} = 310.57 \text{ V}$$

From the expression for $\mathbf{E}_b$, we can find the power angle:

$$\delta = -32.83^\circ$$

Comments: The minus sign indicates that the machine is in the motor mode.

EXAMPLE 17.11 Synchronous Motor Analysis

Problem

Find the stator current, the line current and the induced voltage for a synchronous motor.

Solution

Known Quantities: Motor ratings; motor synchronous impedance.

Find: $\mathbf{I}_S$; $\mathbf{I}_{\text{line}}$; $\mathbf{E}_b$.

Schematics, Diagrams, Circuits, and Given Data: Motor ratings: 208 V; 3 ϕ; 45 kVA; 60 Hz; pf = 0.8 leading; $Z_S = 0 + j2.5\ \Omega$. Friction and windage losses: 1.5 kW; core losses: 1.0 kW. Load power: 15 hp.

Assumptions: Use per-phase analysis.

Analysis: The output power of the motor is 15 hp; that is:

$$P_{\text{out}} = 15 \text{ hp} \times 0.746 \text{ kW/hp} = 11.19 \text{ kW}$$

The electric power supplied to the machine is

$$\begin{aligned}P_{\text{in}} &= P_{\text{out}} + P_{\text{mech}} + P_{\text{core-loss}} + P_{\text{elec-loss}} \\ &= 11.19 \text{ kW} + 1.5 \text{ kW} + 1.0 \text{ kW} + 0 \text{ kW} = 13.69\text{kW}\end{aligned}$$

As discussed in Chapter 7, the resulting line current is

$$I_{\text{line}} = \frac{P_{\text{in}}}{\sqrt{3}V \cos\theta} = \frac{13{,}690 \text{ W}}{\sqrt{3} \times 208 \text{ V} \times 0.8} = 47.5 \text{ A}$$

Because of the Δ connection, the armature current is

$$\mathbf{I}_S = \frac{1}{\sqrt{3}}\mathbf{I}_{\text{line}} = 27.4\angle 36.87^\circ \text{ A}$$

The emf may be found from the equivalent circuit and KVL:

$$\mathbf{E}_b = \mathbf{V}_S - jX_S\mathbf{I}_S$$
$$= 208\angle 0^\circ - j2.5\ \Omega(27.4\angle 36.87^\circ\ \text{A}) = 255\angle -12.4^\circ\ \text{V}$$

The power angle is

$$\delta = -12.4^\circ$$

Synchronous motors are not very commonly used in practice, for various reasons, among which are that they are essentially required to operate at constant speed (unless a variable-frequency AC supply is available) and that they are not self-starting. Further, separate AC and DC supplies are required. It will be seen shortly that the induction motor overcomes most of these drawbacks.

Check Your Understanding

17.11 A synchronous generator has a multipolar construction that permits changing its synchronous speed. If only two poles are energized, at 50 Hz, the speed is 3,000 rev/min. If the number of poles is progressively increased to 4, 6, 8, 10, and 12, find the synchronous speed for each configuration.

17.12 Draw the complete equivalent circuit of a synchronous generator and its phasor diagram.

17.13 Find an expression for the maximum pull-out torque of the synchronous motor.

17.8 THE INDUCTION MOTOR

The induction motor is the most widely used electric machine, because of its relative simplicity of construction. The stator winding of an induction machine is similar to that of a synchronous machine; thus, the description of the three-phase winding of Figure 17.25 also applies to induction machines. The primary advantage of the induction machine, which is almost exclusively used as a motor (its performance as a generator is not very good), is that no separate excitation is required for the rotor. The rotor typically consists of one of two arrangements: a **squirrel cage,** or a **wound rotor.** The former contains conducting bars short-circuited at the end and embedded within it; the latter consists of a multiphase winding similar to that used for the stator, but electrically short-circuited.

In either case, the induction motor operates by virtue of currents induced from the stator field in the rotor. In this respect, its operation is similar to that of a transformer, in that currents in the stator (which acts as a primary coil) induce currents in the rotor (acting as a secondary coil). In most induction motors, no external electrical connection is required for the rotor, thus permitting a simple, rugged construction, without the need for slip rings or brushes. Unlike the synchronous motor, the induction motor does not operate at synchronous speed, but at a somewhat lower speed, which is dependent on the load. Figure 17.37 illustrates the appearance of a squirrel-cage induction motor. The following discussion will focus mainly on this very common configuration.

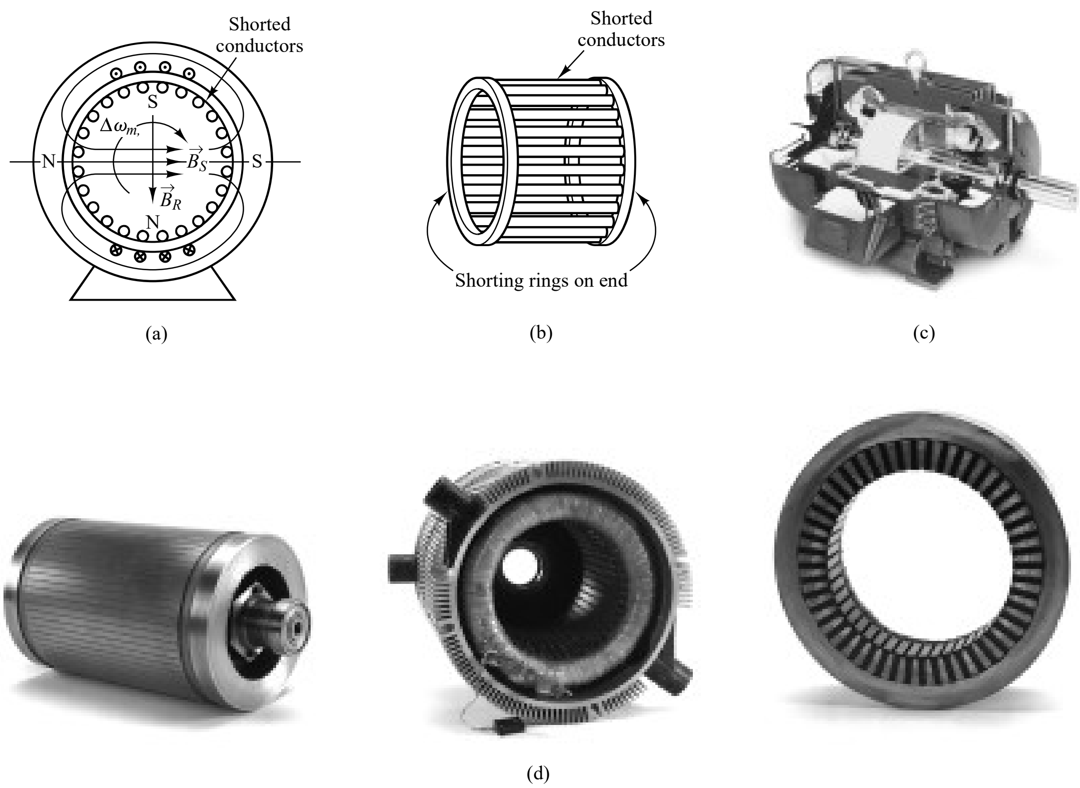


Figure 17.37 (a) Squirrel-cage induction motor; (b) conductors in rotor; (c) photo of squirrel-cage induction motor; (d) views of Smokin' Buckey motor: rotor, stator, and cross section of stator (Courtesy: David H. Koether Photography.)

You are by now acquainted with the notion of a rotating stator magnetic field. Imagine now that a squirrel-cage rotor is inserted in a stator in which such a rotating magnetic field is present. The stator field will induce voltages in the cage conductors, and if the stator field is generated by a three-phase source, the resulting rotor currents—which circulate in the bars of the squirrel cage, with the conducting path completed by the shorting rings at the end of the cage—are also three-phase, and are determined by the magnitude of the induced voltages and by the impedance of the rotor. Since the rotor currents are induced by the stator field, the number of poles and the speed of rotation of the induced magnetic field are the same as those of the stator field, *if the rotor is at rest*. Thus, when a stator field is initially applied, the rotor field is synchronous with it, and the fields are stationary with respect to each other. Thus, according to the earlier discussion, a *starting torque* is generated.

If the starting torque is sufficient to cause the rotor to start spinning, the rotor will accelerate up to its operating speed. However, an induction motor can never reach synchronous speed; if it did, the rotor would appear to be stationary with respect to the rotating stator field, since it would be rotating at the same speed. But in the absence of relative motion between the stator and rotor fields, no voltage would be induced in the rotor. Thus, an induction motor is limited to speeds somewhere below the synchronous speed, n_S. Let the speed of rotation of the rotor be n; then, the rotor is losing ground with respect to the rotation of the stator field at a speed $(n_s - n)$. In effect, this is equivalent to backward motion of the rotor at the **slip speed,** defined by $(n_s - n)$. The **slip,** s, is usually defined as a fraction of n_s:

$$s = \frac{n_s - n}{n_s} \tag{17.73}$$

which leads to the following expression for the rotor speed:

$$n = n_s(1 - s) \tag{17.74}$$

The slip, s, is a function of the load, and the amount of slip in a given motor is dependent on its construction and rotor type (squirrel cage or wound rotor). Since there is a relative motion between the stator and rotor fields, voltages will be induced in the rotor at a frequency called the **slip frequency,** related to the relative speed of the two fields. This gives rise to an interesting phenomenon: the rotor field travels relative to the rotor at the slip speed sn_s, but the rotor is mechanically traveling at the speed $(1 - s)n_s$, so that the net effect is that the rotor field travels at the speed

$$sn_s + (1 - s)n_s = n_s \tag{17.75}$$

that is, at synchronous speed. The fact that the rotor field rotates at synchronous speed—although the rotor itself does not—is extremely important, because it means that the stator and rotor fields will continue to be stationary with respect to each other, and therefore a net torque can be produced.

As in the case of DC and synchronous motors, important characteristics of induction motors are the starting torque, the maximum torque, and the torque-speed curve. These will be discussed shortly, after some analysis of the induction motor is performed in the next few paragraphs.

EXAMPLE 17.12 Induction Motor Analysis

Problem

Find the full load rotor slip and frequency of the induced voltage at rated speed in a four-pole induction motor.

Solution

Known Quantities: Motor ratings.

Find: s; f_R.

Schematics, Diagrams, Circuits, and Given Data: Motor ratings: 230 V; 60 Hz; full-load speed: 1,725 rev/min.

Analysis: The synchronous speed of the motor is

$$n_s = \frac{120f}{p} = \frac{60f}{p/2} = \frac{60 \text{ s/min} \times 60 \text{ rev/s}}{4/2} = 1{,}800 \text{ rev/min}$$

The slip is

$$s = \frac{n_s - n}{n_s} = \frac{1{,}800 \text{ rev/min} - 1{,}725 \text{ rev/min}}{1{,}800 \text{ rev/min}} = 0.0417$$

The rotor frequency, f_R, is

$$f_R = sf = 0.0417 \times 60 \text{ Hz} = 2.5 \text{ Hz}$$

The induction motor can be described by means of an equivalent circuit, which is essentially that of a rotating transformer. (See Chapter 16 for a circuit model of the transformer.) Figure 17.38 depicts such a circuit model, where:

R_S = stator resistance per phase, R_R = rotor resistance per phase

X_S = stator reactance per phase, X_R = rotor reactance per phase

X_m = magnetizing (mutual) reactance

R_C = equivalent core-loss resistance

E_S = per-phase induced voltage in stator windings

E_R = per-phase induced voltage in rotor windings

The primary internal stator voltage, $\mathbf{E}_S$, is coupled to the secondary rotor voltage, $\mathbf{E}_R$, by an ideal transformer with an effective turns ratio α. For the rotor circuit, the induced voltage at any slip will be

$$\mathbf{E}_R = s\mathbf{E}_{R0} \quad \textbf{(17.76)}$$

where $\mathbf{E}_{R0}$ is the induced rotor voltage at the condition in which the rotor is stationary. Also, $X_R = \omega_R L_R = 2\pi f_R L_R = 2\pi s f L_R = s X_{R0}$, where $X_{R0} = 2\pi f L_R$ is the reactance when the rotor is stationary. The rotor current is given by the expression

$$\mathbf{I}_R = \frac{\mathbf{E}_R}{R_R + jX_R} = \frac{s\mathbf{E}_{R0}}{R_R + jsX_{R0}} = \frac{\mathbf{E}_{R0}}{R_R/s + jX_{R0}} \quad \textbf{(17.77)}$$

The resulting rotor equivalent circuit is shown in Figure 17.39.

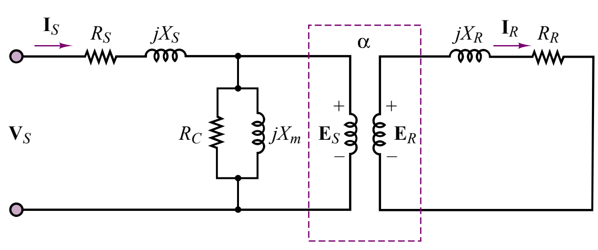


Figure 17.38 Circuit model for induction machine

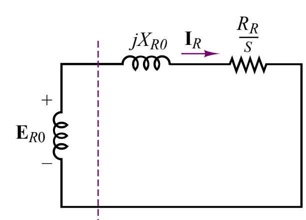


Figure 17.39 Rotor circuit

The voltages, currents, and impedances on the secondary (rotor) side can be reflected to the primary (stator) by means of the effective turns ratio. When this transformation is effected, the transformed rotor voltage is given by

$$\mathbf{E}_2 = \mathbf{E}'_R = \alpha \mathbf{E}_{R0} \quad \textbf{(17.78)}$$

The transformed (reflected) rotor current is

$$\mathbf{I}_2 = \frac{\mathbf{I}_R}{\alpha} \quad \textbf{(17.79)}$$

The transformed rotor resistance can be defined as

$$R_2 = \alpha^2 R_R \tag{17.80}$$

and the transformed rotor reactance can be defined by

$$X_2 = \alpha^2 X_{R0} \tag{17.81}$$

The final per-phase equivalent circuit of the induction motor is shown in Figure 17.40.

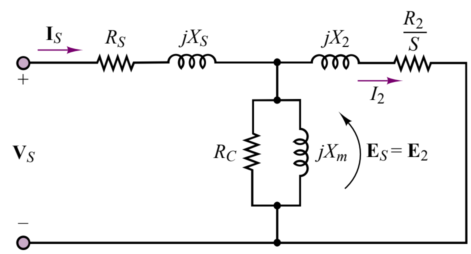


Figure 17.40 Equivalent circuit of an induction machine

The following examples illustrate the use of the circuit model in determining the performance of the induction motor.

EXAMPLE 17.13 Induction Motor Analysis

Problem

Determine the following quantities for an induction motor using the circuit model of Figures 17.38 to 17.40.

1. Speed
2. Stator current
3. Power factor
4. Output torque

Solution

Known Quantities: Motor ratings; circuit parameters.

Find: n; ω_m; $\mathbf{I}_S$; pf; T.

Schematics, Diagrams, Circuits, and Given Data: Motor ratings: 460 V; 60 Hz; four poles; $s = 0.022$; $P = 14$ hp
$R_S = 0.641\ \Omega$; $R_2 = 0.332\ \Omega$; $X_S = 1.106\ \Omega$; $X_S = 0.464\ \Omega$; $X_m = 26.3\ \Omega$

Assumptions: Use per-phase analysis. Neglect core losses ($R_C = 0$).

Analysis:

1. The per-phase equivalent circuit is shown in Figure 17.40. The synchronous speed is found to be

$$n_s = \frac{120f}{p} = \frac{60 \text{ s/min} \times 60 \text{ rev/s}}{4/2} = 1{,}800 \text{ rev/min}$$

or

$$\omega_s = 1{,}800 \text{ rev/min} \times \frac{2\pi \text{ rad}}{60 \text{ s/min}} = 188.5 \text{ rad/s}$$

The rotor mechanical speed is

$$n = (1 - s)n_s \text{ rev/min} = 1{,}760 \text{ rev/min}$$

or

$$\omega_m = (1 - s)\omega_s \text{ rad/s} = 184.4 \text{ rad/s}$$

2. The reflected rotor impedance is found from the parameters of the per-phase circuit to be

$$Z_2 = \frac{R_2}{s} + jX_2 = \frac{0.332}{0.022} + j0.464 \ \Omega$$
$$= 15.09 + j0.464 \ \Omega$$

The combined magnetization plus rotor impedance is therefore equal to

$$Z = \frac{1}{1/jX_m + 1/Z_2} = \frac{1}{-j0.038 + 0.0662\angle -1.76^\circ} = 12.94\angle 31.1^\circ \ \Omega$$

and the total impedance is

$$Z_{\text{total}} = Z_S + Z = 0.641 + j1.106 + 11.08 + j6.68$$
$$= 11.72 + j7.79 = 14.07\angle 33.6^\circ \ \Omega$$

Finally, the stator current is given by

$$\mathbf{I}_S = \frac{\mathbf{V}_S}{Z_{\text{total}}} = \frac{460/\sqrt{3}\angle 0^\circ \text{ V}}{14.07\angle 33.6^\circ \ \Omega} = 18.88\angle -33.6^\circ \text{ A}$$

3. The power factor is

$$\text{pf} = \cos 33.6^\circ = 0.883 \qquad \text{lagging}$$

4. The output power, P_{out}, is

$$P_{\text{out}} = 14 \text{ hp} \times 746 \text{ W/hp} = 10.444 \text{ kW}$$

and the output torque is

$$T = \frac{P_{\text{out}}}{\omega_m} = \frac{10{,}444 \text{ W}}{184.4 \text{ rad/s}} = 56.64 \text{ N-m}$$

EXAMPLE 17.14 Induction Motor Analysis

Problem

Determine the following quantities for a three-phase induction motor using the circuit model of Figures 17.39 to 17.41.

1. Stator current

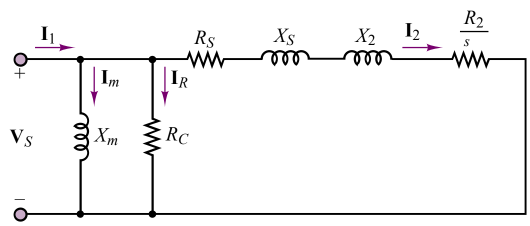


Figure 17.41

2. Power factor
3. Full-load electromagnetic torque

Solution

Known Quantities: Motor ratings; circuit parameters.

Find: $\mathbf{I}_S$; pf; T.

Schematics, Diagrams, Circuits, and Given Data: Motor ratings: 500 V; 3 ϕ; 50 Hz; $p = 8$; $s = 0.05$; $P = 14$ hp.
Circuit parameters: $R_S = 0.13\ \Omega$; $R'_R = 0.32\ \Omega$; $X_S = 0.6\ \Omega$; $X'_R = 1.48\ \Omega$;
$Y_m = G_C + jB_m =$ magnetic branch admittance describing core loss and mutual inductance $= 0.004 - j0.05\ \Omega^{-1}$; stator to rotor turns ratio $= 1 : \alpha = 1 : 1.57$.

Assumptions: Use per-phase analysis. Neglect mechanical losses.

Analysis: The approximate equivalent circuit of the three-phase induction motor on a per-phase basis is shown in Figure 17.41. The parameters of the model are calculated as follows:

$$R_2 = R'_R \times \left(\frac{1}{\alpha}\right)^2 = 0.32 \times \left(\frac{1}{1.57}\right)^2 = 0.13\ \Omega$$

$$X_2 = X'_R \times \left(\frac{1}{\alpha}\right)^2 = 1.48 \times \left(\frac{1}{1.57}\right)^2 = 0.6\ \Omega$$

$$Z = R_S + \frac{R_2}{S} + j(X_S + X_2)$$

$$= 0.13 + \frac{0.13}{0.05} + j(0.6 + 0.6) = 2.73 + j1.2\ \Omega$$

Using the approximate circuit,

$$\mathbf{I}_2 = \frac{\mathbf{V}_S}{Z} = \frac{(500/\sqrt{3})\angle 0^\circ\ \text{V}}{2.73 + j1.2\ \Omega} = 88.8 - j39\ \text{A}$$

$$\mathbf{I}_R = \mathbf{V}_S G_S = 288.7\ \text{V} \times 0.004\ \Omega^{-1} = 1.15\ \text{A}$$

$$\mathbf{I}_m = -j\mathbf{V}_S B_m = 288.7\ \text{V} \times (-j0.05)\Omega = -j14.4\ \text{A}$$

$$\mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_R + \mathbf{I}_m = 89.95 - j53.4\ \text{A}$$

$$\text{Input power factor} = \frac{\text{Re}[\mathbf{I}_1]}{|\mathbf{I}_1|} = \frac{89.95}{104.6} = 0.86\ \text{lagging}$$

$$\text{Torque} = \frac{3P}{\omega_S} = \frac{3I_2^2 R_2/s}{4\pi f/p} = 935\ \text{N-m}$$

Performance of Induction Motors

The performance of induction motors can be described by torque-speed curves similar to those already used for DC motors. Figure 17.42 depicts an induction motor torque-speed curve, with five torque ratings marked *a* through *e*. Point *a* is the *starting torque,* also called **breakaway torque,** and is the torque available with the rotor "locked," that is, in a stationary position. At this condition, the frequency of the voltage induced in the rotor is highest, since it is equal to the frequency of rotation of the stator field; consequently, the inductive reactance of the rotor is greatest. As the rotor accelerates, the torque drops off, reaching a maximum value called the **pull-up torque** (point *b*); this typically occurs somewhere between 25 and 40 percent of synchronous speed. As the rotor speed continues to increase, the rotor reactance decreases further (since the frequency of the induced voltage is determined by the relative speed of rotation of the rotor with respect to the stator field). The torque becomes a maximum when the rotor inductive reactance is equal to the rotor resistance; maximum torque is also called **breakdown torque** (point *c*). Beyond this point, the torque drops off, until it is zero at synchronous speed, as discussed earlier. Also marked on the curve are the *150 percent torque* (point *d*), and the *rated torque* (point *e*).

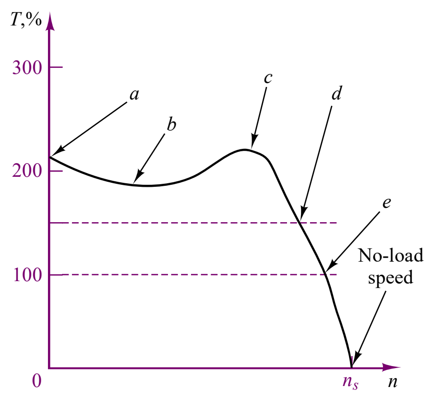


Figure 17.42 Performance curve for induction motor

A general formula for the computation of the induction motor steady-state torque-speed characteristic is

$$T = \frac{1}{\omega_e} \frac{m V_S^2 R_R / s}{[(R_S + \frac{R_R}{s})^2 + (X_S + X_R)^2]} \tag{17.82}$$

where m is the number of phases.

FIND IT ON THE WEB

Different construction arrangements permit the design of **induction motors** with different torque-speed curves, thus permitting the user to select the motor that best suits a given application. Figure 17.43 depicts the four basic classifications, classes A, B, C, and D, as defined by NEMA. The determining features in the classification are the locked-rotor torque and current, the breakdown torque, the pull-up torque, and the percent slip. Class A motors have a higher breakdown torque than class B motors, and a slip of 5 percent or less. Motors in this class are often designed for a specific application. Class B motors are general-purpose motors; this is the

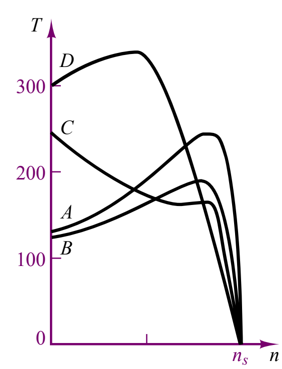


Figure 17.43 Induction motor classification

most commonly used type of induction motor, with typical values of slip of 3 to 5 percent. Class C motors have a high starting torque for a given starting current, and a low slip. These motors are typically used in applications demanding high starting torque but having relatively normal running loads, once running speed has been reached. Class D motors are characterized by high starting torque, high slip, low starting current, and low full-load speed. A typical value of slip is around 13 percent.

Factors that should be considered in the selection of an AC motor for a given application are the *speed range,* both minimum and maximum, and the speed variation. For example, it is important to determine whether constant speed is required; what variation might be allowed, either in speed or in torque; or whether variable-speed operation is required, in which case a variable-speed drive will be needed. The torque requirements are obviously important as well. The starting and running torque should be considered; they depend on the type of load. Starting torque can vary from a small percentage of full-load to several times full-load torque. Furthermore, the excess torque available at start-up determines the *acceleration characteristics* of the motor. Similarly, *deceleration characteristics* should be considered, to determine whether external braking might be required.

Another factor to be considered is the *duty cycle* of the motor. The duty cycle, which depends on the nature of the application, is an important consideration when the motor is used in repetitive, noncontinuous operation, such as is encountered in some types of machine tools. If the motor operates at zero or reduced load for periods of time, the duty cycle—that is, the percentage of the time the motor is loaded—is an important selection criterion. Last, but by no means least, are the *heating properties* of a motor. Motor temperature is determined by internal losses and by ventilation; motors operating at a reduced speed may not generate sufficient cooling, and forced ventilation may be required.

Thus far, we have not considered the dynamic characteristics of induction motors. Among the integral-horsepower induction motors (i.e., motors with horsepower rating greater than one), the most common dynamic problems are associated with starting and stopping and with the ability of the motor to continue operation during supply system transient disturbances. Dynamic analysis methods for induction motors depend to a considerable extent on the nature and complexity of the problem and the associated precision requirements. When the electrical transients in the motor are to be included as well as the motional transients, and especially when the motor is an important element in a large network, the simple transient equivalent circuit of Figure 17.44 provides a good starting approximation. In the circuit model of Figure 17.44, X'_S is called the *transient reactance*. The voltage E'_S is called the *voltage behind the transient reactance* and is assumed to be equal to the initial value of the induced voltage, at the start of the transient. R_S is the stator resistance. The dynamic analysis problem consists of selecting a sufficiently simple but reasonably realistic representation that will not unduly complicate the dynamic analysis, particularly through the introduction of nonlinearities.

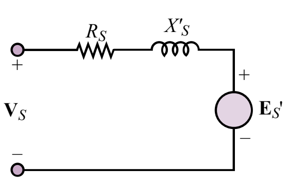


Figure 17.44 Simplified induction motor dynamic model

It should be remarked that the basic equations of the induction machine, as derived from first principles, are quite nonlinear. Thus, an accurate dynamic analysis of the induction motor, without any linearizing approximations, requires the use of computer simulation.

AC Motor Speed and Torque Control

As explained in an earlier section, AC machines are constrained to fixed-speed or near fixed-speed operation when supplied by a constant-frequency source. Several simple methods exist to provide limited **speed control in an AC induction machines;** more complex methods, involving the use of advanced power electronics circuits can be used if the intended application requires wide-bandwidth control of motor speed or torque. In this subsection we provide a general overview of available solutions.

Pole Number Control

The (conceptually) easiest method to implement speed control in an induction machine is by *varying the number of poles.* Equation 17.55 explains the dependence of synchronous speed in an AC machine on the supply frequency and on the number of poles. For machines operated at 60 Hz, the following speeds can be achieved by varying the number of magnetic poles in the stator winding:

Number of poles	2	4	6	8	12
n, rev/min	3,600	1,800	1,200	800	600

Motor stators can be wound so that the number of pole pairs in the stators can be varied by switching between possible winding connections. Such switching requires that care be taken in timing it to avoid damage to the machine.

Slip Control

Since the rotor speed is inherently dependent on the slip, *slip control* is a valid means of achieving some speed variation in an induction machine. Since motor torque falls with the square of the voltage (see equation 17.82), it is possible to change the slip by changing the motor torque through a reduction in motor voltage. This procedure allows for speed control over the range of speeds that allow for stable motor operation. With reference to Figure 17.42, this is possible only above point c, that is, above the *breakdown torque*.

Rotor Control

For motors with wound rotors, it is possible to connect the rotor slip rings to resistors; adding resistance to the rotor increases the losses in the rotor, and therefore causes the rotor speed to decrease. This method is also limited to operation above the *breakdown torque* though it should be noted that the shape of the motor torque-speed characteristic changes when the rotor resistance is changed.

Frequency Regulation

The last two methods cause additional losses to be introduced in the machine. If a variable-frequency supply is used, motor speed can be controlled without any additional losses. As seen in equation 17.55, the motor speed is directly dependent on the supply frequency, as the supply frequency determines the speed of the rotating magnetic field. However, to maintain the same motor torque characteristics over a range of speeds, the motor voltage must change with frequency, to maintain a constant torque. Thus, generally, the V/Hz ratio should be held constant.

This condition is difficult to achieve at start-up and at very low frequencies, in which cases the voltage must be raised above the constant V/Hz ratio that will be appropriate at higher frequency.

Adjustable-Frequency Drives

The advances made in the last two decades in power electronics and microcontrollers (see Chapters 11 and 14) have made AC machines employing **adjustable-frequency drives** well-suited to many common engineering applications that until recently required the use of the more easily speed-controlled DC drives. An adjustable-frequency drive consists of four major subsystems, as shown in Figure 17.45.

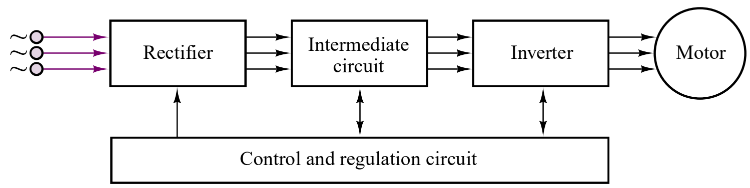


Figure 17.45 General configuration of adjustable-frequency drive

The diagram of Figure 17.45 assumes that a three-phase AC supply is available; the three-phase AC voltage is rectified using a controlled or uncontrolled **rectifier** (see Chapter 8 for a description of uncontrolled rectifiers and Chapter 11 for a description of controlled rectifiers). An **intermediate circuit** is sometimes necessary to further condition the rectified voltage and current. An **inverter** is then used to convert the fixed DC voltage to a variable frequency and variable amplitude AC voltage. This is accomplished via **pulse-amplitude modulation** (PAM) or increasingly, via **pulse-width modulation** (PWM) techniques. Figure 17.46

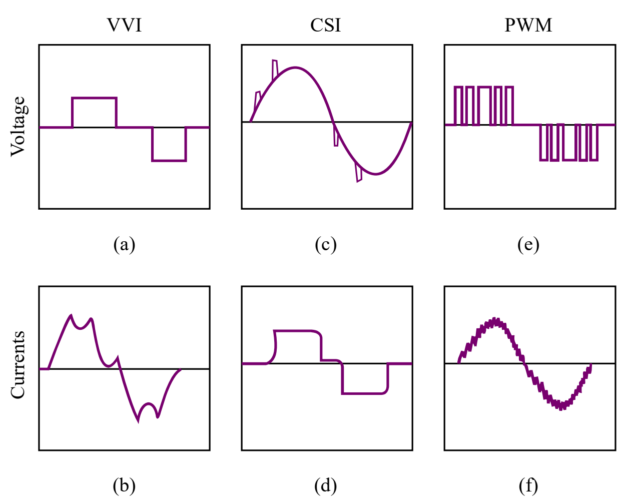


Figure 17.46 Typical adjustable-frequency controller voltage and current waveforms. (Courtesy: Rockwell Automation, Reliance Electric)

illustrates how approximately sinusoidal currents and voltages of variable frequency can be obtained by suitable shaping a train of pulses. It is important to understand that the technology used to generate such wave shapes is based on the simple power switching concepts underlying the voltage-source inverter (VSI) drive described in Chapter 11. DC-AC inverters come in many different configurations; the interested reader will find additional information and resources in the accompanying CD-ROM.

Check Your Understanding

17.14 A three-phase induction motor has six poles. (a) If the line frequency is 60 Hz, calculate the speed of the magnetic field in rev/min. (b) Repeat the calculation if the frequency is changed to 50 Hz.

17.15 A four-pole induction motor operating at a frequency of 60 Hz has a full-load slip of 4 percent. Find the frequency of the voltage induced in the rotor (a) at the instant of starting and (b) at full load.

17.16 A four-pole, 1,746-rev/min, 220-V, 3-phase, 60-Hz, 10-hp, Y-connected induction machine has the following parameters: $R_S = 0.4\ \Omega$, $R_2 = 0.14\ \Omega$, $X_m = 16\ \Omega$, $X_S = 0.35\ \Omega$, $X_2 = 0.35\ \Omega$, $R_C = 0$. Using Figure 16.39, find: (a) the stator current; (b) the rotor current; (c) the motor power factor; and (d) the total stator power input.

CONCLUSION

The principles developed in Chapter 17 can be applied to rotating electric machines, to explain how mechanical energy can be converted to electrical energy, and vice versa. The former function is performed by electric generators, while the latter is provided by electric motors.

- Electric machines are described in terms of their mechanical characteristics, their torque-speed curves, and their electrical characteristics, including current and voltage requirements. Losses and efficiency are an important part of the operation of electric machines, and it should be recognized that there will be electrical losses (due to the resistance of the windings), mechanical losses (friction and windage), and magnetic core losses (eddy currents, hysteresis). The main mechanical components of an electric machine are the stator, rotor, and air gap. Electrically, the important parameters are the armature (load current-carrying) circuit, and the field (magnetizing) circuit. Magnetic fields establish the coupling between the electrical and mechanical systems.
- Electric machines are broadly classified into DC and AC machines; the former use DC excitation for both the field and armature circuits, while the latter may be further subdivided into two classes: synchronous machines, and induction motors. AC synchronous machines are characterized by DC field and AC armature excitation. Induction machines (of the squirrel-cage type), on the other hand, do not require a field excitation, since this is provided by electromagnetic induction. Typically, DC machines have the armature winding on the rotor, while AC machines have it on the stator.
- The performance of electric machines can be approximately predicted with the use of circuit models, or of performance curves. The selection of a particular machine for a given application is driven by many factors, including the availability of suitable electrical supplies (or prime movers), the type of load, and various other concerns, of which heat dissipation and thermal characteristics are probably the most important.

CHECK YOUR UNDERSTANDING ANSWERS

CYU 17.1 (a) $\omega = 55\pi$ rad/s; (b) $n = 1{,}650$ rev/min

CYU 17.2 275 V

CYU 17.3 $T = 40.7$ N-m; volt-amperes = 11,431 VA; $P_{\max} = 11.5$ hp

CYU 17.4 170% of rated torque; 1,700 rev/min

CYU 17.5 $k_a = 5.1$; $E_b = 45.9$ V

CYU 17.6 (a) $E_b = 1{,}040$ V; (b) $k_a = 10.34$; (c) $T = 4{,}138$ N-m

CYU 17.7 (a) 400 A; (b) 5 A; (c) 405 A; (d) 270.25 V

CYU 17.8 (a) 144 N-m; (b) 132 N-m

CYU 17.9 $T = \dfrac{P}{\omega_m} = 9.93$ N-m

CYU 17.10 Increasing the armature voltage leads to an increase in armature current. Consequently, the motor torque increases until it exceeds the load torque, causing the speed to increase as well. The corresponding increase in back emf, however, causes the armature current to drop and the motor torque to decrease until a balance condition is reached between motor and load torque and the motor runs at constant speed.

CYU 17.11 1,500 rev/min; 1,000 rev/min; 750 rev/min; 600 rev/min; 500 rev/min

CYU 17.13 $T_{\max} = \dfrac{3V_S E_b}{\omega_m X_S}$

CYU 17.14 (a) $n = 1{,}200$ rev/min; (b) $n = 1{,}000$ rev/min

CYU 17.15 (a) $f_R = 60$ Hz; (b) $f_R = 2.4$ Hz

CYU 17.16 (a) $25.92\angle -22.45^\circ$ A; (b) $24.39\angle -6.51^\circ$ A; (c) 0.9243; (d) 8,476 W

HOMEWORK PROBLEMS

Section 1: Electric Machine Fundamentals

17.1 Calculate the force exerted by each conductor, 6 in. long, on the armature of a DC motor when it carries a current of 90 A and lies in a field the density of which is 5.2×10^{-4} Wb per square in.

17.2 In a DC machine, the air-gap flux density is 4 Wb/m^2. The area of the pole face is 2 cm × 4 cm. Find the flux per pole in the machine.

17.3 The power rating of a motor can be modified to account for different ambient temperature, according to the following table:

Ambient temperature	30°C	35°C	40°C
Variation of rated power	+8%	+5%	0
Ambient temperature	**45°C**	**50°C**	**55°C**
Variation of rated power	−5%	−12.5%	−25%

A motor with $P_e = 10$ kW is rated up to 85°C. Find the actual power for each of the following conditions:

a. Ambient temperature is 50°C.
b. Ambient temperature is 25°C.

17.4 The speed-torque characteristic of an induction motor has been empirically determined as follows:

Speed	1,470	1,440	1,410	1,300	1,100
Torque	3	6	9	13	15
Speed	**900**	**750**	**350**	**0**	**rev/min**
Torque	13	11	7	5	N-m

The motor will drive a load requiring a starting torque of 4 N-m and increase linearly with speed to 8 N-m at 1,500 rev/min.

a. Find the steady-state operating point of the motor.
b. Equation 17.82 predicts that the motor speed can be regulated in the face of changes in load torque by adjusting the stator voltage. Find the change in voltage required to maintain the speed at the operating point of part a if the load torque increases to 10 N-m.

Section 2: Direct-Current Machines

17.5 A 220-V shunt motor has an armature resistance of 0.32 Ω and a field resistance of 110 ohms. At no load the armature current is 6 A and the speed is 1,800 rpm.

Assume that the flux does not vary with load and calculate:

a. The speed of the motor when the line current is 62 A (assume a 2-volt brush drop).
b. The speed regulation of the motor.

17.6 A 120-V, 10-A shunt generator has an armature resistance of 0.6 Ω. The shunt field current is 2 A. Determine the voltage regulation of the generator.

17.7 A 50-hp, 550-volt shunt motor has an armature resistance, including brushes, of 0.36 ohm. When operating at rated load and speed, the armature takes 75 amp. What resistance should be inserted in the armature circuit to obtain a 20 percent speed reduction when the motor is developing 70 percent of rated torque? Assume that there is no flux change.

17.8 A 20-kW, 230-V separately excited generator has an armature resistance of 0.2 Ω and a load current of 100 A. Find:

a. The generated voltage when the terminal voltage is 230 V.
b. The output power.

17.9 A 10-kW, 120-VDC series generator has an armature resistance of 0.1 Ω and a series field resistance of 0.05 Ω. Assuming that it is delivering rated current at rated speed, find (a) the armature current and (b) the generated voltage.

17.10 The armature resistance of a 30-kW, 440-V shunt generator is 0.1 Ω. Its shunt field resistance is 200 Ω. Find

a. The power developed at rated load.
b. The load, field, and armature currents.
c. The electrical power loss.

17.11 A four-pole, 450-kW, 4.6-kV shunt generator has armature and field resistances of 2 and 333 Ω. The generator is operating at the rated speed of 3,600 rev/min. Find the no-load voltage of the generator and terminal voltage at half load.

17.12 A shunt DC motor has a shunt field resistance of 400 Ω and an armature resistance of 0.2 Ω. The motor nameplate rating values are 440 V, 1,200 rev/min, 100 hp, and full-load efficiency of 90 percent. Find:

a. The motor line current.
b. The field and armature currents.
c. The counter emf at rated speed.
d. The output torque.

17.13 A 30-kW, 240-V generator is running at half load at 1,800 rev/min with efficiency of 85 percent. Find the total losses and input power.

17.14 A 240-volt series motor has an armature resistance of 0.42 Ω and a series-field resistance of 0.18 Ω. If the speed is 500 rev/min when the current is 36 A, what will be the motor speed when the load reduces the line current to 21 A? (Assume a 3-volt brush drop and that the flux is proportional to the current.)

17.15 A 220-VDC shunt motor has an armature resistance of 0.2 Ω and a rated armature current of 50 A. Find

a. The voltage generated in the armature.
b. The power developed.

17.16 A 550-volt series motor takes 112 A and operates at 820 rev/min when the load is 75 hp. If the effective armature-circuit resistance is 0.15 Ω, calculate the horsepower output of the motor when the current drops to 84 A, assuming that the flux is reduced by 15 percent.

17.17 A 200-VDC shunt motor has the following parameters:

$$R_a = 0.1\ \Omega \qquad R_f = 100\ \Omega$$

When running at 1,100 rev/min with no load connected to the shaft, the motor draws 4 A from the line. Find E and the rotational losses at 1,100 rev/min (assuming that the stray-load losses can be neglected).

17.18 A 230-VDC shunt motor has the following parameters:

$$R_a = 0.5\ \Omega \qquad R_f = 75\ \Omega$$

$$P_{\text{rot}} = 500\ \text{W (at 1,120 rev/min)}$$

When loaded, the motor draws 46 A from the line. Find

a. The speed, P_{dev}, and T_{sh}.
b. If $L_f = 25$ H, $L_a = 0.008$ H, and the terminal voltage has a 115-V change, find $i_a(t)$ and $\omega_m(t)$.

17.19 A 200-VDC shunt motor with an armature resistance of 0.1 Ω and a field resistance of 100 Ω draws a line current of 5 A when running with no load at 955 rev/min. Determine the motor speed, the motor efficiency, the total losses (i.e., rotational and I^2R losses), and the load torque (i.e., T_{sh}) that will result when the motor draws 40 A from the line. Assume rotational power losses are proportional to the square of shaft speed.

17.20 A self-excited DC shunt generator is delivering 20 A to a 100-V line when it is driven at 200 rad/s. The magnetization characteristic is shown in Figure P17.20. It is known that $R_a = 1.0\ \Omega$ and $R_f = 100\ \Omega$. When the generator is disconnected from the line, the drive motor speeds up to 220 rad/s. What is the terminal voltage?

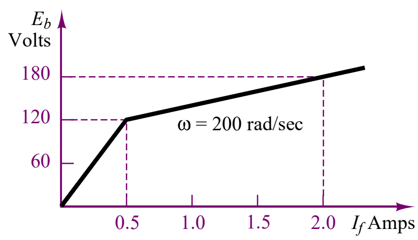


Figure P17.20

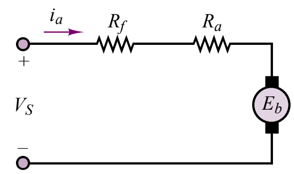


Figure P17.23

17.21 A 50-hp, 230-volt shunt motor has a field resistance of 17.7 Ω and operates at full load when the line current is 181 A at 1,350 rev/min. To increase the speed of the motor to 1,600 rev/min, a resistance of 5.3 Ω is "cut in" via the field rheostat; the line current then increases to 190 A. Calculate:

a. The power loss in the field and its percentage of the total power input for the 1,350 rev/min speed.
b. The power losses in the field and the field rheostat for the 1,600 rev/min speed.
c. The percent losses in the field and in the field rheostat at 1,600 rev/min.

17.22 A 10-hp, 230-volt shunt-wound motor has rated speed of 1,000 rev/min and full-load efficiency of 86 percent. Armature circuit resistance is 0.26 Ω; field-circuit resistance is 225 Ω. If this motor is operating under rated load and the field flux is very quickly reduced to 50 percent of its normal value, what will be the effect upon counter emf, armature current and torque? What effect will this change have upon the operation of the motor, and what will be its speed when stable operating conditions have been regained?

17.23 The machine of example 17.7 is being used in a series connection. That is, the field coil is connected in series with the armature, as shown in Figure P17.23. The machine is to be operated under the same conditions as in the previous example, that is, $n =$ 120 rev/min, $I_a = 8$ A. In the operating region, $\phi = kI_f$, and $k = 200$. The armature resistance is 0.2 Ω, and the resistance of the field winding is negligible.

a. Find the number of field winding turns necessary for full-load operation.
b. Find the torque output for the following speeds:

1. $n' = 2n$	3. $n' = n/2$
2. $n' = 3n$	4. $n' = n/4$

c. Plot the speed-torque characteristic for the conditions of part b.

17.24 With reference to Example 17.9, assume that the load torque applied to the PM DC motor is zero. Determine the response of the motor to a step change in input voltage. Derive expressions for the natural frequency and damping ratio of the second order system. What determines whether the system is over- or underdamped?

17.25 A motor with polar moment of inertia J develops torque according to the relationship $T = a\omega + b$. The motor drives a load defined by the torque-speed relationship $T_L = c\omega^2 + d$. If the four coefficients are all positive constants, determine the equilibrium speeds of the motor-load pair, and whether these speeds are stable.

17.26 Assume that a motor has known friction and windage losses described by the equation $T_{FW} = b\omega$. Sketch the T-ω characteristic of the motor if the load torque, T_L, is constant, and the T_L-ω characteristic if the motor torque is constant. Assume that T_{FW} at full speed is equal to 30 percent of the load torque.

17.27 A PM DC motor is rated at 6 V, 3350 rev/min, and has the following parameters: $r_a = 7$ Ω, $L_a =$ 120 mH, $k_T = 7 \times 10^{-3}$ N-m/A, $J = 1 \times 10^{-6}$ kg-m^2. The no-load armature current is 0.15 A.

a. In the steady-state no-load condition, the magnetic torque must be balanced by an internal damping torque; find the damping coefficient, b. Now sketch a model of the motor, write the dynamic equations, and determine the transfer function from armature voltage to motor speed. What is the approximate 3-dB bandwidth of the motor?
b. Now let the motor be connected to a pump with inertia $J_L = 1 \times 10^{-4}$ kg-m^2, damping coefficient $b_L = 5 \times 10^{-3}$ N-m-s, and load torque $T_L = 3.5 \times 1{-}^{-3}$ N-m. Sketch the model describing the motor-load configuration, and write the dynamic equations for this system; determine the new transfer function from armature voltage to motor speed. What is the approximate 3-dB bandwidth of the motor/pump system?

17.28 A PM DC motor with torque constant k_{PM} is used to power a hydraulic pump; the pump is a positive displacement type and generates a flow proportional to the pump velocity: $q_p = k_p\omega$. The fluid travels through a conduit of negligible resistance; an accumulator is included to smooth out the pulsations of the pump. A hydraulic load (modelled by a fluid resistance, R) is connected between the pipe and a reservoir (assumed at zero pressure). Sketch the motor-pump circuit. Derive the dynamic equations for the system and determine the transfer function between motor voltage and the pressure across the load.

17.29 A shunt motor in Figure P17.29 is characterized by a field coefficient $k_f = 1.2$ V-s/A-rad, such that the back emf is given by the expression $E_b = k_f I_f \omega$, and the motor torque by the expression $T = k_f I_f I_a$. The motor drives an inertia/viscous friction load with parameters $J = 0.8$ kg-m^2 and $b = 0.6$ N-m-s/rad. The field equation may be approximated by $V_S = R_f I_f$. The armature resistance is $R_a = 0.75\ \Omega$, and the field resistance is $R_f = 60\ \Omega$. The system is perturbed around the nominal operating point $V_{S0} = 150$ V, $\omega_0 = 200$ rad/s, $I_{a0} = 40$ A, respectively.

a. Derive the dynamic system equations in *symbolic form*.
b. Linearize the equations you obtained in part a.

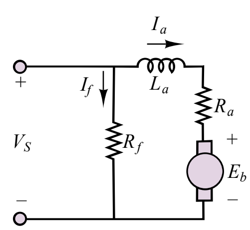


Figure P17.29

17.30 A PM DC motor is rigidly coupled to a fan; the fan load torque is described by the expression $T_L = 5 + 0.05\omega + 0.001\omega^2$ where torque is in N-m and speed in rad/s. The motor has $k_a\phi = k_T\phi = 2.42$. $R_a = 0.2\ \Omega$, and the inductance is negligible. If the motor voltage is 50 V, what will be the speed of rotation of the motor and fan?

17.31 A separately excited DC motor has the following parameters:

$R_a = 0.1\ \Omega$	$R_f = 100\ \Omega$	$L_a = 0.2$ H
$L_f = 0.02$ H	$K_a = 0.8$	$K_f = 0.9$

The motor load is an inertia with $J = 0.5$ kg-m^2, $b =$ 2 N-m-rad/s. No external load torque is applied.

a. Sketch a diagram of the system and derive the (three) differential equations.
b. Sketch a simulation block diagram of the system (you should have three integrators).
c. Code the diagram using Simulink.
d. Run the following simulations:

Armature control. Assume a constant field with $V_f = 100$ V; now simulate the response of the system when the armature voltage changes in step fashion from 50 V to 75 V. Save and plot the current and angular speed responses.
Field control. Assume a constant armature voltage with $V_a = 100$ V; now simulate the response of the system when the field voltage changes in step fashion from 75 V to 50 V. This procedure is called *field weakening*. Save and plot the current and angular speed responses.

17.32 Determine the transfer functions from *input voltage* to *angular velocity* and from *load torque* to *angular velocity* for a PM DC motor rigidly connected to an inertial load. Assume resistance and inductance parameters R_a, L_a let the armature constant be k_a. Assume ideal energy conversion, so that $k_a = k_T$. The motor has inertia J_m and damping coefficient b_m, and is rigidly connected to an inertial load with inertia J and damping coefficient b. The load torque T_L acts on the load inertia to oppose the magnetic torque.

17.33 Assume that the coupling between the motor and the inertial load of the proceding problem is flexible (e.g., a long shaft). This can be modeled by adding a torsional spring between the motor inertia and the load inertia. Now we can no longer lump the two inertias and damping coefficients as if they were one, but we need to write separate equations for the two inertias. In total, there will be three equations in this system:

1. the motor electrical equation
2. the motor mechanical equation (J_m and B_m)
3. the load mechanical equation (J and B)

a. Sketch a diagram of the system.
b. Use free-body diagrams to write each of the two mechanical equations. Set up the equations in matrix form.
c. Compute the transfer function from input voltage to load inertia speed using the method of determinants.

17.34 A wound DC motor is connected in both a shunt and series configuration. Assume generic resistance and inductance parameters R_a, R_f, L_a, L_f; let the field magnetization constant be k_f and the armature constant be k_a. Assume ideal energy conversion, so that $k_a = k_T$. The motor has inertia J_m and damping coefficient b_m, and is rigidly connected to an inertial load with inertia J and damping coefficient b.

a. Sketch a system-level diagram of the two configurations that illustrates both the mechanical and electrical systems.
b. Write an expression for the torque-speed curve of the motor in each configuration.
c. Write the differential equations of the motor-load system in each configuration.
d. Determine whether the differential equations of each system are linear; if one (or both) are nonlinear, could they be made linear with some

simple assumption? Explain clearly under what conditions this would be the case.

Section 3: AC Synchronous Machines

17.35 A non-salient pole, Y-connected, three-phase, two-pole synchronous machine has a synchronous reactance of 7 Ω and negligible resistance and rotational losses. One point on the open-circuit characteristic is given by $V_o = 400$ V (phase voltage) for a field current of 3.32 A. The machine is to be operated as a motor, with a terminal voltage of 400 V (phase voltage). The armature current is 50 A, with power factor 0.85, leading. Determine E_b, field current, torque developed, and power angle δ.

17.36 A factory load of 900 kW at 0.6 power factor lagging is to be increased by the addition of a synchronous motor that takes 450 kW. At what power factor must this motor operate, and what must be its KVA input if the overall power factor is to be 0.9 lagging?

17.37 A non-salient pole, Y-connected, three-phase, two-pole synchronous generator is connected to a 400-V (line to line), 60-Hz, three-phase line. The stator impedance is $0.5 + j1.6\ \Omega$ (per phase). The generator is delivering rated current (36 A) at unity power factor to the line. Determine the power angle for this load and the value of E_b for this condition. Sketch the phasor diagram, showing $\mathbf{E}_b$, $\mathbf{I}_S$, and $\mathbf{V}_S$.

17.38 A non-salient pole, three-phase, two-pole synchronous motor is connected in parallel with a three-phase, Y-connected load so that the per-phase equivalent circuit is as shown in Figure P17.38. The parallel combination is connected to a 220-V (line to line), 60-Hz, three-phase line. The load current $\mathbf{I}_L$ is 25 A at a power factor of 0.866 inductive. The motor has $X_S = 2\ \Omega$ and is operating with $I_f = 1$ A and $T = 50$ N-m at a power angle of $-30°$. (Neglect all losses for the motor.) Find $\mathbf{I}_S$, P_{in} (to the motor), the overall power factor (i.e., angle between $\mathbf{I}_1$ and $\mathbf{V}_S$), and the total power drawn from the line.

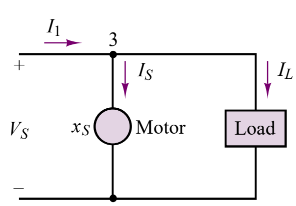


Figure P17.38

17.39 An automotive alternator is rated 500 V-A and 20 V. It delivers its rated V-A at a power factor of 0.85. The resistance per phase is 0.05 Ω, and the field takes 2 A at 12 V. If friction and windage loss is 25 W and core loss is 30 W, calculate the percent efficiency under rated conditions.

17.40 A four-pole, three-phase, Y-connected, non–salient pole synchronous motor has a synchronous reactance of 10 Ω. This motor is connected to a $230\sqrt{3}$ V (line to line), 60-Hz, three-phase line and is driving a load such that $T_{shaft} = 30$ N-m. The line current is 15 A, leading the phase voltage. Assuming that all losses can be neglected, determine the power angle δ and E for this condition. If the load is removed, what is the line current, and is it leading or lagging the voltage?

17.41 A 10-hp, 230-V, 60 Hz, three-phase wye-connected synchronous motor delivers full load at a power factor of 0.8 leading. The synchronous reactance is 6 Ω, the rotational loss is 230 W, and the field loss is 50 W. Find

a. The armature current.
b. The motor efficiency.
c. The power angle.

Neglect the stator winding resistance.

17.42 The circuit of Figure P17.42 represents a voltage regulator for a car alternator. Briefly, explain the function of Q, D, Z, and SCR. Note that unlike other alternators, a car alternator is *not* driven at constant speed.

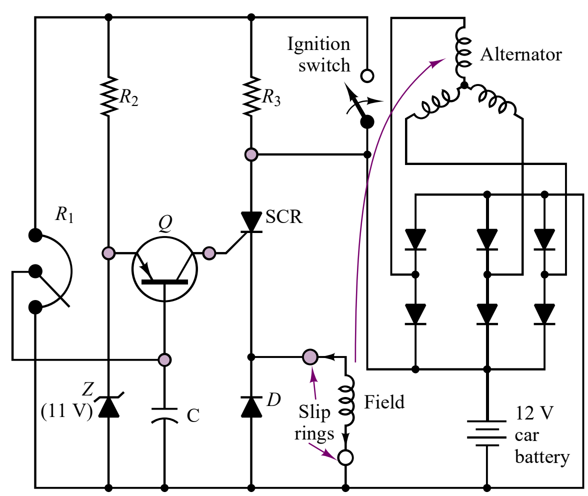


Figure P17.42

17.43 It has been determined by test that the synchronous reactance, X_s, and armature resistance, r_a, of a 2,300-V, 500-KVA, three-phase synchronous generator are 8.0 Ω and 0.1 Ω, respectively. If the machine is operating at rated load and voltage at a power factor of 0.867 lagging, find the generated voltage per phase and the torque angle.

17.44 A 2,000-hp, unity power factor, three-phase, Y-connected, 2,300-V, 30-pole, 60-Hz synchronous motor has a synchronous reactance of 1.95 Ω per phase. Neglect all losses. Find the maximum power and torque.

17.45 A 1,200-V, three-phase, wye-connected synchronous motor takes 110 kW (exclusive of field winding loss) when operated under a certain load at 1,200 rev/min. The back emf of the motor is 2,000 V. The synchronous reactance is 10 Ω per phase, with negligible winding resistance. Find the line current and the torque developed by the motor.

17.46 The per-phase impedance of a 600-V, three-phase, Y-connected synchronous motor is $5 + j50\ \Omega$. The motor takes 24 kW at a leading power factor of 0.707. Determine the induced voltage and the power angle of the motor.

Section 4: AC Induction Machines

17.47 A 74.6-kW, three-phase, 440-V (line to line), four-pole, 60-Hz induction motor has the following (per-phase) parameters referred to the stator circuit:

$$R_S = 0.06\ \Omega \qquad X_S = 0.3\ \Omega \qquad X_m = 5\ \Omega$$
$$R_R = 0.08\ \Omega \qquad X_R = 0.3\ \Omega$$

The no-load power input is 3,240 W at a current of 45 A. Determine the line current, the input power, the developed torque, the shaft torque, and the efficiency at $s = 0.02$.

17.48 A 60-Hz, four-pole, Y-connected induction motor is connected to a 400-V (line to line), three-phase, 60-Hz line. The equivalent circuit parameters are:

$$R_S = 0.2\ \Omega \qquad R_R = 0.1\ \Omega$$
$$X_S = 0.5\ \Omega \qquad X_R = 0.2\ \Omega$$
$$X_m = 20\ \Omega$$

When the machine is running at 1,755 rev/min, the total rotational and stray-load losses are 800 W. Determine the slip, input current, total input power, mechanical power developed, shaft torque, and efficiency.

17.49 A three-phase, 60-Hz induction motor has eight poles and operates with a slip of 0.05 for a certain load. Determine

a. The speed of the rotor with respect to the stator.
b. The speed of the rotor with respect to the stator magnetic field.
c. The speed of the rotor magnetic field with respect to the rotor.
d. The speed of the rotor magnetic field with respect to the stator magnetic field.

17.50 A three-phase, two-pole, 400-V (per phase), 60-Hz induction motor develops 37 kW (total) of mechanical power (P_m) at a certain speed. The rotational loss at this speed is 800 W (total). (Stray-load loss is negligible.)

a. If the total power transferred to the rotor is 40 kW, determine the slip and the output torque.
b. If the total power into the motor (P_{in}) is 45 kW and R_S is 0.5 Ω find I_S and the power factor.

17.51 The nameplate speed of a 25-Hz induction motor is 720 rev/min. If the speed at no load is 745 rev/min, calculate:

a. The slip
b. The percent regulation

17.52 The name plate of a squirrel-cage four-pole induction motor has the following information: 25 hp, 220 volts, three-phase, 60 Hz, 830 rev/min, 64A line current. If the motor draws 20,800 watts when operating at full load, calculate:

a. Slip
b. Percent regulation if the no-load speed is 895 rpm
c. Power factor
d. Torque
e. Efficiency

17.53 A 60-Hz, four-pole, Y-connected induction motor is connected to a 200-V (line to line), three-phase, 60 Hz line. The equivalent circuit parameters are:

$R_S = 0.48\ \Omega$ Rotational loss torque = 3.5 N-m
$X_S = 0.8\ \Omega$ $R_R = 0.42\ \Omega$ (referred to the stator)
$X_m = 30\ \Omega$ $X_R = 0.8\ \Omega$ (referred to the stator)

The motor is operating at slip $s = 0.04$. Determine the input current, input power, mechanical power, and shaft torque (assuming that stray-load losses are negligible).

17.54

a. A three-phase, 220-V, 60-Hz induction motor runs at 1,140 rev/min. Determine the number of poles (for minimum slip), the slip, and the frequency of the rotor currents.
b. To reduce the starting current, a three-phase squirrel-cage induction motor is started by reducing the line voltage to $V_s/2$. By what factor are the starting torque and the starting current reduced?

17.55 A six-pole induction motor for vehicle traction has a 50-kW rating and is 85 percent efficient. If the supply is 220 V at 60 Hz, compute the motor speed and torque at a slip of 0.04.

17.56 An AC induction machine has six poles and is designed for 60-Hz, 240-V (rms) operation. When the machine operates with 10 percent slip, it produces 60 N-m of torque.

a. The machine is now used in conjunction with a friction load which opposes a torque of 50 N-m. Determine the speed and slip of the machine when used with the above mentioned load.
b. If the machine has an efficiency of 92 percent, what minimum rms current is required for operation with the load of part a)?

[*Hint:* you may assume that the speed torque curve is approximately linear in the region of interest.]

17.57 A blocked-rotor test was performed on a 5-hp, 220-V, four-pole, 60 Hz, three-phase induction motor. The following data were obtained: $V = 48$ V, $I = 18$ A, $P = 610$ W. Calculate:

a. The equivalent stator resistance per phase, R_S

b. The equivalent rotor resistance per phase, R_R

c. The equivalent blocked-rotor reactance per phase, X_R

17.58 Calculate the starting torque of the motor of Problem 17.58 when it is started at:

a. 220 V

b. 110 V

The starting torque equation is:

$$T = \frac{m}{\omega_e} \cdot V_S^2 \cdot \frac{R_R}{(R_R + R_S)^2 + (X_R + X_S)^2}$$

17.59 Find the speed of the rotating field of a six-pole, three-phase motor connected to (a) a 60-Hz line and (b) a 50-Hz line, in rev/min and rad/s.

17.60 A six-pole, three-phase, 440-V, 60-Hz induction motor has the following model impedances:

$$R_S = 0.8\ \Omega \qquad X_S = 0.7\ \Omega$$
$$R_R = 0.3\ \Omega \qquad X_R = 0.7\ \Omega$$
$$X_m = 35\ \Omega$$

Calculate the input current and power factor of the motor for a speed of 1,200 rev/min.

17.61 An eight-pole, three-phase, 220-V, 60-Hz induction motor has the following model impedances:

$$R_S = 0.78\ \Omega \qquad X_S = 0.56\ \Omega \qquad X_m = 32\ \Omega$$
$$R_R = 0.28\ \Omega \qquad X_R = 0.84\ \Omega$$

Find the input current and power factor of this motor for $s = 0.02$.

17.62 A nameplate is given in Example 17.2. Find the rated torque, rated volt amperes, and maximum continuous output power for this motor.

17.63 A 3-phase induction motor, at rated voltage and frequency, has a starting torque of 140 percent and a maximum torque of 210 percent of full-load torque. Neglect stator resistance and rotational losses and assume constant rotor resistance. Determine:

a. The slip at full load.

b. The slip at maximum torque.

c. The rotor current at starting as a percent of full-load rotor current.