Department of Electrical and Electronics Engineering

Higher Colleges of Technology

EEL 3003/ELE-3323 Electric Machines

LO 1: Analyze the Construction and Operational Principles of Electric Machines

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HIGHER COLLEGES OF TECHNOLOGY

CHAPTER 16

Principles of Electromechanics

The objective of this chapter is to introduce the fundamental notions of electromechanical energy conversion, leading to an understanding of the operation of various electromechanical transducers. The chapter also serves as an introduction to the material on electric machines to be presented in Chapters 17 and 18. The foundations for the material introduced in this chapter will be found in the circuit analysis chapters (1–7). In addition, the material on power electronics (Chapter 11) is also relevant, especially with reference to Chapters 17 and 18.

The subject of electromechanical energy conversion is one that should be of particular interest to the *non–electrical* engineer, because it forms one of the important points of contact between electrical engineering and other engineering disciplines. Electromechanical transducers are commonly used in the design of industrial and aerospace control systems and in biomedical applications, and they form the basis of many common appliances. In the course of our exploration of electromechanics, we shall illustrate the operation of practical devices, such as loudspeakers, relays, solenoids, sensors for the measurement of position and velocity, and other devices of practical interest.

Upon completion of the chapter, you should be able to:

- Analyze simple magnetic circuits, to determine electrical and mechanical performance and energy requirements.

- Size a relay or solenoid for a given application.
- Describe the energy-conversion process in electromechanical systems.
- Perform a simplified linear analysis of electromechanical transducers.

16.1 ELECTRICITY AND MAGNETISM

The notion that the phenomena of electricity and magnetism are interconnected was first proposed in the early 1800s by H. C. Oersted, a Danish physicist. Oersted showed that an electric current produces magnetic effects (more specifically, a magnetic field). Soon after, the French scientist André Marie Ampère expressed this relationship by means of a precise formulation, known as *Ampère's law*. A few years later, the English scientist Faraday illustrated how the converse of Ampère's law also holds true, that is, that a magnetic field can generate an electric field; in short, *Faraday's law* states that a changing magnetic field gives rise to a voltage. We shall undertake a more careful examination of both Ampère's and Faraday's laws in the course of this chapter.

As will be explained in the next few sections, the magnetic field forms a necessary connection between electrical and mechanical energy. Ampère's and Faraday's laws will formally illustrate the relationship between electric and magnetic fields, but it should already be evident from your own individual experience that the magnetic field can also convert magnetic energy to mechanical energy (for example, by lifting a piece of iron with a magnet). In effect, the devices we commonly refer to as *electromechanical* should more properly be referred to as electro*magneto*mechanical, since they almost invariably operate through a conversion from electrical to mechanical energy (or vice versa) by means of a magnetic field. Chapters 16 through 18 are concerned with the use of electricity and magnetic materials for the purpose of converting electrical energy to mechanical, and back.

The Magnetic Field and Faraday's Law

The quantities used to quantify the strength of a magnetic field are the **magnetic flux,** ϕ, in units of **webers** (Wb); and the **magnetic flux density, B,** in units of webers per square meter (Wb/m^2), or **teslas** (T). The latter quantity, as well as the associated **magnetic field intensity, H** (in units of amperes per meter, or A/m) are vectors.[1] Thus, the density of the magnetic flux and its intensity are in general described in vector form, in terms of the components present in each spatial direction (e.g., on the x, y, and z axes). In discussing magnetic flux density and field intensity in this chapter and the next, we shall almost always assume that the field is a *scalar field*, that is, that it lies in a single spatial direction. This will simplify many explanations.

It is customary to represent the magnetic field by means of the familiar *lines of force* (a concept also due to Faraday); we visualize the strength of a magnetic field by observing the density of these lines in space. You probably know from a previous course in physics that such lines are closed in a magnetic field, that is, that they form continuous loops exiting at a magnetic north pole (by definition)

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[1] We will use the boldface symbols **B** and **H** to denote the vector forms of B and H; the standard typeface will represent the scalar flux density or field intensity in a given direction.

and entering at a magnetic south pole. The relative strengths of the magnetic fields generated by two magnets could be depicted as shown in Figure 16.1.

Magnetic fields are generated by electric charge in motion, and their effect is measured by the force they exert on a moving charge. As you may recall from previous physics courses, the vector force **f** exerted on a charge of q moving at velocity **u** in the presence of a magnetic field with flux density **B** is given by the equation

$$\mathbf{f} = q\mathbf{u} \times \mathbf{B} \qquad \textbf{(16.1)}$$

where the symbol $\times$ denotes the (vector) cross product. If the charge is moving at a velocity **u** in a direction that makes an angle θ with the magnetic field, then the magnitude of the force is given by

$$f = quB \sin\theta \qquad \textbf{(16.2)}$$

and the direction of this force is at right angles with the plane formed by the vectors **B** and **u**. This relationship is depicted in Figure 16.2.

The magnetic flux, ϕ, is then defined as the integral of the flux density over some surface area. For the simplified (but often useful) case of magnetic flux lines perpendicular to a cross-sectional area A, we can see that the flux is given by the following integral:

$$\phi = \int_A B\,dA \qquad \textbf{(16.3)}$$

in webers (Wb), where the subscript A indicates that the integral is evaluated over the surface A. Furthermore, if the flux were to be uniform over the cross-sectional area A (a simplification that will be useful), the preceding integral could be approximated by the following expression:

$$\boxed{\phi = B \cdot A} \qquad \textbf{(16.4)}$$

Figure 16.3 illustrates this idea, by showing hypothetical magnetic flux lines traversing a surface, delimited in the figure by a thin conducting wire.

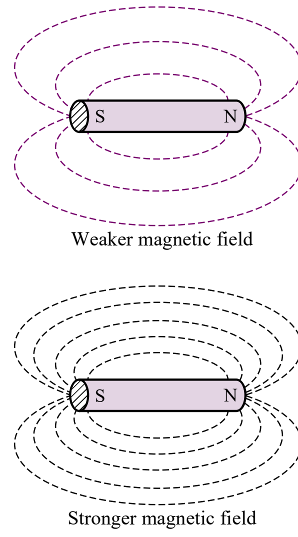


Figure 16.1 Lines of force in a magnetic field

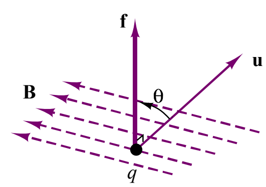


Figure 16.2 Charge moving in a constant magnetic field

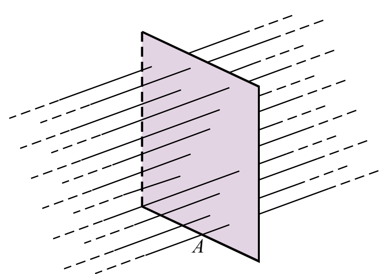
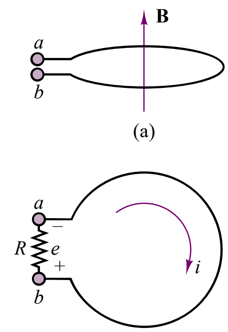


Figure 16.3 Magnetic flux lines crossing a surface

Faraday's law states that if the imaginary surface A were bounded by a conductor—for example, the thin wire of Figure 16.3—then a *changing* magnetic field would induce a voltage, and therefore a current, in the conductor. More



(a)

Current generating a magnetic flux opposing the increase in flux due to **B**

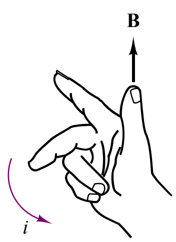
(b)

Figure 16.4 Flux direction

precisely, Faraday's law states that a time-varying flux causes an induced **electromotive force,** or **emf,** e, as follows:

$$e = -\frac{d\phi}{dt} \tag{16.5}$$

A little discussion is necessary at this point to explain the meaning of the minus sign in equation 16.5. Consider the one-turn coil of Figure 16.4, which forms a circular cross-sectional area, in the presence of a magnetic field with flux density **B** oriented in a direction perpendicular to the plane of the coil. If the magnetic field, and therefore the flux within the coil, is constant, no voltage will exist across terminals a and b; if, however, the flux were increasing and terminals a and b were connected—for example, by means of a resistor, as indicated in Figure 16.4(b)—current would flow in the coil in such a way that *the magnetic flux generated by the current would oppose the increasing flux.* Thus, the flux induced by such a current would be in the direction opposite to that of the original flux density vector, **B.** This principle is known as **Lenz's law.** The reaction flux would then point downward in Figure 16.4(a), or into the page in Figure 16.4(b). Now, by virtue of the **right-hand rule,** this reaction flux would induce a current flowing clockwise in Figure 16.4(b), that is, a current that flows out of terminal b and into terminal a. The resulting voltage across the hypothetical resistor R would then be negative. If, on the other hand, the original flux were decreasing, current would be induced in the coil so as to reestablish the initial flux; but this would mean that the current would have to generate a flux in the upward direction in Figure 16.4(a) (or out of the page in Figure 16.4(b)). Thus, the resulting voltage would change sign.



Right-hand rule

The polarity of the induced voltage can usually be determined from physical considerations; therefore the minus sign in equation 16.5 is usually left out. We will use this convention throughout the chapter.

In practical applications, the size of the voltages induced by the changing magnetic field can be significantly increased if the conducting wire is coiled many times around, so as to multiply the area crossed by the magnetic flux lines many times over. For an N-turn coil with cross-sectional area A, for example, we have the emf

$$e = N\frac{d\phi}{dt} \tag{16.6}$$

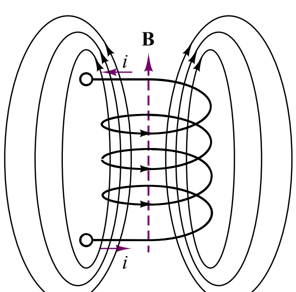


Figure 16.5 Concept of flux linkage

Figure 16.5 shows an N-turn coil *linking* a certain amount of magnetic flux; you can see that if N is very large and the coil is tightly wound (as is usually the case in the construction of practical devices), it is not unreasonable to presume that each turn of the coil links the same flux. It is convenient, in practice, to define the **flux linkage,** λ, as

$$\lambda = N\phi \tag{16.7}$$

so that

$$\boxed{e = \frac{d\lambda}{dt}} \tag{16.8}$$

Note that equation 16.8, relating the derivative of the flux linkage to the induced emf, is analogous to the equation describing current as the derivative of charge:

$$i = \frac{dq}{dt} \tag{16.9}$$

In other words, flux linkage can be viewed as the dual of charge in a circuit analysis sense, provided that we are aware of the simplifying assumptions just stated in the preceding paragraphs, namely, a uniform magnetic field perpendicular to the area delimited by a tightly wound coil. These assumptions are not at all unreasonable when applied to the inductor coils commonly employed in electric circuits.

What, then, are the physical mechanisms that can cause magnetic flux to change, and therefore to induce an electromotive force? Two such mechanisms are possible. The first consists of physically moving a permanent magnet in the vicinity of a coil—for example, so as to create a time-varying flux. The second requires that we first produce a magnetic field by means of an electric current (how this can be accomplished is discussed later in this section) and then vary the current, thus varying the associated magnetic field. The latter method is more practical in many circumstances, since it does not require the use of permanent magnets and allows variation of field strength by varying the applied current; however, the former method is conceptually simpler to visualize. The voltages induced by a moving magnetic field are called **motional voltages;** those generated by a time-varying magnetic field are termed **transformer voltages.** We shall be interested in both in this chapter, for different applications.

In the analysis of linear circuits in Chapter 4, we implicitly assumed that the relationship between flux linkage and current was a linear one:

$$\lambda = Li \tag{16.10}$$

so that the effect of a time-varying current was to induce a transformer voltage across an inductor coil, according to the expression

$$v = L\frac{di}{dt} \tag{16.11}$$

This is, in fact, the defining equation for the ideal **self-inductance,** L. In addition to self-inductance, however, it is also important to consider the **magnetic coupling** that can occur between neighboring circuits. Self-inductance measures the voltage induced in a circuit by the magnetic field generated by a current flowing in the same circuit. It is also possible that a second circuit in the vicinity of the first may experience an induced voltage as a consequence of the magnetic field generated in the first circuit. As we shall see in Section 16.4, this principle underlies the operation of all transformers.

Self- and Mutual Inductance

Figure 16.6 depicts a pair of coils, one of which, L_1, is excited by a current, i_1, and therefore develops a magnetic field and a resulting induced voltage, v_1. The second coil, L_2, is not energized by a current, but links some of the flux generated by the current i_1 around L_1 because of its close proximity to the first coil. The magnetic coupling between the coils established by virtue of their proximity is described by a quantity called **mutual inductance** and defined by the symbol M. The mutual inductance is defined by the equation

$$v_2 = M\frac{di_1}{dt} \tag{16.12}$$

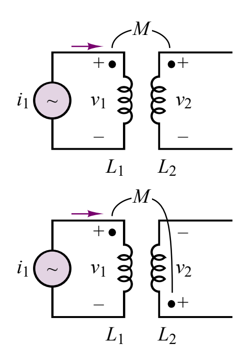


Figure 16.6 Mutual inductance

The dots shown in the two figures indicate the polarity of the coupling between the coils. If the dots are at the same end of the coils, the voltage induced in coil 2 by a current in coil 1 has the same polarity as the voltage induced by the same current in coil 1; otherwise, the voltages are in opposition, as shown in the lower part of Figure 16.6. Thus, the presence of such dots indicates that magnetic coupling is present between two coils. It should also be pointed out that if a current (and therefore a magnetic field) were present in the second coil, an additional voltage would be induced across coil 1. The voltage induced across a coil is, in general, equal to the sum of the voltages induced by self-inductance and mutual inductance.

FOCUS ON MEASUREMENTS

Linear Variable Differential Transformer (LVDT)

FIND IT ON THE WEB

The **linear variable differential transformer** (LVDT) is a displacement transducer based on the mutual inductance concept just discussed. Figure 16.7 shows a simplified representation of an LVDT, which consists of a primary coil, subject to AC excitation (v_{ex}), and of a pair of identical secondary coils, which are connected so as to result in the output voltage

$$v_{out} = v_1 - v_2$$

The ferromagnetic core between the primary and secondary coils can be displaced in proportion to some external motion, x, and determines the magnetic coupling between primary and secondary coils. Intuitively, as the core is displaced upward, greater coupling will occur between the primary coil and the top secondary coil, thus inducing a greater voltage in the top secondary coil. Hence, $v_{out} > 0$ for positive displacements. The converse is true for negative displacements. More formally, if the primary coil has resistance R_p and self-inductance L_p, we can write

$$iR_p + L_p \frac{di}{dt} = v_{ex}$$

and the voltages induced in the secondary coils are given by

$$v_1 = M_1 \frac{di}{dt}$$

$$v_2 = M_2 \frac{di}{dt}$$

so that

$$v_{out} = (M_1 - M_2) \frac{di}{dt}$$

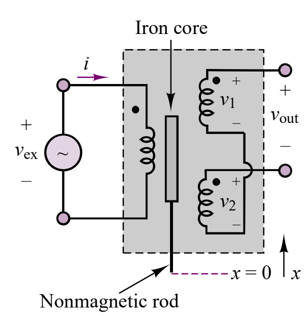


Figure 16.7 Linear variable differential transformer

where M_1 and M_2 are the mutual inductances between the primary and the respective secondary coils. It should be apparent that each of the mutual inductances is dependent on the position of the iron core. For example, with the core at the *null position*, $M_1 = M_2$ and $v_{out} = 0$. The LVDT is typically designed so that $M_1 - M_2$ is linearly related to the displacement of the core, x.

Because the excitation is by necessity an AC signal (why?), the output voltage is actually given by the difference of two sinusoidal voltages at the same frequency, and is therefore itself a sinusoid, whose amplitude and phase depend on the displacement, x. Thus, v_{out} is an *amplitude-modulated* (AM) signal, similar to the one discussed in "Focus on Measurements: Capacitive Displacement Transducer" in Chapter 4. To recover a signal proportional to the actual displacement, it is therefore necessary to use a demodulator circuit, such as the one discussed in "Focus on Measurements: Peak Detector for Capacitive Displacement Transducer" in Chapter 8.

In practical electromagnetic circuits, the self-inductance of a circuit is not necessarily constant; in particular, the inductance parameter, L, is not constant, in general, but depends on the strength of the magnetic field intensity, so that it will not be possible to use such a simple relationship as $v = L\, di/dt$, with L constant. If we revisit the definition of the transformer voltage,

$$e = N\frac{d\phi}{dt} \tag{16.13}$$

we see that in an inductor coil, the inductance is given by

$$L = \frac{N\phi}{i} = \frac{\lambda}{i} \tag{16.14}$$

This expression implies that the relationship between current and flux in a magnetic structure is linear (the inductance being the slope of the line). In fact, the properties of ferromagnetic materials are such that the flux-current relationship is nonlinear, as we shall see in Section 16.3, so that the simple linear inductance parameter used in electric circuit analysis is not adequate to represent the behavior of the magnetic circuits of the present chapter. In any practical situation, the relationship between the flux linkage, λ, and the current is nonlinear, and might be described by a curve similar to that shown in Figure 16.8. Whenever the i-λ curve is not a straight line, it is more convenient to analyze the magnetic system in terms of energy calculations, since the corresponding circuit equation would be nonlinear.

In a magnetic system, the energy stored in the magnetic field is equal to the integral of the instantaneous power, which is the product of voltage and current, just as in a conventional electrical circuit:

$$W_m = \int ei\, dt' \tag{16.15}$$

However, in this case, the voltage corresponds to the induced emf, according to Faraday's law:

$$e = \frac{d\lambda}{dt} = N\frac{d\phi}{dt} \tag{16.16}$$

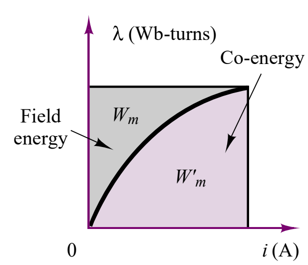


Figure 16.8 Relationship between flux linkage, current, energy, and co-energy.

and is therefore related to the rate of change of the magnetic flux. The energy stored in the magnetic field could therefore be expressed in terms of the current by the integral

$$W_m = \int ei\, dt' = \int \frac{d\lambda}{dt} i\, dt' = \int i\, d\lambda' \qquad \textbf{(16.17)}$$

It should be straightforward to recognize that this energy is equal to the area above the λ-i curve of Figure 16.8. From the same figure, it is also possible to define a fictitious (but sometimes useful) quantity called **co-energy,** equal to the area under the curve and identified by the symbol W'_m. From the figure, it is also possible to see that the co-energy can be expressed in terms of the stored energy by means of the following relationship:

$$W'_m = i\lambda - W_m \qquad \textbf{(16.18)}$$

Example 16.1 illustrates the calculation of energy, co-energy, and induced voltage using the concepts developed in these paragraphs.

The calculation of the energy stored in the magnetic field around a magnetic structure will be particularly useful later in the chapter, when the discussion turns to practical electromechanical transducers and it will be necessary to actually compute the forces generated in magnetic structures.

EXAMPLE 16.1 Energy and Co-Energy Calculation for an Inductor

Problem

Compute the energy, co-energy, and incremental linear inductance for an iron core inductor with a given λ-i relationship. Also compute the voltage across the terminals given the current through the coil.

Solution

Known Quantities: λ-i relationship; nominal value of λ; coil resistance; coil current.

Find: W_m; W'_m; L_Δ; v.

Schematics, Diagrams, Circuits, and Given Data: $i = \lambda + 0.5\lambda^2$; $\lambda_0 = 0.5$ V · s; $R = 1\ \Omega$; $i(t) = 0.625 + 0.01 \sin(400t)$.

Assumptions: Assume that the magnetic equation can be linearized and use the linear model in all circuit calculations.

Analysis:

1. *Calculation of energy and co-energy.* From equation 16.17, we calculate the energy as follows.

$$W_m = \int_0^\lambda i(\lambda')d\lambda' = \frac{\lambda^2}{2} + \frac{\lambda^3}{6}$$

The above expression is valid in general; in our case, the inductor is operating at a nominal flux linkage $\lambda_0 = 0.5$ V-s and we can therefore evaluate the energy to be:

$$W_m(\lambda = \lambda_0) = \left(\frac{\lambda^2}{2} + \frac{\lambda^3}{6}\right)\bigg|_{\lambda=0.5} = 0.1458 \text{ J}$$

Thus, after equation 16.18, the co-energy is given by:

$$W'_m = i\lambda - W_m$$

where

$$i = \lambda + 0.5\lambda^2 = 0.625 \text{ A}$$

and

$$W'_m = i\lambda - W_m = (0.625)(0.5) - (0.1458) = 0.1667 \text{ J}$$

2. *Calculation of incremental inductance.* If we know the nominal value of flux linkage (i.e., the operating point), we can calculate a linear inductance L_Δ, valid around values of λ close to the operating point λ_0:

$$L_\Delta = \left.\frac{d\lambda}{di}\right|_{\lambda=\lambda_0} = \left.\frac{1}{1+\lambda}\right|_{\lambda=0.5} = 0.667 \text{ H}$$

   The above expressions can be used to analyze the circuit behavior of the inductor when the flux linkage is around 0.5 V · s, or, equivalently, when the current through the inductor is around 0.625 A.

3. *Circuit analysis using linearized model of inductor.* We can use the incremental linear inductance calculated above to compute the voltage across the inductor in the presence of a current $i(t) = 0.625 + 0.01\sin(400t)$. Using the basic circuit definition of an inductor with series resistance R, the voltage across the inductor is given by:

$$v = iR + L_\Delta \frac{di}{dt} = [0.625 + 0.01\sin(400t)] \times 1 + 0.667 \times 4\cos(400t)$$
$$= 0.625 + 0.01\sin(400t) + 2.668\cos(400t) = 0.625 + 2.668\sin(400t + 89.8^\circ)$$

Comments: The linear approximation in this case is not a bad one: the small sinusoidal current is oscillating around a much larger average current. In this type of situation, it is reasonable to assume that the inductor behaves linearly. This example explains why the linear inductor model introduced in Chapter 4 is an acceptable approximation in most circuit analysis problems.

Ampère's Law

As explained in the previous section, Faraday's law is one of two fundamental laws relating electricity to magnetism. The second relationship, which forms a counterpart to Faraday's law, is **Ampère's law.** Qualitatively, Ampère's law states that the magnetic field intensity, **H,** in the vicinity of a conductor is related to the current carried by the conductor; thus Ampère's law establishes a dual relationship with Faraday's law.

In the previous section, we described the magnetic field in terms of its flux density, **B,** and flux ϕ. To explain Ampère's law and the behavior of magnetic materials, we need to define a relationship between the magnetic field intensity, **H,** and the flux density, **B.** These quantities are related by:

$$\mathbf{B} = \mu\mathbf{H} = \mu_r\mu_0\mathbf{H} \qquad \text{Wb/m}^2 \text{ or T} \tag{16.19}$$

where the parameter μ is a scalar constant for a particular physical medium (at least, for the applications we consider here) and is called the **permeability** of the medium. The permeability of a material can be factored as the product of the permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ H/m, times the relative permeability, μ_r, which varies greatly according to the medium. For example, for air and for most electrical conductors and insulators, μ_r is equal to 1. For ferromagnetic materials, the value of μ_r can take values in the hundreds or thousands. The size of μ_r represents a measure of the magnetic properties of the material. A consequence of Ampère's law is that, the larger the value of μ, the smaller the current required to produce a large flux density in an electromagnetic structure. Consequently, many electromechanical devices make use of ferromagnetic materials, called iron cores, to enhance their magnetic properties. Table 16.1 gives approximate values of μ_r for some common materials.

Table 16.1 Relative permeabilities for common materials

Material	μ_r
Air	1
Permalloy	100,000
Cast steel	1,000
Sheet steel	4,000
Iron	5,195

Conversely, the reason for introducing the magnetic field intensity is that it is independent of the properties of the materials employed in the construction of magnetic circuits. Thus, a given magnetic field intensity, **H,** will give rise to different flux densities in different materials. It will therefore be useful to define *sources* of magnetic energy in terms of the magnetic field intensity, so that different magnetic structures and materials can then be evaluated or compared for a given source. In analogy with electromotive force, this "source" will be termed **magnetomotive force (mmf).** As stated earlier, both the magnetic flux density and field intensity are vector quantities; however, for ease of analysis, scalar fields will be chosen by appropriately selecting the orientation of the fields, wherever possible.

Ampère's law states that the integral of the vector magnetic field intensity, **H,** around a closed path is equal to the total current linked by the closed path, i:

$$\oint \mathbf{H} \cdot d\mathbf{l} = \sum i \tag{16.20}$$

where $d\mathbf{l}$ is an increment in the direction of the closed path. If the path is in the same direction as the direction of the magnetic field, we can use scalar quantities to state that

$$\int H\,dl = \sum i \tag{16.21}$$

Figure 16.9 illustrates the case of a wire carrying a current i, and of a circular path of radius r surrounding the wire. In this simple case, you can see that the magnetic field intensity, **H,** is determined by the familiar right-hand rule. This rule states that if the direction of the current i points in the direction of the thumb of one's right hand, the resulting magnetic field encircles the conductor in the direction in which the other four fingers would encircle it. Thus, in the case of Figure 16.9, the closed-path integral becomes equal to $H \cdot (2\pi r)$, since the path and the magnetic field are in the same direction, and therefore the magnitude of the magnetic field intensity is given by

$$H = \frac{i}{2\pi r} \tag{16.22}$$

Now, the magnetic field intensity is unaffected by the material surrounding the conductor, but the flux density depends on the material properties, since $B = \mu H$. Thus, the density of flux lines around the conductor would be far greater in the presence of a magnetic material than if the conductor were surrounded by air. The field generated by a single conducting wire is not very strong; however, if we arrange the wire into a tightly wound coil with many turns, we can greatly increase

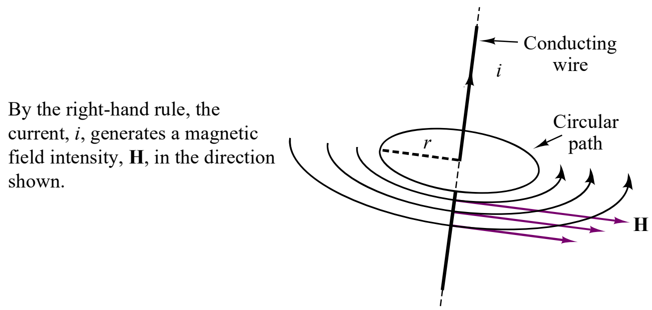


Figure 16.9 Illustration of Ampère's law

the strength of the magnetic field. For such a coil, with N turns, one can verify visually that the lines of force associated with the magnetic field link all of the turns of the conducting coil, so that we have effectively increased the current linked by the flux lines N-fold. The product $N \cdot i$ is a useful quantity in electromagnetic circuits, and is called the magnetomotive force,[2] $\mathcal{F}$ (often abbreviated mmf), in analogy with the electromotive force defined earlier:

$$\boxed{\mathcal{F} = Ni} \qquad \text{ampere-turns } (A \cdot t) \qquad \textbf{(16.23)}$$

Figure 16.10 illustrates the magnetic flux lines in the vicinity of a coil. The magnetic field generated by the coil can be made to generate a much greater flux density if the coil encloses a magnetic material. The most common ferromagnetic materials are steel and iron; in addition to these, many alloys and oxides of iron—as well as nickel—and some artificial ceramic materials called **ferrites** also exhibit magnetic properties. Winding a coil around a ferromagnetic material accomplishes two useful tasks at once: it forces the magnetic flux to be concentrated near the coil and—if the shape of the magnetic material is appropriate—completely confines the flux within the magnetic material, thus forcing the closed path for the flux lines to be almost entirely enclosed within the ferromagnetic material. Typical arrangements are the iron-core inductor and the toroid of Figure 16.11. The flux densities for these inductors are given by the expressions

$$B = \frac{\mu N i}{l} \qquad \text{Flux density for tightly wound circular coil} \qquad \textbf{(16.24)}$$

$$B = \frac{\mu N i}{2\pi r_2} \qquad \text{Flux density for toroidal coil} \qquad \textbf{(16.25)}$$

Intuitively, the presence of a high-permeability material near a source of magnetic flux causes the flux to preferentially concentrate in the high-μ material, rather than in air, much as a conducting path concentrates the current produced by an electric field in an electric circuit. In the course of this chapter, we shall

[2]Note that, although dimensionally equal to amperes, the units of magnetomotive force are ampere-turns.

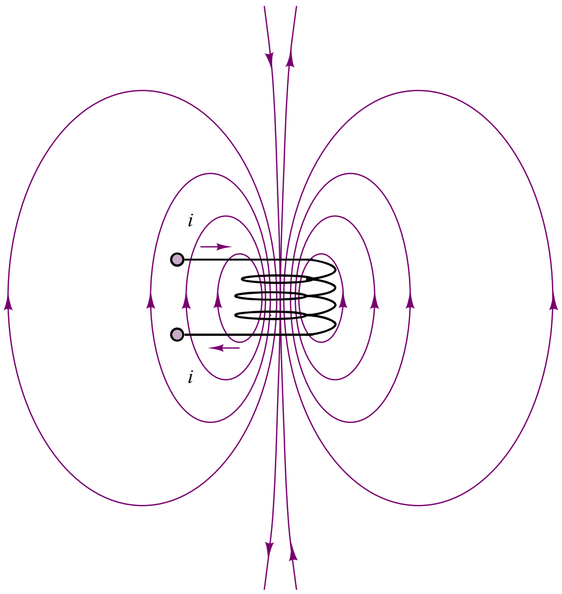


Figure 16.10 Magnetic field in the vicinity of a current-carrying coil

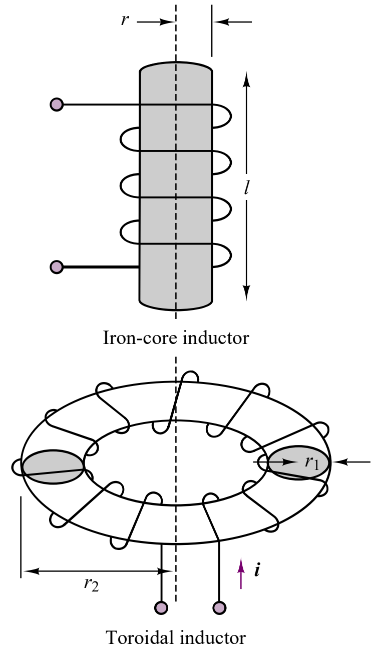


Figure 16.11 Practical inductors

continue to develop this analogy between electric circuits and magnetic circuits. Figure 16.12 depicts an example of a simple electromagnetic structure, which, as we shall see shortly, forms the basis of the practical transformer.

Table 16.2 summarizes the variables introduced thus far in the discussion of electricity and magnetism.

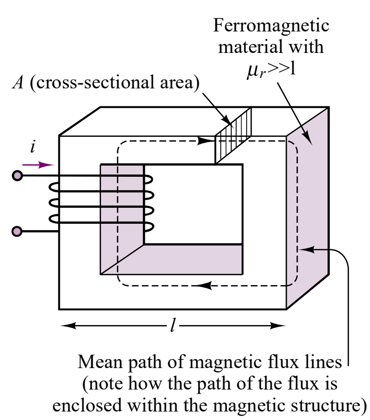


Figure 16.12 A simple electromagnetic structure

Table 16.2 Magnetic variables and units

Variable	Symbol	Units
Current	I	A
Magnetic flux density	B	$\text{Wb/m}^2 = \text{T}$
Magnetic flux	ϕ	Wb
Magnetic field intensity	H	A/m
Electromotive force	e	V
Magnetomotive force	$\mathcal{F}$	$\text{A} \cdot \text{t}$
Flux linkage	λ	$\text{Wb} \cdot \text{t}$

Check Your Understanding

16.1 A coil having 100 turns is immersed in a magnetic field that is varying uniformly from 80 mWb to 30 mWb in 2 seconds. Find the induced voltage in the coil.

16.2 The magnitude of **H** at a radius of 0.5 m from a long linear conductor is $1\ \text{A} \cdot \text{m}^{-1}$. Find the current in the wire.

16.3 The relation between the flux linkages and the current for a magnetic material is given by $\lambda = 6i/(2i + 1)$ Wb · t. Determine the energy stored in the magnetic field for $\lambda = 2$ Wb · t.

16.4 Verify that for the linear case, where the flux is proportional to the mmf, the energy stored in the magnetic field is $\frac{1}{2}Li^2$.

16.2 MAGNETIC CIRCUITS

It is possible to analyze the operation of electromagnetic devices such as the one depicted in Figure 16.12 by means of magnetic equivalent circuits, similar in many respects to the equivalent electrical circuits of the earlier chapters. Before we can present this technique, however, we need to make a few simplifying approximations. The first of these approximations assumes that there exists a **mean path** for the magnetic flux, and that the corresponding mean flux density is approximately constant over the cross-sectional area of the magnetic structure. Thus, a coil wound around a core with cross-sectional area A will have flux density

$$B = \frac{\phi}{A} \tag{16.26}$$

where A is assumed to be perpendicular to the direction of the flux lines. Figure 16.12 illustrates such a mean path and the cross-sectional area, A. Knowing the flux density, we obtain the field intensity:

$$H = \frac{B}{\mu} = \frac{\phi}{A\mu} \tag{16.27}$$

But then, knowing the field intensity, we can relate the mmf of the coil, $\mathcal{F}$, to the product of the magnetic field intensity, H, and the length of the magnetic (mean) path, l, for one leg of the structure:

$$\mathcal{F} = N \cdot i = H \cdot l \tag{16.28}$$

In summary, the mmf is equal to the magnetic flux times the length of the magnetic path, divided by the permeability of the material times the cross-sectional area:

$$\mathcal{F} = \phi \frac{l}{\mu A} \tag{16.29}$$

A review of this formula reveals that the magnetomotive force, $\mathcal{F}$, may be viewed as being analogous to the voltage source in a series electrical circuit, and that the flux, ϕ, is then equivalent to the electrical current in a series circuit and the term $l/\mu A$ to the *magnetic resistance* of one leg of the magnetic circuit. You will note that the term $l/\mu A$ is very similar to the term describing the resistance of a cylindrical conductor of length l and cross-sectional area A, where the permeability, μ, is analogous to the conductivity, σ. The term $l/\mu A$ occurs frequently enough to be assigned the name of **reluctance,** and the symbol $\mathcal{R}$. It is also important to recognize the relationship between the reluctance of a magnetic structure and its inductance. This can be derived easily starting from equation 16.14:

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{N}{i}\frac{Ni}{\mathcal{R}} = \frac{N^2}{\mathcal{R}} \quad \text{(H)} \tag{16.30}$$

In summary, when an N-turn coil carrying a current i is wound around a magnetic core such as the one indicated in Figure 16.12, the mmf, $\mathcal{F}$, generated by the coil produces a flux, ϕ, that is *mostly* concentrated within the core and is assumed to be uniform across the cross section. Within this simplified picture, then, the analysis of a magnetic circuit is analogous to that of resistive electrical circuits. This analogy is illustrated in Table 16.3 and in the examples in this section.

Table 16.3 Analogy between electric and magnetic circuits

Electrical quantity	Magnetic quantity
Electrical field intensity, E, V/m	Magnetic field intensity, H, A · t/m
Voltage, v, V	Magnetomotive force, $\mathcal{F}$, A · t
Current, i, A	Magnetic flux, ϕ, Wb
Current density, J, A/m^2	Magnetic flux density, B, Wb/m^2
Resistance, R, Ω	Reluctance, $\mathcal{R} = l/\mu A$, A · t/Wb
Conductivity, σ, 1/Ω · m	Permeability, μ, Wb/A · m

The usefulness of the magnetic circuit analogy can be emphasized by analyzing a magnetic core similar to that of Figure 16.12, but with a slightly modified geometry. Figure 16.13 depicts the magnetic structure and its equivalent circuit analogy. In the figure, we see that the mmf, $\mathcal{F} = Ni$, excites the magnetic circuit, which is composed of four legs: two of mean path length l_1 and cross-sectional area $A_1 = d_1 w$, and the other two of mean length l_2 and cross section $A_2 = d_2 w$. Thus, the reluctance encountered by the flux in its path around the magnetic core is given by the quantity $\mathcal{R}_{\text{series}}$, with

$$\mathcal{R}_{\text{series}} = 2\mathcal{R}_1 + 2\mathcal{R}_2$$

and

$$\mathcal{R}_1 = \frac{l_1}{\mu A_1} \qquad \mathcal{R}_2 = \frac{l_2}{\mu A_2}$$

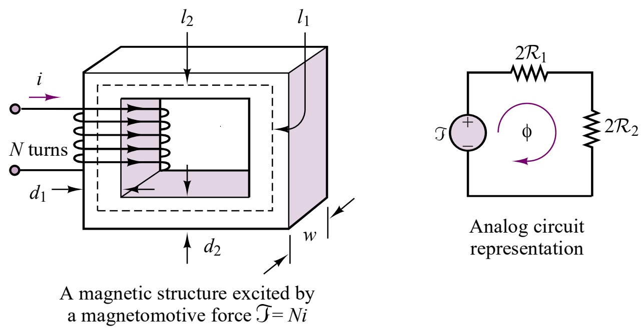


Figure 16.13 Analogy between magnetic and electric circuits

It is important at this stage to review the assumptions and simplifications made in analyzing the magnetic structure of Figure 16.13:

1. All of the magnetic flux is linked by all of the turns of the coil.
2. The flux is confined exclusively within the magnetic core.
3. The density of the flux is uniform across the cross-sectional area of the core.

You can probably see intuitively that the first of these assumptions might not hold true near the ends of the coil, but that it might be more reasonable if the coil is tightly wound. The second assumption is equivalent to stating that the relative permeability of the core is infinitely higher than that of air (presuming that this is the medium surrounding the core): if this were the case, the flux would indeed be confined within the core. It is worthwhile to note that we make a similar assumption when we treat wires in electric circuits as perfect conductors: the conductivity of copper is substantially greater than that of free space, by a factor of approximately 10^{15}. In the case of magnetic materials, however, even for the best alloys, we have a relative permeability only on the order of 10^3 to 10^4. Thus, an approximation that is quite appropriate for electric circuits is not nearly as good in the case of magnetic circuits. Some of the flux in a structure such as those of Figures 16.12 and 16.13 would thus not be confined within the core (this is usually referred to as **leakage flux**). Finally, the assumption that the flux is uniform across the core cannot hold for a finite-permeability medium, but it is very helpful in giving an approximate *mean* behavior of the magnetic circuit.

The magnetic circuit analogy is therefore far from being exact. However, short of employing the tools of electromagnetic field theory and of vector calculus, or advanced numerical simulation software, it is the most convenient tool at the engineer's disposal for the analysis of magnetic structures. In the remainder of this chapter, the approximate analysis based on the electric circuit analogy will be used to obtain approximate solutions to problems involving a variety of useful magnetic circuits, many of which you are already familiar with. Among these will be the loudspeaker, solenoids, automotive fuel injectors, sensors for the measurement of linear and angular velocity and position, and other interesting applications.

EXAMPLE 16.2 Analysis of Magnetic Structure and Equivalent Magnetic Circuit

Problem

Calculate the flux, flux density, and field intensity on the magnetic structure of Figure 16.14.

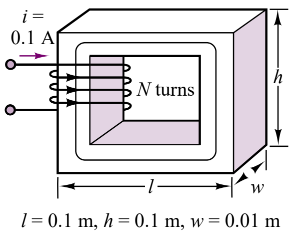


Figure 16.14

Solution

Known Quantities: Relative permeability; number of coil turns; coil current; structure geometry.

Find: ϕ; B; H.

Schematics, Diagrams, Circuits, and Given Data: $\mu_r = 1{,}000$; $N = 500$ turns; $i = 0.1$ A. The magnetic circuit geometry is defined in Figures 16.14 and 16.15.

Assumptions: All magnetic flux is linked by the coil; the flux is confined to the magnetic core; the flux density is uniform.

Analysis:

1. *Calculation of magnetomotive force.* From equation 16.28, we calculate the magnetomotive force:

$$\mathcal{F} = \text{mmf} = Ni = (500 \text{ turns})(0.1 \text{ A}) = 50 \text{ A} \cdot \text{t}$$

2. *Calculation of mean path.* Next, we estimate the mean path of the magnetic flux. On the basis of the assumptions, we can calculate a mean path that runs through the geometric center of the magnetic structure, as shown in Figure 16.15. The path length is:

$$l_c = 4 \times 0.09 \text{ m} = 0.36 \text{ m}$$

The cross sectional area is $A = w^2 = (0.01)^2 = 0.0001 \text{ m}^2$.

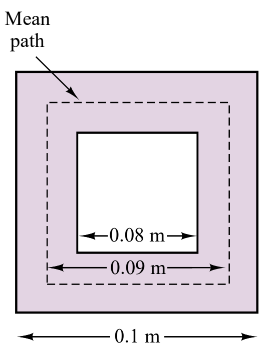


Figure 16.15

3. *Calculation of reluctance.* Knowing the magnetic path length and cross sectional area we can calculate the reluctance of the circuit:

$$\mathcal{R} = \frac{l_c}{\mu A} = \frac{l_c}{\mu_r \mu_0 A} = \frac{0.36}{1{,}000 \times 4\pi \times 10^{-7} \times 0.0001} = 2.865 \times 10^6 \text{ A} \cdot \text{t/Wb}$$

The corresponding equivalent magnetic circuit is shown in Figure 16.16.

4. *Calculation of magnetic flux, flux density and field intensity.* On the basis of the assumptions, we can now calculate the magnetic flux:

$$\phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{50 \text{ A} \cdot \text{t}}{2.865 \times 10^6 \text{ A} \cdot \text{t/Wb}} = 1.75 \times 10^{-5} \text{ Wb}$$

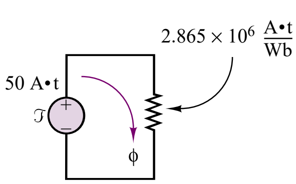


Figure 16.16

the flux density:

$$B = \frac{\phi}{A} = \frac{\phi}{w^2} = \frac{1.75 \times 10^{-5} \text{ Wb}}{0.0001 \text{ m}^2} = 0.175 \text{ Wb/m}^2$$

and the magnetic field intensity:

$$H = \frac{B}{\mu} = \frac{B}{\mu_r \mu_0} = \frac{0.175 \text{ Wb/m}^2}{1{,}000 \times 4\pi \times 10^{-7} \text{ H/m}} = 139 \text{ A} \cdot \text{t/m}$$

Comments: This example has illustrated all the basic calculations that pertain to magnetic structures. Remember that the assumptions stated in this example (and earlier in the chapter) simplify the problem and make its approximate numerical solution possible in a few simple steps. In reality, flux leakage, fringing, and uneven distribution of flux across the structure would require the solution of three-dimensional equations using finite-element methods. These methods are not discussed in this book, but are necessary for practical engineering designs.

The usefulness of these approximate methods is that you can, for example, quickly calculate the approximate magnitude of the current required to generate a given magnetic flux or flux density. You shall soon see how these calculations can be used to determine electromagnetic energy and magnetic forces in practical structures.

The methodology described in this example is summarized in the following methodology box.

FOCUS ON METHODOLOGY

Magnetic Structures and Equivalent Magnetic Circuits

Direct Problem:

Given—The structure geometry and the coil parameters (number of turns, current).

Calculate—The magnetic flux in the structure.

1. Compute the mmf.
2. Determine the length and cross section of the magnetic path for each continuous *leg* or section of the path.
3. Calculate the equivalent reluctance of the *leg*.
4. Generate the equivalent magnetic circuit diagram and calculate the total equivalent reluctance.
5. Calculate the flux, flux density, and magnetic field intensity, as needed.

Inverse Problem:

Given—The desired flux or flux density and structure geometry.

Calculate—The necessary coil current and number of turns.

1. Calculate the total equivalent reluctance of the structure from the desired flux.
2. Generate the equivalent magnetic circuit diagram.
3. Determine the mmf required to establish the required flux.
4. Choose the coil current and number of turns required to establish the desired mmf.

Consider the analysis of the same simple magnetic structure when an **air gap** is present. Air gaps are very common in magnetic structures; in rotating machines, for example, air gaps are necessary to allow for free rotation of the inner core of the machine. The magnetic circuit of Figure 16.17(a) differs from

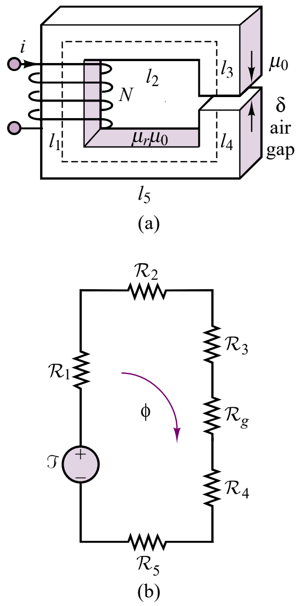


Figure 16.17 (a) Magnetic circuit with air gap; (b) Equivalent representation of magnetic circuit with an air gap

the circuit analyzed in Example 16.2 simply because of the presence of an air gap; the effect of the gap is to break the continuity of the high-permeability path for the flux, adding a high-reluctance component to the equivalent circuit. The situation is analogous to adding a very large series resistance to a series electrical circuit. It should be evident from Figure 16.17(a) that the basic concept of reluctance still applies, although now two different permeabilities must be taken into account.

The equivalent circuit for the structure of Figure 16.17(a) may be drawn as shown in Figure 16.17(b), where $\mathcal{R}_n$ is the reluctance of path l_n, for $n = 1, 2, \ldots, 5$, and $\mathcal{R}_g$ is the reluctance of the air gap. The reluctances can be expressed as follows, if we assume that the magnetic structure has a uniform cross-sectional area, A:

$$\mathcal{R}_1 = \frac{l_1}{\mu_r\mu_0 A} \qquad \mathcal{R}_2 = \frac{l_2}{\mu_r\mu_0 A} \qquad \mathcal{R}_3 = \frac{l_3}{\mu_r\mu_0 A}$$
$$\mathcal{R}_4 = \frac{l_4}{\mu_r\mu_0 A} \qquad \mathcal{R}_5 = \frac{l_5}{\mu_r\mu_0 A} \qquad \mathcal{R}_g = \frac{\delta}{\mu_0 A_g} \tag{16.31}$$

Note that in computing $\mathcal{R}_g$, the length of the gap is given by δ and the permeability is given by μ_0, as expected, but A_g is different from the cross-sectional area, A, of the structure. The reason is that the flux lines exhibit a phenomenon known as **fringing** as they cross an air gap. The flux lines actually *bow out* of the gap defined by the cross section, A, not being contained by the high-permeability material any longer. Thus, it is customary to define an area A_g that is greater than A, to account for this phenomenon. Example 16.3 describes in more detail the procedure for finding A_g and also discusses the phenomenon of fringing.

EXAMPLE 16.3 Magnetic Structure with Air Gaps

Problem

Compute the equivalent reluctance of the magnetic circuit of Figure 16.18 and the flux density established in the bottom bar of the structure.

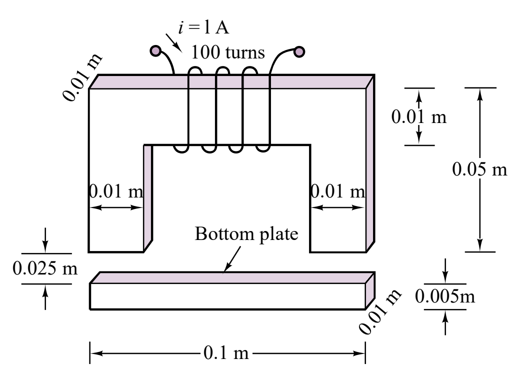


Figure 16.18 Electromagnetic structure with air gaps

Solution

Known Quantities: Relative permeability; number of coil turns; coil current; structure geometry.

Find: $\mathcal{R}_{eq}$; B_{bar}.

Schematics, Diagrams, Circuits, and Given Data: $\mu_r = 10{,}000$; $N = 100$ turns; $i = 1$ A.

Assumptions: All magnetic flux is linked by the coil; the flux is confined to the magnetic core; the flux density is uniform.

Analysis:

1. *Calculation of magnetomotive force.* From equation 16.28, we calculate the magnetomotive force:

$$\mathcal{F} = \text{mmf} = Ni = (100 \text{ turns})(1 \text{ A}) = 100 \text{ A} \cdot \text{t}$$

2. *Calculation of mean path.* Figure 16.19 depicts the geometry. The path length is:

$$l_c = l_1 + l_2 + l_3 + l_4 + l_5 + l_6 + l_g + l_g$$

However, the path must be broken into three legs: the upside-down U-shaped element, the air gaps, and the bar. We cannot treat these three parts as one because the relative permeability of the magnetic material is very different from that of the air gap. Thus, we define the following three paths, neglecting the very small (half bar thickness) lengths l_5 and l_6:

$$l_U = l_1 + l_2 + l_3 \qquad l_{\text{bar}} = l_4 + l_5 + l_6 \approx l_4 \qquad l_{\text{gap}} = l_g + l_g$$

where

$$l_U = 0.18 \text{ m} \qquad l_{\text{bar}} = 0.09 \text{ m} \qquad l_{\text{gap}} = 0.05 \text{ m}.$$

Next, we compute the cross-sectional area. For the magnetic structure, we calculate the square cross section to be: $A = w^2 = (0.01)^2 = 0.0001 \text{ m}^2$. For the air gap, we will make an empirical adjustment to account for the phenomenon of *fringing,* that is, to account for the tendency of the magnetic flux lines to bow out of the magnetic path, as illustrated in Figure 16.20. A rule of thumb used to account for fringing is to add the length of the gap to the actual cross-sectional area. Thus:

$$A_{\text{gap}} = (0.01 \text{ m} + l_g)^2 = (0.0125)^2 = 0.15625 \times 10^{-3} \text{ m}^2$$

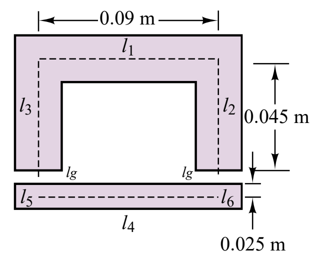


Figure 16.19

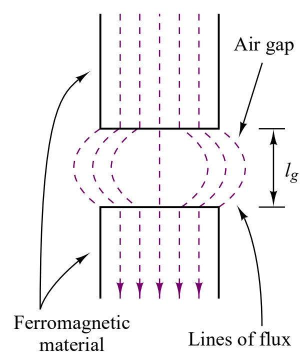


Figure 16.20 Fringing effects in air gap

3. *Calculation of reluctance.* Knowing the magnetic path length and cross sectional area we can calculate the reluctance of each of the legs of the circuit:

$$\mathcal{R}_U = \frac{l_U}{\mu_U A} = \frac{l_U}{\mu_r \mu_0 A} = \frac{0.18}{10{,}000 \times 4\pi \times 10^{-7} \times 0.0001}$$
$$= 1.43 \times 10^5 \text{ A} \cdot \text{t/Wb}$$

$$\mathcal{R}_{\text{bar}} = \frac{l_{\text{bar}}}{\mu_{\text{bar}} A} = \frac{l_{\text{bar}}}{\mu_r \mu_0 A} = \frac{0.09}{10{,}000 \times 4\pi \times 10^{-7} \times 0.0001}$$
$$= 0.715 \times 10^5 \text{ A} \cdot \text{t/Wb}$$

$$\mathcal{R}_{\text{gap}} = \frac{l_{\text{gap}}}{\mu_{\text{gap}} A_{\text{gap}}} = \frac{l_{\text{gap}}}{\mu_0 A_{\text{gap}}} = \frac{0.05}{4\pi \times 10^{-7} \times 0.0001} = 2.55 \times 10^7 \text{ A} \cdot \text{t/Wb}$$

Note that the reluctance of the air gap is dominant with respect to that of the magnetic structure, in spite of the small dimension of the gap. This is because the relative permeability of the air gap is much smaller than that of the magnetic material.

The equivalent reluctance of the structure is:

$$\mathcal{R}_{eq} = \mathcal{R}_U + \mathcal{R}_{bar} + \mathcal{R}_{gap} = 1.43 \times 10^5 + 0.715 \times 10^5 + 2.55 \times 10^7$$
$$= 2.57 \times 10^7$$

Thus,

$$\mathcal{R}_{eq} \approx \mathcal{R}_{gap}$$

Since the gap reluctance is two orders of magnitude greater than the reluctance of the magnetic structure, it is reasonable to neglect the magnetic structure reluctance and work only with the gap reluctance in calculating the magnetic flux.

4. *Calculation of magnetic flux and flux density in the bar.* From the result of the preceding sub-section, we calculate the flux

$$\phi = \frac{\mathcal{F}}{\mathcal{R}_{eq}} \approx \frac{\mathcal{F}}{\mathcal{R}_{gap}} = \frac{100 \text{ A} \cdot \text{t}}{2.55 \times 10^7 \text{ A} \cdot \text{t/Wb}} = 3.92 \times 10^{-6} \text{ Wb}$$

and the flux density in the bar:

$$B_{bar} = \frac{\phi}{A} = \frac{3.92 \times 10^{-6} \text{ Wb}}{0.0001 \text{ m}^2} = 39.2 \times 10^{-3} \text{ Wb/m}^2$$

Comments: It is very common to neglect the reluctance of the magnetic material sections in these approximate calculations. We shall make this assumption very frequently in the remainder of the chapter.

EXAMPLE 16.4 Magnetic Structure of Electric Motor

Problem

Figure 16.21 depicts the configuration of an electric motor. The electric motor consists of a *stator* and of a *rotor.* Compute the air gap flux and flux density.

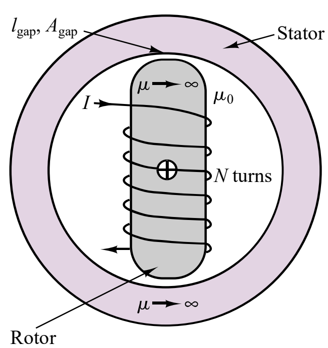


Figure 16.21 Cross-sectional view of synchronous motor

Solution

Known Quantities: Relative permeability; number of coil turns; coil current; structure geometry.

Find: ϕ_{gap}; B_{gap}.

Schematics, Diagrams, Circuits, and Given Data: $\mu_r \rightarrow \infty$; $N = 1{,}000$ turns; $i = 10$ A; $l_{gap} = 0.01$ m; $A_{gap} = 0.1$ m^2. The magnetic circuit geometry is defined in Figure 16.21.

Assumptions: All magnetic flux is linked by the coil; the flux is confined to the magnetic core; the flux density is uniform. The reluctance of the magnetic structure is negligible.

Analysis:

1. *Calculation of magnetomotive force.* From equation 16.28, we calculate the magnetomotive force:

$$\mathcal{F} = \text{mmf} = Ni = (1{,}000 \text{ turns})(10 \text{ A}) = 10{,}000 \text{ A} \cdot \text{t}$$

2. *Calculation of reluctance.* Knowing the magnetic path length and cross sectional area, we can calculate the equivalent reluctance of the two gaps:

$$\mathcal{R}_{gap} = \frac{l_{gap}}{\mu_{gap} A_{gap}} = \frac{l_{gap}}{\mu_0 A_{gap}} = \frac{0.01}{4\pi \times 10^{-7} \times 0.2} = 3.97 \times 10^4 \text{ A} \cdot \text{t/Wb}$$

$$\mathcal{R}_{eq} = 2\mathcal{R}_{gap} = 7.94 \times 10^4 \text{ A} \cdot \text{t/Wb}$$

3. *Calculation of magnetic flux and flux density.* From the results of steps 1 and 2, we calculate the flux

$$\phi = \frac{\mathcal{F}}{\mathcal{R}_{eq}} = \frac{10{,}000 \text{ A} \cdot \text{t}}{7.94 \times 10^4 \text{ A} \cdot \text{t/Wb}} = 0.126 \text{ Wb}$$

and the flux density:

$$B_{bar} = \frac{\phi}{A} = \frac{0.126 \text{ Wb}}{0.1 \text{ m}^2} = 1.26 \text{ Wb/m}^2$$

Comments: Note that the flux and flux density in this structure are significantly larger than in the preceding example because of the larger mmf and larger gap area of this magnetic structure.

The subject of electric motors will be formally approached in Chapter 17.

EXAMPLE 16.5 Equivalent Circuit of Magnetic Structure with Multiple Air Gaps

Problem

Figure 16.23 depicts the configuration of a magnetic structure with two air gaps. Determine the equivalent circuit of the structure.

Solution

Known Quantities: Structure geometry.

Find: Equivalent circuit diagram.

Assumptions: All magnetic flux is linked by the coil; the flux is confined to the magnetic core; the flux density is uniform. The reluctance of the magnetic structure is negligible.

Analysis:

1. *Calculation of magnetomotive force.*

$$\mathcal{F} = \text{mmf} = Ni$$

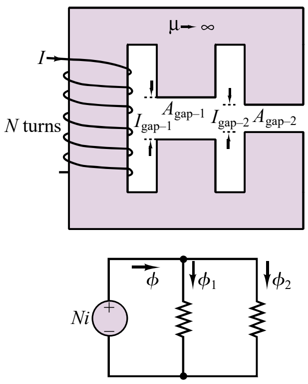


Figure 16.22 Magnetic structure with two air gaps

2. *Calculation of reluctance.* Knowing the magnetic path length and cross sectional area we can calculate the equivalent reluctance of the two gaps:

$$\mathcal{R}_{gap-1} = \frac{l_{gap-1}}{\mu_{gap-1} A_{gap-1}} = \frac{l_{gap-1}}{\mu_0 A_{gap-1}}$$

$$\mathcal{R}_{gap-1} = \frac{l_{gap-2}}{\mu_{gap-2} A_{gap-2}} = \frac{l_{gap-2}}{\mu_0 A_{gap-2}}$$

3. *Calculation of magnetic flux and flux density.* Note that the flux must now divide between the two legs, and that a different air-gap flux will exist in each leg. Thus:

$$\phi_1 = \frac{Ni}{\mathcal{R}_{gap-1}} = \frac{Ni\mu_0 A_{gap-1}}{l_{gap-1}}$$

$$\phi_2 = \frac{Ni}{\mathcal{R}_{gap-2}} = \frac{Ni\mu_0 A_{gap-2}}{l_{gap-2}}$$

and the total flux generated by the coil is $\phi = \phi_1 + \phi_2$.

The equivalent circuit is shown in the bottom half of Figure 16.22.

Comments: Note that the two legs of the structure act like resistors in a parallel circuit.

EXAMPLE 16.6 Inductance, Stored Energy, and Induced Voltage

Problem

1. Determine the inductance and the magnetic stored energy for the structure of Fig. 16.17(a). The structure is identical to that of Example 16.2 except for the air gap.
2. Assume that the flux density in the air gap varies sinusoidally as $B(t) = B_0 \sin(\omega t)$. Determine the induced voltage across the coil, e.

Solution

Known Quantities: Relative permeability; number of coil turns; coil current; structure geometry; flux density in air gap.

Find: L; W_m; e.

Schematics, Diagrams, Circuits, and Given Data: $\mu_r \to \infty$; $N = 500$ turns; $i = 0.1$ A. The magnetic circuit geometry is defined in Figures 16.14 and 16.15. The air gap has $l_g = 0.002$ m. $B_0 = 0.6$ Wb/m^2.

Assumptions: All magnetic flux is linked by the coil; the flux is confined to the magnetic core; the flux density is uniform. The reluctance of the magnetic structure is negligible.

Analysis:

1. *Part 1.* To calculate the inductance of this magnetic structure, we use equation 16.30:

$$L = \frac{N^2}{\mathcal{R}}$$

Thus, we need to first calculate the reluctance. Assuming that the reluctance of the structure is negligible, we have:

$$\mathcal{R}_{gap} = \frac{l_{gap}}{\mu_{gap} A_{gap}} = \frac{l_{gap}}{\mu_0 A_{gap}} = \frac{0.002}{4\pi \times 10^{-7} \times 0.0001} = 1.59 \times 10^7 \text{ A} \cdot \text{t/Wb}$$

and

$$L = \frac{N^2}{\mathcal{R}} = \frac{500^2}{1.59 \times 10^7} = 0.157 \text{ H}$$

Finally, we can calculate the stored magnetic energy as follows:

$$W_m = \frac{1}{2}Li^2 = \frac{1}{2} \times (0.157 \text{ H}) \times (0.1 \text{ A})^2 = 0.785 \times 10^{-3} \text{ J}$$

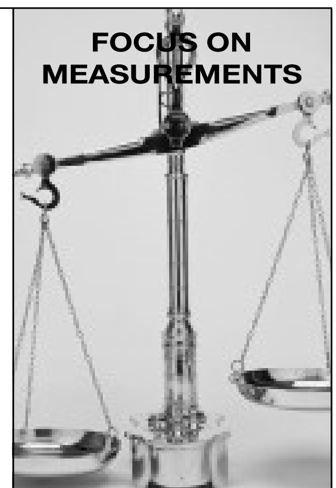
2. *Part 2.* To calculate the induced voltage due to a time-varying magnetic flux, we use equation 16.16:

$$e = \frac{d\lambda}{dt} = N\frac{d\phi}{dt} = NA\frac{dB}{dt} = NAB_0\omega\cos(\omega t)$$

$$= 500 \times 0.0001 \times 0.6 \times 377\cos(377t) = 11.31\cos(377t) \text{ V}$$

Comments: The voltage induced across a coil in an electromagnetic transducer is a very important quantity called *back electromotive force,* or back emf. We shall make use of this quantity in Sec. 16.5.

FOCUS ON MEASUREMENTS



Magnetic Reluctance Position Sensor

FIND IT ON THE WEB

A simple magnetic structure, very similar to those examined in the previous examples, finds very common application in the so-called **variable-reluctance position sensor,** which, in turn, finds widespread application in a variety of configurations for the measurement of linear and angular velocity. Figure 16.23 depicts one particular configuration that is used in many applications. In this structure, a permanent magnet with a coil of wire wound around it forms the sensor; a steel disk (typically connected to a rotating shaft) has a number of tabs that pass between the pole pieces of the sensor. The area of the tab is assumed equal to the area of the cross section of the pole pieces and is equal to a^2. The reason for the name *variable-reluctance sensor* is that the reluctance of the magnetic structure is variable, depending on whether or not a ferromagnetic tab lies between the pole pieces of the magnet.

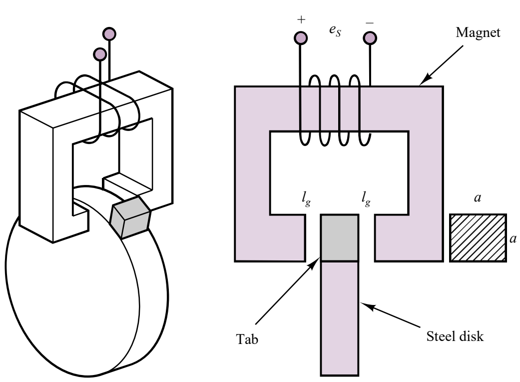


Figure 16.23 Variable-reluctance position sensor

The principle of operation of the sensor is that an electromotive force, e_S, is induced across the coil by the change in magnetic flux caused by the passage of the tab between the pole pieces when the disk is in motion. As the tab enters the volume between the pole pieces, the flux will increase, because of the lower reluctance of the configuration, until it reaches a maximum when the tab is centered between the poles of the magnet. Figure 16.24 depicts the approximate shape of the resulting voltage, which, according to Faraday's law, is given by

$$e_S = -\frac{d\phi}{dt}$$

The rate of change of flux is dictated by the geometry of the tab and of the pole pieces, and by the speed of rotation of the disk. It is important to note that, since the flux is changing only if the disk is rotating, this sensor cannot detect the static position of the disk.

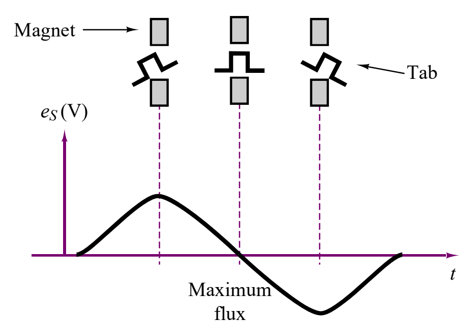


Figure 16.24 Variable-reluctance position sensor waveform

One common application of this concept is in the measurement of the speed of rotation of rotating machines, including electric motors and internal combustion engines. In these applications, use is made of a *60-tooth wheel*, which permits the conversion of the speed rotation directly to units of revolutions per minute. The output of a variable-reluctance position sensor magnetically coupled to a rotating disk equipped with 60 tabs (teeth) is processed through a comparator or Schmitt trigger circuit (see Chapter 15). The voltage waveform generated by the sensor is nearly sinusoidal when the teeth are closely spaced, and it is characterized by one sinusoidal cycle for each tooth on the disk. If a negative zero-crossing detector (see Chapter 15) is employed, the trigger circuit will generate a pulse corresponding to the passage of each tooth, as shown in Figure 16.25. If the time between any two pulses is measured by means of a high-frequency clock, the speed of the engine can be directly determined in units of rev/min by means of a digital counter (see Chapter 14).

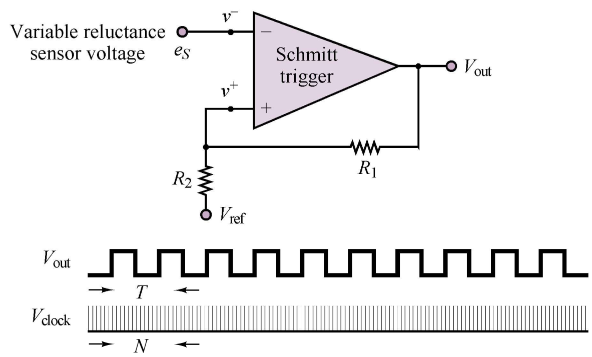
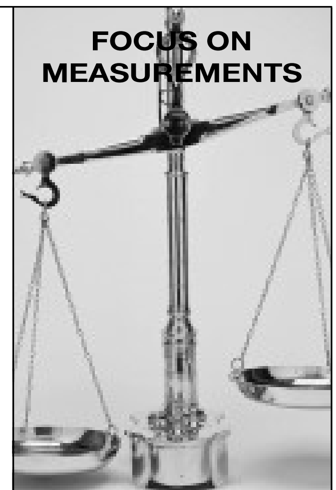


Figure 16.25 Signal processing for a 60-tooth-wheel RPM sensor



FOCUS ON MEASUREMENTS

Voltage Calculation in Magnetic Reluctance Position Sensor

This example illustrates the calculation of the voltage induced in a magnetic reluctance sensor by a rotating toothed wheel. In particular, we will find an approximate expression for the reluctance and the induced voltage for the position sensor shown in Figure 16.26, and show that the induced voltage is speed-dependent. It will be assumed that the reluctance of the core and fringing at the air gaps are both negligible.

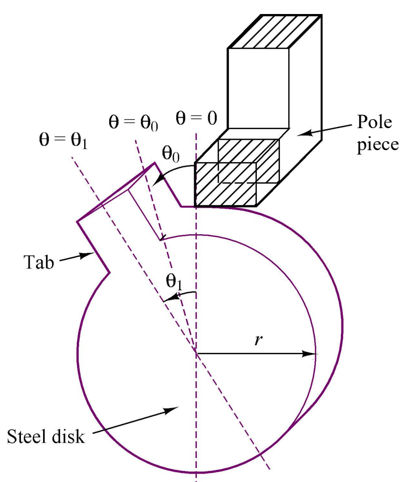


Figure 16.26 Reluctance sensor for measurement of angular position

Solution:

From the geometry shown in the preceding "Focus on Measurements," the equivalent reluctance of the magnetic structure is twice that of one gap, since the permeability of the tab and magnetic structure are assumed infinite (i.e., they have negligible reluctance). When the tab and the poles are aligned, the angle θ is zero, as shown in Figure 16.26, and the area of the air gap is maximum. For angles greater than $2\theta_0$, the magnetic length of the air gaps is so large that the magnetic field may reasonably be taken as zero.

To model the reluctance of the gaps, we assume the following simplified expression, where the area of overlap of the tab with the magnetic poles is assumed proportional to the angular displacement:

$$\mathcal{R} = \frac{2l_g}{\mu_0 A} = \frac{2l_g}{\mu_0 ar(\theta_1 - \theta)} \qquad \text{for} \qquad 0 < \theta < \theta_1$$

Naturally, this is an approximation; however, the approximation captures the essential idea of this transducer, namely, that the reluctance will decrease with increasing overlap area until it reaches a minimum, and then it will increase as the overlap area decreases. For $\theta = \theta_1$, that is, with the tab outside the magnetic pole pieces, we have $\mathcal{R}_{\text{max}} \to \infty$. For $\theta = 0$, that is, with the tab perfectly aligned with the pole pieces, we have $\mathcal{R}_{\text{min}} = 2l_g/\mu_0 ar\theta_1$. The flux ϕ may therefore be computed as follows:

$$\phi = \frac{Ni}{\mathcal{R}} = \frac{Ni\mu_0 ar(\theta_1 - \theta)}{2l_g}$$

The induced voltage e_S is found by

$$e_S = \frac{d\phi}{dt} = -\frac{d\phi}{d\theta}\frac{d\theta}{dt} = \frac{Ni\mu_0 ar}{2l_g} \times \omega$$

where $\omega = d\theta/dt$ is the rotational speed of the steel disk. It should be evident that the induced voltage is speed-dependent. For $a = 1$ cm, $r =$ 10 cm, $l_g = 0.1$ cm, $N = 100$ turns, $i = 10$ mA, $\theta_1 = 6^\circ \approx 0.1$ rad, and $\omega = 400$ rad/s (approximately 3,800 rev/min), we have

$$\mathcal{R}_{\text{max}} = \frac{2 \times 0.1 \times 10^{-2}}{4\pi \times 10^{-7} \times 1 \times 10^{-2} \times 10 \times 10^{-2} \times 0.1}$$
$$= 1.59 \times 10^7 \text{ A} \cdot \text{t/Wb}$$

$$e_{S\ \text{peak}} = \frac{1{,}000 \times 10 \times 10^{-3} \times 4\pi \times 10^{-7} \times 1 \times 10^{-2} \times 10^{-1}}{2 \times 0.1 \times 10^{-2}} \times 400$$
$$= 2.5 \text{ mV}$$

That is, the peak amplitude of e_S will be 2.5 mV.

Check Your Understanding

16.5 If $\mathcal{R}_{\text{eq}} = 2\mathcal{R}_{\text{gap}}$ in Example 16.3, calculate ϕ and B.

16.6 Determine the equivalent reluctance of the structure of Figure 16.27 as seen by the "source" if μ_r for the structure is 1,000, $l = 5$ cm, and all of the legs are 1 cm on a side.

16.7 Find the equivalent reluctance of the magnetic circuit shown in Figure 16.28 if μ_r of the structure is infinite, $\delta = 2$ mm, and the physical cross section of the core is 1 cm^2. Do not neglect fringing.

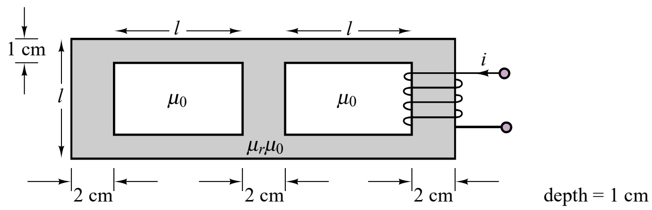


Figure 16.27

16.8 Find the equivalent magnetic circuit of the structure of Figure 16.29 if μ_r is infinite. Give expressions for each of the circuit values if the physical cross-sectional area of each of the legs is given by

$$A = l \times w$$

Do not neglect fringing.

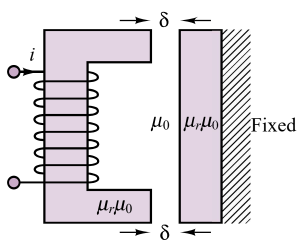


Figure 16.28

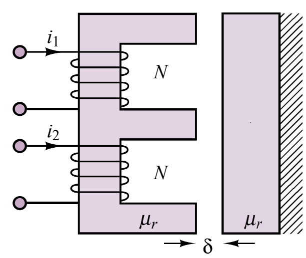


Figure 16.29

16.3 MAGNETIC MATERIALS AND *B-H* CURVES

In the analysis of magnetic circuits presented in the previous sections, the relative permeability, μ_r, was treated as a constant. In fact, the relationship between the magnetic flux density, **B,** and the associated field intensity, **H,**

$$\mathbf{B} = \mu\mathbf{H} \tag{16.32}$$

is characterized by the fact that the relative permeability of magnetic materials is not a constant, but is a function of the magnetic field intensity. In effect, all magnetic materials exhibit a phenomenon called **saturation,** whereby the flux density increases in proportion to the field intensity until it cannot do so any longer. Figure 16.30 illustrates the general behavior of all magnetic materials. You will note that since the B-H curve shown in the figure is nonlinear, the value of μ (which is the slope of the curve) depends on the intensity of the magnetic field.

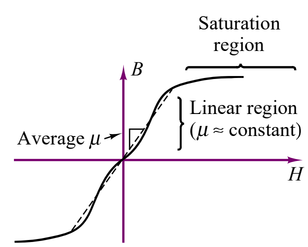


Figure 16.30 Permeability and magnetic saturation effects

To understand the reasons for the saturation of a magnetic material, we need to briefly review the mechanism of magnetization. The basic idea behind magnetic materials is that the spin of electrons constitutes motion of charge, and therefore leads to magnetic effects, as explained in the introductory section of this chapter. In most materials, the electron spins cancel out, on the whole, and no net effect remains. In ferromagnetic materials, on the other hand, atoms can align so that the electron spins cause a net magnetic effect. In such materials, there exist small regions with strong magnetic properties (called **magnetic domains**), the effects of which are neutralized in unmagnetized material by other, similar regions that are oriented differently, in a random pattern. When the material is magnetized, the magnetic domains tend to align with each other, to a degree that is determined by the intensity of the applied magnetic field.

In effect, the large number of miniature magnets within the material are *polarized* by the external magnetic field. As the field increases, more and more domains become aligned. When all of the domains have become aligned, any further increase in magnetic field intensity does not yield an increase in flux density beyond the increase that would be caused in a nonmagnetic material. Thus, the relative permeability, μ_r, approaches 1 in the saturation region. It should be apparent that an exact value of μ_r cannot be determined; the value of μ_r used in the earlier examples is to be interpreted as an average permeability, for intermediate values of flux density. As a point of reference, commercial magnetic steels saturate at flux densities around a few teslas. Figure 16.33, shown later in this section, will provide some actual B-H curves for common ferromagnetic materials.

The phenomenon of saturation carries some interesting implications with regard to the operation of magnetic circuits: the results of the previous section would seem to imply that an increase in the mmf (that is, an increase in the current driving the coil) would lead to a proportional increase in the magnetic flux. This is true in the *linear region* of Figure 16.30; however, as the material reaches saturation, further increases in the driving current (or, equivalently, in the mmf) do not yield further increases in the magnetic flux.

There are two more features that cause magnetic materials to further deviate from the ideal model of the linear B-H relationship: **eddy currents** and **hysteresis.** The first phenomenon consists of currents that are caused by any time-varying flux in the core material. As you know, a time-varying flux will induce a voltage, and therefore a current. When this happens inside the magnetic core, the induced voltage will cause "eddy" currents (the terminology should be self-explanatory) in the core, which depend on the resistivity of the core. Figure 16.31 illustrates the phenomenon of eddy currents. The effect of these currents is to dissipate energy in the form of heat. Eddy currents are reduced by selecting high-resistivity core materials, or by *laminating* the core, introducing tiny, discontinuous air gaps between core layers (see Figure 16.31). Lamination of the core reduces eddy currents greatly without affecting the magnetic properties of the core.

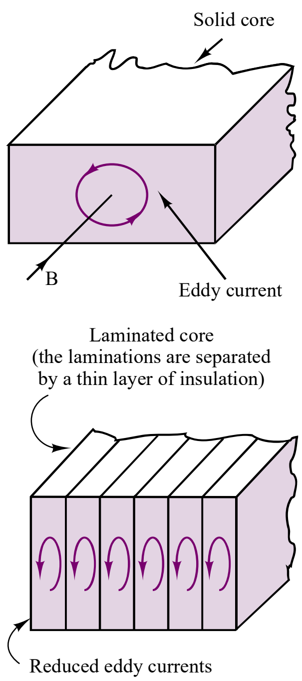


Figure 16.31 Eddy currents in magnetic structures

It is beyond the scope of this chapter to quantify the losses caused by induced eddy currents, but it will be important in Chapters 17 and 18 to be aware of this source of energy loss.

Hysteresis is another loss mechanism in magnetic materials; it displays a rather complex behavior, related to the magnetization properties of a material. The curve of Figure 16.32 reveals that the B-H curve for a magnetic material during magnetization (as H is increased) is displaced with respect to the curve that is measured when the material is demagnetized. To understand the hysteresis process, consider a core that has been energized for some time, with a field intensity of $H_1 A \cdot$ t/m. As the current required to sustain the mmf corresponding to H_1 is decreased, we follow the hysteresis curve from the point α to the point β. When the mmf is exactly zero, the material displays the **remanent** (or **residual**)

magnetization B_r. To bring the flux density to zero, we must further decrease the mmf (i.e., produce a negative current), until the field intensity reaches the value $-H_0$ (point γ on the curve). As the mmf is made more negative, the curve eventually reaches the point α'. If the excitation current to the coil is now increased, the magnetization curve will follow the path $\alpha' = \beta' = \gamma' = \alpha$, eventually returning to the original point in the B-H plane, but via a different path.

The result of this process, by which an *excess magnetomotive force* is required to magnetize or demagnetize the material, is a net energy loss. It is difficult to evaluate this loss exactly; however, it can be shown that it is related to the area between the curves of Figure 16.32. There are experimental techniques that enable the approximate measurement of these losses.

Figures 16.33(a)–(c) depict magnetization curves for three very common ferromagnetic materials: cast iron, cast steel, and sheet steel. These curves will be useful in solving some of the homework problems.

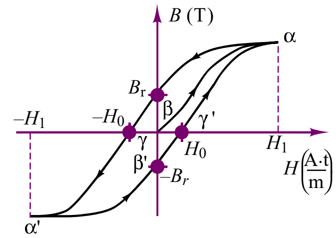


Figure 16.32 Hysteresis in magnetization curves

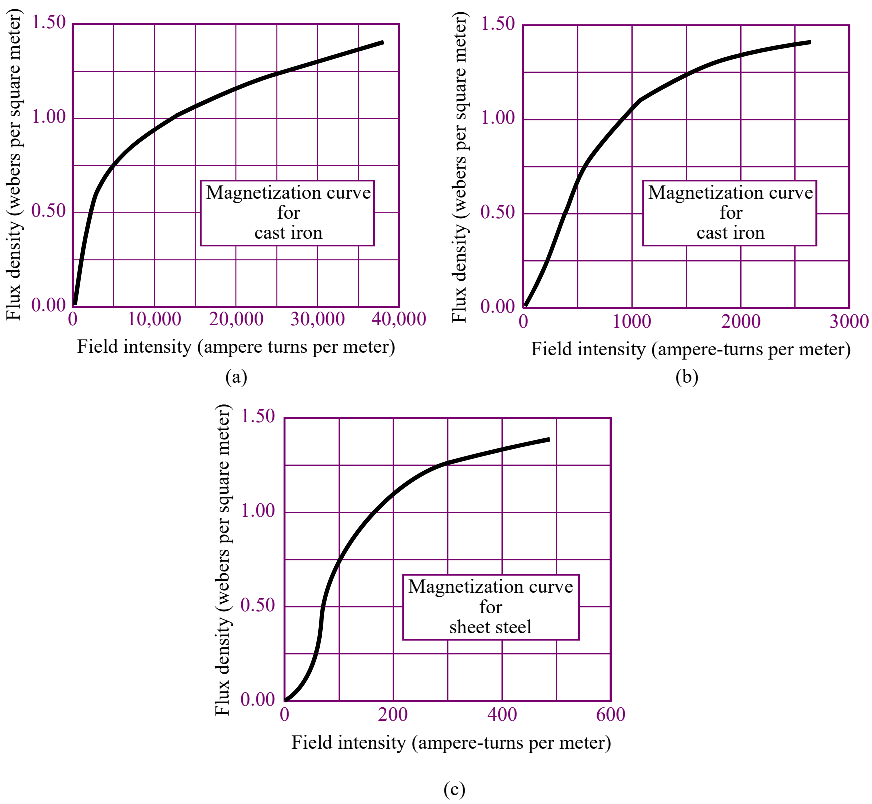


Figure 16.33 (a) Magnetization curve for cast iron; (b) Magnetization curve for cast steel; (c) Magnetization curve for sheet steel

16.4 TRANSFORMERS

One of the more common magnetic structures in everyday applications is the **transformer.** The ideal transformer was introduced in Chapter 7 as a device that can step an AC voltage up or down by a fixed ratio, with a corresponding decrease or increase in current. The structure of a simple magnetic transformer is shown in

Figure 16.34, which illustrates that a transformer is very similar to the magnetic circuits described earlier in this chapter. Coil L_1 represents the input side of the transformer, while coil L_2 is the output coil; both coils are wound around the same magnetic structure, which we show here to be similar to the "square doughnut" of the earlier examples.

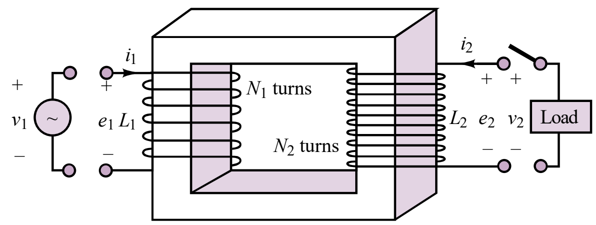


Figure 16.34 Structure of a transformer

The ideal transformer operates on the basis of the same set of assumptions we made in earlier sections: the flux is confined to the core, the flux links all turns of both coils, and the permeability of the core is infinite. The last assumption is equivalent to stating that an arbitrarily small mmf is sufficient to establish a flux in the core. In addition, we assume that the ideal transformer coils offer negligible resistance to current flow.

The operation of a transformer requires a time-varying current; if a time-varying voltage is applied to the primary side of the transformer, a corresponding current will flow in L_1; this current acts as an mmf and causes a (time-varying) flux in the structure. But the existence of a changing flux will induce an emf across the secondary coil! Without the need for a direct electrical connection, the transformer can couple a source voltage at the primary to the load; the coupling occurs by means of the magnetic field acting on both coils. Thus, a transformer operates by converting electric energy to magnetic, and then back to electric. The following derivation illustrates this viewpoint in the ideal case (no loss of energy), and compares the result with the definition of the ideal transformer in Chapter 7.

If a time-varying voltage source is connected to the input side, then by virtue of Faraday's law, a corresponding time-varying flux $d\phi/dt$ is established in coil L_1:

$$e_1 = N_1 \frac{d\phi}{dt} = v_1 \qquad \textbf{(16.33)}$$

But since the flux thus produced also links coil L_2, an emf is induced across the output coil as well:

$$e_2 = N_2 \frac{d\phi}{dt} = v_2 \qquad \textbf{(16.34)}$$

This induced emf can be measured as the voltage v_2 at the output terminals, and one can readily see that the ratio of the open-circuit output voltage to input terminal voltage is

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} \qquad \textbf{(16.35)}$$

If a load current i_2 is now required by the connection of a load to the output circuit (by closing the switch in the figure), the corresponding mmf is $\mathcal{F}_2 = N_2 i_2$. This mmf, generated by the load current i_2, would cause the flux in the core to change; however, this is not possible, since a change in ϕ would cause a corresponding change in the voltage induced across the input coil. But this voltage is determined (fixed) by the source v_1 (and is therefore $d\phi/dt$), so that the input coil is forced to generate a **counter mmf** to oppose the mmf of the output coil; this is accomplished as the input coil draws a current i_1 from the source v_1 such that

$$i_1 N_1 = i_2 N_2 \tag{16.36}$$

or

$$\frac{i_2}{i_1} = \frac{N_1}{N_2} = \alpha \tag{16.37}$$

where α is the ratio of primary to secondary turns (the transformer ratio) and N_1 and N_2 are the primary and secondary turns, respectively. If there were any net difference between the input and output mmf, flux balance required by the input voltage source would not be satisfied. Thus, the two mmf's must be equal. As you can easily verify, these results are the same as in Chapter 7; in particular, the ideal transformer does not dissipate any power, since

$$v_1 i_1 = v_2 i_2 \tag{16.38}$$

Note the distinction we have made between the induced voltages (emf's), e, and the terminal voltages, v. In general, these are not the same.

The results obtained for the ideal case do not completely represent the physical nature of transformers. A number of loss mechanisms need to be included in a practical transformer model, to account for the effects of leakage flux, for various magnetic core losses (e.g., hysteresis), and for the unavoidable resistance of the wires that form the coils.

Commercial transformer ratings are usually given on the so-called **nameplate,** which indicates the normal operating conditions. The nameplate includes the following parameters:

- Primary-to-secondary voltage ratio
- Design frequency of operation
- (Apparent) rated output power

For example, a typical nameplate might read 480:240 V, 60 Hz, 2 kVA. The voltage ratio can be used to determine the turns ratio, while the rated output power represents the continuous power level that can be sustained without overheating. It is important that this power be rated as the apparent power in kVA, rather than real power in kW, since a load with low power factor would still draw current and therefore operate near rated power. Another important performance characteristic of a transformer is its **power efficiency,** defined by:

$$\text{Power efficiency} = \eta = \frac{\text{Output power}}{\text{Input power}} \tag{16.39}$$

The following examples illustrate the use of the nameplate ratings and the calculation of efficiency in a practical transformer, in addition to demonstrating the application of the circuit models.

EXAMPLE 16.7 Transformer Nameplate

Problem

Determine the turns ratio and the rated currents of a transformer from nameplate data.

Solution

Known Quantities: Nameplate data.

Find: $\alpha = N_1/N_2$; I_1; I_2.

Schematics, Diagrams, Circuits, and Given Data: Nameplate data: 120 V/480 V; 48 kVA; 60 Hz.

Assumptions: Assume an ideal transformer.

Analysis: The first element in the nameplate data is a pair of voltages, indicating the primary and secondary voltages for which the transformer is rated. The ratio, α, is found as follows:

$$\alpha = \frac{N_1}{N_2} = \frac{480}{120} = 4$$

To find the primary and secondary currents, we use the kVA rating (apparent power) of the transformer:

$$I_1 = \frac{|S|}{V_1} = \frac{48 \text{ kVA}}{480 \text{ V}} = 100 \text{ A} \qquad I_2 = \frac{|S|}{V_2} = \frac{48 \text{ kVA}}{120 \text{ V}} = 400 \text{ A}$$

Comments: In computing the rated currents, we have assumed that no losses take place in the transformer; in fact, there will be losses due to coil resistance and magnetic core effects. These losses result in heating of the transformer, and limit its rated performance.

EXAMPLE 16.8 Impedance Transformer

Problem

Find the equivalent load impedance seen by the voltage source (i.e., reflected from secondary to primary) for the transformer of Figure 16.35.

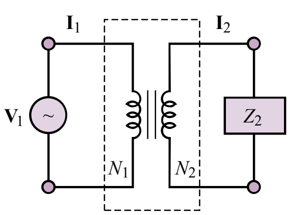


Figure 16.35 Ideal transformer

Solution

Known Quantities: Transformer turns ratio, α.

Find: Reflected impedance, Z_2'.

Assumptions: Assume an ideal transformer.

Analysis: By definition, the load impedance is equal to the ratio of secondary phasor voltage and current:

$$Z_2 = \frac{\mathbf{V}_2}{\mathbf{I}_2}$$

To find the reflected impedance we can express the above ratio in terms of primary voltage and current:

$$Z_2 = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{\frac{\mathbf{V}_1}{\alpha}}{\alpha \mathbf{I}_1} = \frac{1}{\alpha^2}\frac{\mathbf{V}_1}{\mathbf{I}_1}$$

where the ratio $\mathbf{V}_1/\mathbf{I}_1$ is the impedance seen by the source at the primary coil, that is, the *reflected load impedance* seen by the primary (source) side of the circuit. Thus, we can write the load impedance, Z_2, in terms of the primary circuit voltage and current; we call this the reflected impedance, Z_2':

$$Z_2 = \frac{1}{\alpha^2}\frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{1}{\alpha^2} Z_1 = \frac{1}{\alpha^2} Z_2'.$$

Thus, $Z_2' = \alpha^2 Z^2$. Figure 16.36 depicts the equivalent circuit with the load impedance reflected back to the primary.

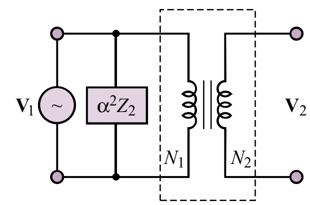


Figure 16.36

Comments: The equivalent reflected circuit calculations are convenient because all circuit elements can be referred to a single set of variable (i.e., only primary or secondary voltages and currents).

Check Your Understanding

16.9 The high-voltage side of a transformer has 500 turns, and the low-voltage side has 100 turns. When the transformer is connected as a step-down transformer, the load current is 12 A. Calculate: (a) the turns ratio α; (b) the primary current.

16.10 Calculate the turns ratio if the transformer in Check Your Understanding 16.9 is used as a step-up transformer.

16.11 The output of a transformer under certain conditions is 12 kW. The copper losses are 189 W and the core losses are 52 W. Calculate the efficiency of this transformer.

16.12 The output impedance of a servo amplifier is 250 Ω. The servomotor that the amplifier must drive has an impedance of 2.5 Ω. Calculate the turns ratio of the transformer required to match these impedances.

16.5 ELECTROMECHANICAL ENERGY CONVERSION

From the material developed thus far, it should be apparent that electromagnetomechanical devices are capable of converting mechanical forces and displacements to electromagnetic energy, and that the converse is also possible. The objective of this section is to formalize the basic principles of energy conversion in electromagnetomechanical systems, and to illustrate its usefulness and potential for application by presenting several examples of **energy transducers.** A transducer is a device that can convert electrical to mechanical energy (in this case, it is often called an **actuator**), or vice versa (in which case it is called a **sensor**).

Several physical mechanisms permit conversion of electrical to mechanical energy and back, the principal phenomena being the **piezoelectric effect,**[3] consisting of the generation of a change in electric field in the presence of strain in

[3] See "Focus on Measurements: Charge Amplifiers" in Chapter 12.

certain crystals (e.g., quartz), and **electrostriction** and **magnetostriction,** in which changes in the dimension of certain materials lead to a change in their electrical (or magnetic) properties. Although these effects lead to many interesting applications, this chapter is concerned only with transducers in which electrical energy is converted to mechanical energy through the coupling of a magnetic field. It is important to note that all rotating machines (motors and generators) fit the basic definition of electromechanical transducers we have just given.

Forces in Magnetic Structures

Mechanical forces can be converted to electrical signals, and vice versa, by means of the coupling provided by energy stored in the magnetic field. In this subsection, we discuss the computation of mechanical forces and of the corresponding electromagnetic quantities of interest; these calculations are of great practical importance in the design and application of electromechanical actuators. For example, a problem of interest is the computation of the current required to generate a given force in an electromechanical structure. This is the kind of application that is likely to be encountered by the engineer in the selection of an electromechanical device for a given task.

As already seen in this chapter, an electromechanical system includes an electrical system and a mechanical system, in addition to means through which the two can interact. The principal focus of this chapter has been the coupling that occurs through an electromagnetic field common to both the electrical and the mechanical system; to understand electromechanical energy conversion, it will be important to understand the various energy storage and loss mechanisms in the electromagnetic field. Figure 16.37 illustrates the coupling between the electrical and mechanical systems. In the mechanical system, energy loss can occur because of the heat developed as a consequence of *friction*, while in the electrical system, analogous losses are incurred because of *resistance*. Loss mechanisms are also present in the magnetic coupling medium, since *eddy current losses* and *hysteresis losses* are unavoidable in ferromagnetic materials. Either system can supply energy, and either system can store energy. Thus, the figure depicts the flow of energy from the electrical to the mechanical system, accounting for these various losses. The same flow could be reversed if mechanical energy were converted to electrical form.

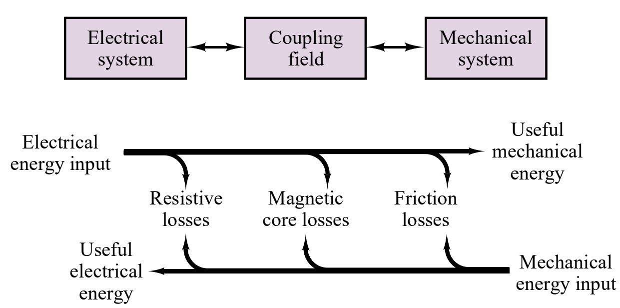


Figure 16.37

Moving-Iron Transducers

One important class of electromagnetomechanical transducers is that of **moving-iron transducers.** The aim of this section is to derive an expression for the mag-

netic forces generated by such transducers and to illustrate the application of these calculations to simple, yet common devices such as electromagnets, solenoids, and relays. The simplest example of a moving-iron transducer is the **electromagnet** of Figure 16.38, in which the U-shaped element is fixed and the bar is movable. In the following paragraphs, we shall derive a relationship between the current applied to the coil, the displacement of the movable bar, and the magnetic force acting in the air gap.

The principle that will be applied throughout the section is that in order for a mass to be displaced, some work needs to be done; this work corresponds to a change in the energy stored in the electromagnetic field, which causes the mass to be displaced. With reference to Figure 16.38, let f_e represent the magnetic force acting on the bar and x the displacement of the bar, in the direction shown. Then the net work into the electromagnetic field, W_m, is equal to the sum of the work done by the electrical circuit plus the work done by the mechanical system. Therefore, for an incremental amount of work, we can write

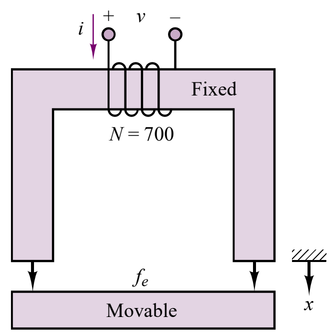


Figure 16.38

$$dW_m = ei\,dt - f_e\,dx \tag{16.40}$$

where e is the electromotive force across the coil and the negative sign is due to the sign convention indicated in Figure 16.38. Recalling that the emf e is equal to the derivative of the flux linkage (equation 16.16), we can further expand equation 16.40 to obtain

$$dW_m = ei\,dt - f_e\,dx = i\frac{d\lambda}{dt}dt - f_e\,dx = i\,d\lambda - f_e\,dx \tag{16.41}$$

or

$$f_e\,dx = i\,d\lambda - dW_m \tag{16.42}$$

Now we must observe that the flux in the magnetic structure of Figure 16.38 depends on two variables, which are in effect independent: the current flowing through the coil, and the displacement of the bar. Each of these variables can cause the magnetic flux to change. Similarly, the energy stored in the electromagnetic field is also dependent on both current and displacement. Thus we can rewrite equation 16.42 as follows:

$$f_e = i\left(\frac{\partial\lambda}{\partial i}di + \frac{\partial\lambda}{\partial x}dx\right) - \left(\frac{\partial W_m}{\partial i}di + \frac{\partial W_m}{\partial x}dx\right) \tag{16.43}$$

Since i and x are independent variables, we can write

$$f_e = i\frac{\partial\lambda}{\partial x} - \frac{\partial W_m}{\partial x} \qquad \text{and} \qquad 0 = i\frac{\partial\lambda}{\partial i} - \frac{\partial W_m}{\partial i} \tag{16.44}$$

From the first of the expressions in equation 16.44, we obtain the relationship

$$f_e = \frac{\partial}{\partial x}(i\lambda - W_m) = \frac{\partial}{\partial x}(W_c) \tag{16.45}$$

where the term W_c corresponds to W'_m, defined as the co-energy in equation 16.18. Finally, we observe that the force acting to *pull* the bar toward the electromagnet structure, which we will call f, is of opposite sign relative to f_e, and therefore we can write

$$f = -f_e = -\frac{\partial}{\partial x}(W_c) = -\frac{\partial W_m}{\partial x} \tag{16.46}$$

Equation 16.46 includes a very important assumption: that the energy is equal to the co-energy. If you make reference to Figure 16.8, you will realize that in general this is not true. Energy and co-energy are equal only if the λ-i relationship is linear. Thus, the useful result of equation 16.46, stating that the magnetic force acting on the moving iron is proportional to the rate of change of stored energy with displacement, applies only for *linear magnetic structures*.

Thus, in order to determine the forces present in a magnetic structure, it will be necessary to compute the energy stored in the magnetic field. To simplify the analysis, it will be assumed hereafter that the structures analyzed are magnetically linear. This is, of course, only an approximation, in that it neglects a number of practical aspects of electromechanical systems (for example, the nonlinear λ-i curves described earlier, and the core losses typical of magnetic materials), but it permits relatively simple analysis of many useful magnetic structures. Thus, although the analysis method presented in this section is only approximate, it will serve the purpose of providing a feeling for the direction and the magnitude of the forces and currents present in electromechanical devices. On the basis of a linear approximation, it can be shown that the stored energy in a magnetic structure is given by the expression

$$W_m = \frac{\phi \mathcal{F}}{2} \qquad \textbf{(16.47)}$$

and since the flux and the mmf are related by the expression

$$\phi = \frac{Ni}{\mathcal{R}} = \frac{\mathcal{F}}{\mathcal{R}} \qquad \textbf{(16.48)}$$

the stored energy can be related to the reluctance of the structure according to

$$W_m = \frac{\phi^2 \mathcal{R}(x)}{2} \qquad \textbf{(16.49)}$$

where the reluctance has been explicitly shown to be a function of displacement, as is the case in a moving-iron transducer. Finally, then, we shall use the following approximate expression to compute the magnetic force acting on the moving iron:

$$f = -\frac{dW_m}{dx} = -\frac{\phi^2}{2}\frac{d\mathcal{R}(x)}{dx} \qquad \textbf{(16.50)}$$

The following examples illustrate the application of this approximate technique for the computation of forces and currents (the two problems of practical engineering interest to the user of such electromechanical systems) in some common devices.

EXAMPLE 16.9 An Electromagnet

Problem

An electromagnet is used to support a solid piece of steel, as shown in Figure 16.38. Determine the minimum coil current required to support the weight for a given air gap.

Solution

Known Quantities: Force required to support weight; cross-sectional area of magnetic core; air gap dimension, number of coil turns.

Find: Coil current, i.

Schematics, Diagrams, Circuits, and Given Data: $F = 8{,}900$ N; $A = 0.01$ m^2; $x = 0.0015$ m.

Assumptions: Assume that the reluctance of the iron is negligible; neglect fringing.

Analysis: To compute the current we need to derive an expression for the force in the air gap. Using equation 16.50, we see that we need to compute the reluctance of the structure and the magnetic flux to derive an expression for the force.

Since we are neglecting the iron reluctance, we can write the expression for the reluctance as follows:

$$\mathcal{R}(x) = \frac{2x}{\mu_0 A} = \frac{2x}{4\pi \times 10^7 \times 0.01} = \frac{2x}{4\pi \times 10^{-7} \times 0.01} = 1.59 \times 10^8 x = \alpha x \text{ A} \cdot \text{t/Wb}$$

Knowing the reluctance we can calculate the magnetic flux in the structure as a function of the coil current:

$$\phi = \frac{Ni}{\mathcal{R}(x)} = \frac{Ni}{\alpha x}$$

and the magnitude of the force in the air gap is given by the expression

$$|f| = \frac{\phi^2}{2}\frac{d\mathcal{R}(x)}{dx} = \frac{(Ni)^2}{2\alpha^2 x^2}\alpha = \frac{N^2 i^2}{2\alpha x^2}$$

Solving for the current, we calculate:

$$i^2 = \frac{2\alpha x^2 |f|}{N^2} = \frac{2 \times 1.59 \times 10^8 \times (0.0015)^2 \times 8{,}900}{700^2} = 13 \text{ A}$$

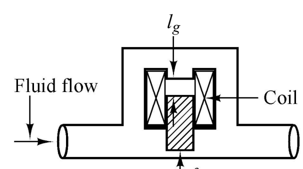
$$i = \pm 3.6 \text{ A}$$

Comments: As the air gap becomes smaller, the reluctance of the air gap decreases, to the point where the reluctance of the iron cannot be neglected. When the air gap is zero, the required, or *holding*, current is a minimum. Conversely, if the bar is initially positioned at a substantial distance from the electromagnet, the initial current required to exert the required force will be significantly larger than that computed in this example.

One of the more common practical applications of the concepts discussed in this section is the **solenoid.** Solenoids find application in a variety of electrically controlled valves. The action of a solenoid valve is such that when it is energized, the plunger moves in such a direction as to permit the flow of a fluid through a conduit, as shown schematically in Figure 16.39.

FIND IT ON THE WEB

The following examples illustrate the calculations involved in the determination of forces and currents in a solenoid.



Force acting on plunger with coil energized

Figure 16.39 Application of the solenoid as a valve

EXAMPLE 16.10 A Solenoid

Problem

Figure 16.40 depicts a simplified representation of a solenoid. The restoring force for the plunger is provided by a spring.

1. Derive a general expression for the force exerted on the plunger as a function of the plunger position, x.

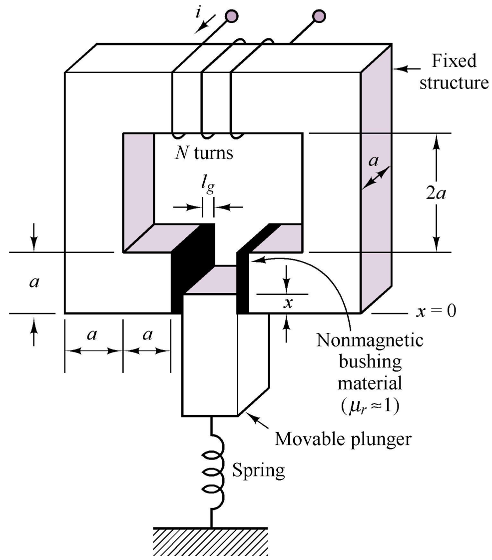


Figure 16.40 A solenoid

2. Determine the mmf required to pull the plunger to its end position ($x = a$).

Solution

Known Quantities: Geometry of magnetic structure; spring constant.

Find: f; mmf.

Schematics, Diagrams, Circuits, and Given Data: $a = 0.01$ m; $l_{gap} = 0.001$ m; $k = 1$ N/m.

Assumptions: Assume that the reluctance of the iron is negligible; neglect fringing. At $x = 0$ the plunger is in the gap by an infinitesimal displacement, ε.

Analysis:

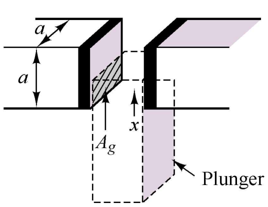


Figure 16.41

1. *Force on the plunger.* To compute a general expression for the magnetic force exerted on the plunger, we need to derive an expression for the force in the air gap. Using equation 16.50, we see that we need to compute the reluctance of the structure and the magnetic flux to derive an expression for the force.

   Since we are neglecting the iron reluctance, we can write the expression for the reluctance as follows. Note that the area of the gap is variable, depending on the position of the plunger, as shown in Figure 16.41.

$$\mathcal{R}_{gap}(x) = 2 \times \frac{l_{gap}}{\mu_0 A_{gap}} = \frac{2 l_{gap}}{\mu_0 a x}$$

   The derivative of the reluctance with respect to the displacement of the plunger can then be computed to be:

$$\frac{d\mathcal{R}_{gap}(x)}{dx} = \frac{-2 l_{gap}}{\mu_0 a x^2}$$

   Knowing the reluctance, we can calculate the magnetic flux in the structure as a function of the coil current:

$$\phi = \frac{Ni}{\mathcal{R}(x)} = \frac{Ni\mu_0 ax}{2l_{gap}}$$

The force in the air gap is given by the expression

$$f_{gap} = \frac{\phi^2}{2}\frac{d\mathcal{R}(x)}{dx} = \frac{(Ni\mu_0 ax)^2}{8l_{gap}^2}\frac{(-2l_{gap})}{\mu_0 ax^2} = -\frac{\mu_0 a(Ni)^2}{4l_{gap}}$$

Thus, the force in the gap is proportional to the square of the current, and does not vary with plunger displacement.

2. *Calculation of magnetomotive force.* To determine the required magnetomotive force, we observe that the magnetic force must overcome the mechanical (restoring) force generated by the spring. Thus, $f_{gap} = kx = ka$. For the stated values, $f_{gap} = (10\text{ N/m}) \times (0.01\text{ m}) = 0.1\text{ N}$, and

$$Ni = \sqrt{\frac{4l_{gap}f_{gap}}{\mu_0 a}} = \sqrt{\frac{4 \times 0.001 \times 0.1}{4\pi \times 10^{-7} \times 0.01}} = 56.4\text{ A}\cdot\text{t}$$

The required mmf can be most effectively realized by keeping the current value relatively low, and using a large number of turns.

Comments: The same mmf can be realized with an infinite number of combinations of current and number of turns; however, there are trade-offs involved. If the current is very large (and the number of turns small), the required wire diameter will be very large. Conversely, a small current will require a small wire diameter and a large number of turns. A homework problem explores this trade-off.

EXAMPLE 16.11 Transient Response of a Solenoid

Problem

Analyze the current response of the solenoid of Example 16.10 to a step change in excitation voltage. Plot the force and current as a function of time.

Solution

Known Quantities: Coil inductance and resistance; applied current.

Find: Current and force response as a function of time.

Schematics, Diagrams, Circuits, and Given Data: See Example 16.10. $N = 1000$ turns. $V = 12$ V. $R_{coil} = 5\ \Omega$.

Assumptions: The inductance of the solenoid is approximately constant, and is equal to the midrange value (plunger displacement equal to $a/2$).

Analysis: From Example 16.10, we have an expression for the reluctance of the solenoid:

$$\mathcal{R}_{gap}(x) = \frac{2l_{gap}}{\mu_0 ax}$$

Using equation 16.30, and assuming $x = a/2$, we calculate the inductance of the structure:

$$L \approx \frac{N^2}{\mathcal{R}_{gap}|_{x=a/2}} = \frac{N^2\mu_0 a^2}{4l_{gap}} = \frac{10^6 \times 4\pi \times 10^{-7} \times 10^{-4}}{4 \times 10^{-3}} = 31.4\text{ mH}$$

The equivalent solenoid circuit is shown in Figure 16.42. When the switch is closed, the solenoid current rises exponentially with time constant $\tau = L/R = 6.3$ ms. As shown in Chapter 5, the response is of the form:

$$i(t) = \frac{V}{R}(1 - e^{-t/\tau}) = \frac{V}{R}(1 - e^{-Rt/L}) = \frac{12}{5}(1 - e^{-t/6.3\times 10^{-3}}) \qquad \text{A}$$

To determine how the magnetic force responds during the turn-on transient, we return to the expression for the force derived in Example 16.10:

$$f_{\text{gap}}(t) = \frac{\mu_0 a (Ni)^2}{4l_{\text{gap}}} = \frac{4\pi \times 10^{-7} \times 10^{-2} \times 10^{6}}{4 \times 10^{-3}} i^2(t) = \pi i^2(t)$$

$$= \pi \left[\frac{12}{5}(1 - e^{-t/6.3\times 10^{-3}}) \right]^2$$

The two curves are plotted in Figure 16.42(b).

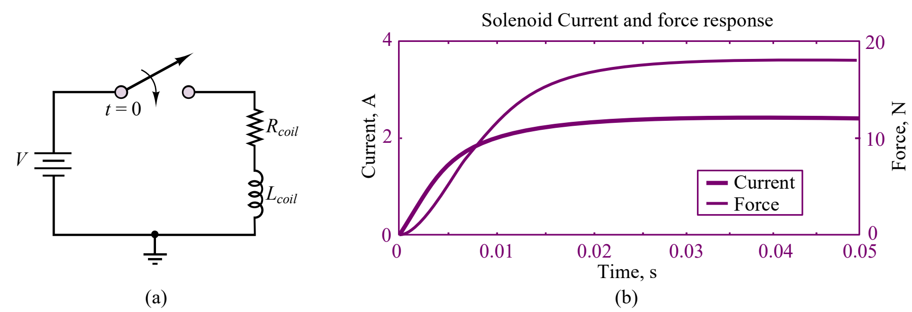


Figure 16.42 Solenoid equivalent electrical circuit and step response

Comments: The assumption that the inductance is approximately constant is not quite accurate. The reluctance (and therefore the inductance) of the structure will change as the plunger moves into position. However, allowing for the inductance to be a function of plunger displacement causes the problem to become nonlinear, and requires numerical solution of the differential equation (i.e., the transient response results of Chapter 5 no longer apply). This issue is explored in a homework problem.

Practical Facts About Solenoids

Solenoids can be used to produce linear or rotary motion, either in the *push* or *pull* mode. The most common solenoid types are listed below:

1. *Single-action linear* (push or pull). Linear stroke motion, with a restoring force (from a spring, for example) to return the solenoid to the neutral position.
2. *Double-acting linear.* Two solenoids back to back can act in either direction. Restoring force is provided by another mechanism (e.g., a spring).
3. *Mechanical latching solenoid* (bistable). An internal latching mechanism holds the solenoid in place against the load.
4. *Keep solenoid.* Fitted with a permanent magnet so that no power is needed to hold the load in the pulled-in position. Plunger is released by applying a current pulse of opposite polarity to that required to pull in the plunger.
5. *Rotary solenoid.* Constructed to permit rotary travel. Typical range is 25 to 95°. Return action via mechanical means (e.g., a spring).
6. *Reversing rotary solenoid.* Rotary motion is from one end to the other; when the solenoid is energized again it reverses direction.

Solenoid power ratings are dependent primarily on the current required by the coil, and on the coil resistance. I^2R is the primary power sink, and solenoids are therefore limited by the heat they can dissipate. Solenoids can operated in continuous or pulsed mode. The power rating depends on the mode of operation, and can be increased by adding *hold-in resistors* to the circuit to reduce the *holding current* required for continuous operation. The hold resistor is switched into the circuit once the *pull-in* current required to pull the plunger has been applied, and the plunger has moved into place. The holding current can be significantly smaller than the pull-in current.

A common method to reduce the solenoid holding current employs a normally closed (NC) switch in parallel with a hold-in resistor. In Figure 16.43, when the push button (PB) closes the circuit, full voltage is applied to the solenoid coil, bypassing the resistor through the NC switch, connecting the resistor in series with the coil. The resistor will now limit the current to the value required to hold the solenoid in position.

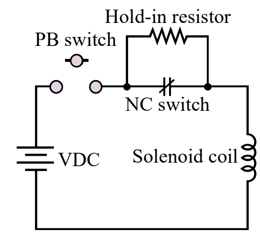


Figure 16.43

Another electromechanical device that finds common application in industrial practice is the **relay.** The relay is essentially an electromechanical switch that permits the opening and closing of electrical contacts by means of an electromagnetic structure similar to those discussed earlier in this section.

A relay such as would be used to start a high-voltage single-phase motor is shown in Figure 16.44. The magnetic structure has dimensions equal to 1 cm on all sides, and the transverse dimension is 8 cm. The relay works as follows. When the push button is pressed, an electrical current flows through the coil and generates a field in the magnetic structure. The resulting force draws the movable part toward the fixed part, causing an electrical contact to be made. The advantage of the relay is that a relatively low-level current can be used to control the opening and closing

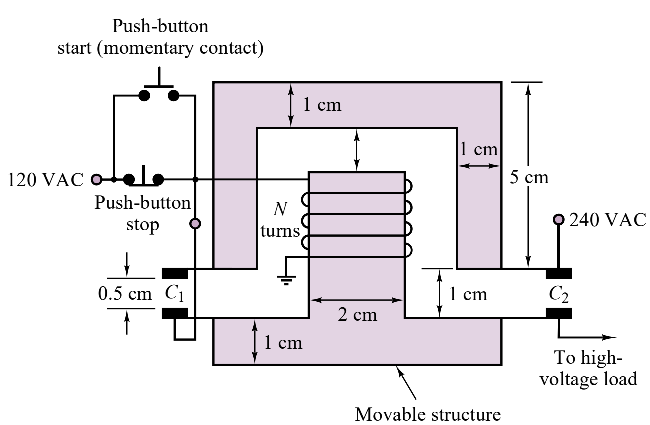
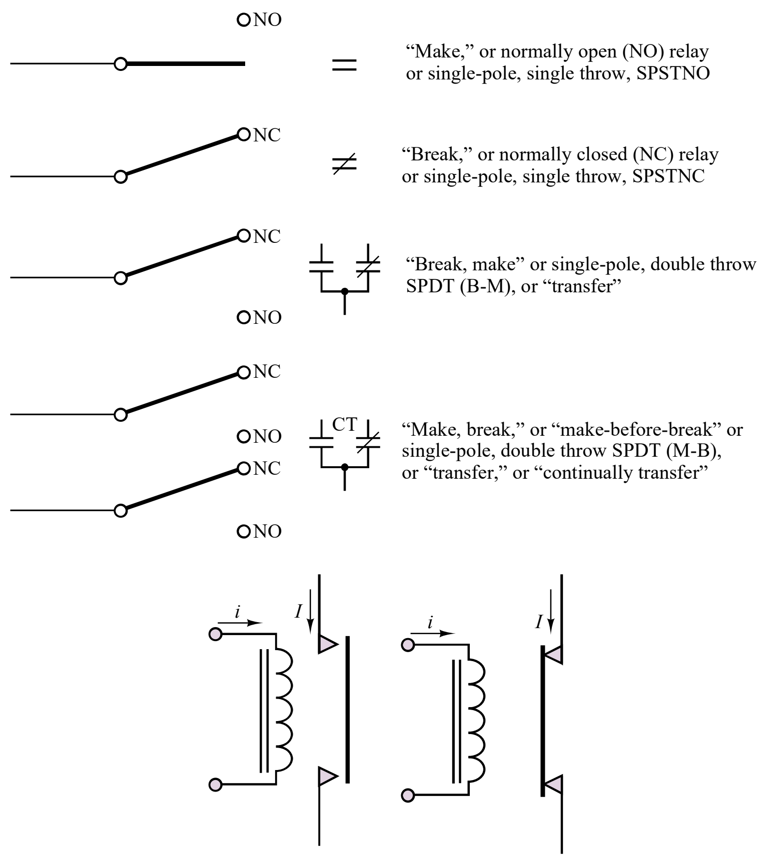


Figure 16.44 A relay

of a circuit that can carry large currents. In this particular example, the relay is energized by a 120-VAC contact, establishing a connection in a 240-VAC circuit. Such relay circuits are commonly employed to remotely switch large industrial loads.

Circuit symbols for relays are shown in Figure 16.45. An example of the calculations that would typically be required in determining the mechanical and electrical characteristics of a simple relay are given in Example 16.12.



Basic operation of the electromechanical relay: The (small) coil current i causes the relay to close (or open) and enables (interrupts) the larger current, I.
On the left: SPSTNO relay (magnetic field causes relay to close).
On the right: SPSTNC relay (magnetic field causes relay to open).

Figure 16.45 Circuit symbols and basic operation of relays

EXAMPLE 16.12 A Relay

Problem

Figure 16.46 depicts a simplified representation of a relay. Determine the current required for the relay to make contact (i.e., pull in the ferromagnetic plate) from a distance x.

Solution

Known Quantities: Relay geometry; restoring force to be overcome; distance between bar and relay contacts; number of coil turns.

Find: i.

Schematics, Diagrams, Circuits, and Given Data: $A_{gap} = (0.01\text{ m})^2$; $x = 0.05$ m; $f_{restore} = 5$ N; $N = 10{,}000$.

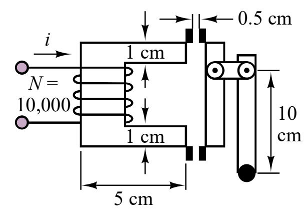


Figure 16.46

Assumptions: Assume that the reluctance of the iron is negligible; neglect fringing.

Analysis:

$$\mathcal{R}_{gap}(x) = \frac{2x}{\mu_0 A_{gap}}$$

The derivative of the reluctance with respect to the displacement of the plunger can then be computed to be:

$$\frac{d\mathcal{R}_{gap}(x)}{dx} = \frac{2}{\mu_0 A_{gap}}$$

Knowing the reluctance, we can calculate the magnetic flux in the structure as a function of the coil current:

$$\phi = \frac{Ni}{\mathcal{R}(x)} = \frac{Ni\mu_0 A_{gap}}{2}$$

and the force in the air gap is given by the expression

$$f_{gap} = \frac{\phi^2}{2}\frac{d\mathcal{R}(x)}{dx} = \frac{(Ni\mu_0 A_{gap})^2}{8}\frac{2}{\mu_0 A_{gap}} = \frac{\mu_0 A_{gap}(Ni)^2}{4}$$

The magnetic force must overcome a mechanical holding force of 5 N, thus,

$$f_{gap} = \frac{\mu_0 A_{gap}(Ni)^2}{4} = f_{restore} = 5\text{ N}$$

or

$$i = \frac{1}{N}\sqrt{\frac{4f_{restore}}{\mu_0 A_{gap}}} = \frac{1}{10{,}000}\sqrt{\frac{20}{4\pi \times 10^{-7} \times 0.0001}} = 39.9\text{ A}$$

Comments: The current required to close the relay is much larger than that required to hold the relay closed, because the reluctance of the structure is much smaller once the gap is reduced to zero.

Moving-Coil Transducers

Another important class of electromagnetomechanical transducers is that of **moving-coil transducers.** This class of transducers includes a number of common devices, such as microphones, loudspeakers, and all electric motors and generators. The aim of this section is to explain the relationship between a fixed magnetic field, the emf across the moving coil, and the forces and motions of the moving element of the transducer.

The basic principle of operation of electromechanical transducers was presented in Section 16.1, where we stated that a magnetic field exerts a force on a charge moving through it. The equation describing this effect is

$$\mathbf{f} = q\mathbf{u} \times \mathbf{B} \quad \textbf{(16.51)}$$

which is a vector equation, as explained earlier. In order to correctly interpret equation 16.51, we must recall the right-hand rule and apply it to the transducer, illustrated in Figure 16.47, depicting a structure consisting of a sliding bar which makes contact with a fixed conducting frame. Although this structure does not represent a practical actuator, it will be a useful aid in explaining the operation of moving-coil transducers such as motors and generators. In Figure 16.47, and in all similar figures in this section, a small cross represents the "tail" of an arrow pointing into the page, while a dot represents an arrow pointing out of the page; this convention will be useful in visualizing three-dimensional pictures.

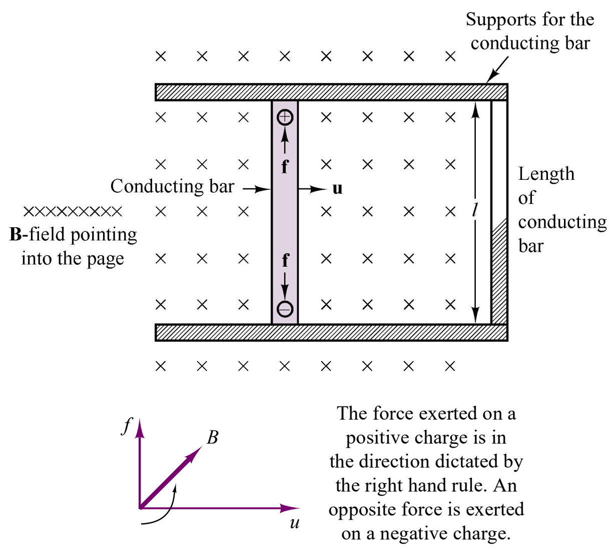


Figure 16.47 A simple electromechanical motion transducer

Motor Action

A moving-coil transducer can act as a motor when an externally supplied current flowing through the electrically conducting part of the transducer is converted into a force that can cause the moving part of the transducer to be displaced. Such a current would flow, for example, if the support of Figure 16.47 were made of conducting material, so that the conductor and the right-hand side of the support "rail" were to form a loop (in effect, a 1-turn coil). In order to understand the effects of this current flow in the conductor, one must consider the fact that a charge moving at a velocity u' (along the conductor and perpendicular to the velocity of the conducting bar, as shown in Figure 16.48) corresponds to a current $i = dq/dt$ along the length l of the conductor. This fact can be explained by considering the current i along a differential element dl and writing

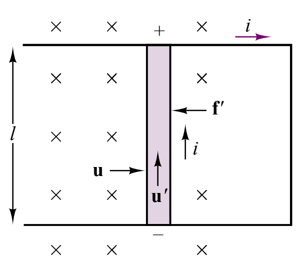


Figure 16.48

$$i\,dl = \frac{dq}{dt} \cdot u'\,dt \qquad \textbf{(16.52)}$$

since the differential element dl would be traversed by the current in time dt at a velocity u'. Thus we can write

$$i\,dl = dq\,u' \qquad \textbf{(16.53)}$$

or

$$il = qu' \tag{16.54}$$

for the geometry of Figure 16.48. From Section 16.1, the force developed by a charge moving in a magnetic field is, in general, given by

$$\mathbf{f} = q\mathbf{u} \times \mathbf{B} \tag{16.55}$$

For the term $q\mathbf{u}'$ we can substitute $i\mathbf{l}$, to obtain

$$\mathbf{f}' = i\mathbf{l} \times \mathbf{B} \tag{16.56}$$

Using the right-hand rule, we determine that the force $\mathbf{f}'$ generated by the current i is in the direction that would push the conducting bar to the left. The magnitude of this force is $f' = Bli$ if the magnetic field and the direction of the current are perpendicular. If they are not, then we must consider the angle γ formed by $\mathbf{B}$ and $\mathbf{l}$; in the more general case,

$$f' = Bli \sin\gamma \tag{16.57}$$

The phenomenon we have just described is sometimes referred to as the "***Bli*** **law.**"

Generator Action

The other mode of operation of a moving-coil transducer occurs when an external force causes the coil (i.e., the moving bar, in Figure 16.47) to be displaced. This external force is converted to an emf across the coil, as will be explained in the following paragraphs.

Since positive and negative charges are forced in opposite directions in the transducer of Figure 16.47, a potential difference will appear across the conducting bar; this potential difference is the electromotive force, or emf. The emf must be equal to the force exerted by the magnetic field. In short, the electric force per unit charge (or electric field) e/l must equal the magnetic force per unit charge $f/q = Bu$. Thus, the relationship

$$e = Blu \tag{16.58}$$

which holds whenever **B, l,** and **u** are mutually perpendicular, as in Figure 16.49. If equation 16.58 is analyzed in greater depth, it can be seen that the product lu (length times velocity) is the area crossed per unit time by the conductor. If one visualizes the conductor as "cutting" the flux lines into the base in Figure 16.48, it can be concluded that the electromotive force is equal to the *rate at which the conductor "cuts" the magnetic lines of flux*. It will be useful for you to carefully absorb this notion of conductors cutting lines of flux, since this will greatly simplify understanding the material in this section and in the next chapter.

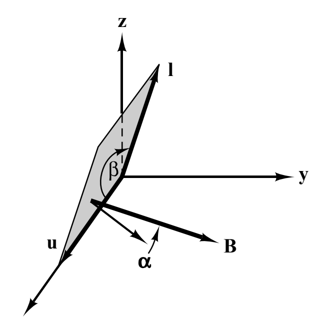


Figure 16.49

In general, **B, l,** and **u** are not necessarily perpendicular. In this case one needs to consider the angles formed by the magnetic field with the normal to the plane containing **l** and **u,** and the angle between **l** and **u.**. The former is the angle α of Figure 16.49, the latter the angle β in the same figure. It should be apparent that the optimum values of α and β are 0° and 90°, respectively. Thus, most practical devices are constructed with these values of α and β. Unless otherwise noted, it will be tacitly assumed that this is the case. The "***Bli* law**" just illustrated explains how a moving conductor in a magnetic field can generate an electromotive force.

To summarize the electromechanical energy conversion that takes place in the simple device of Figure 16.47, we must note now that the presence of a current in the loop formed by the conductor and the rail requires that the conductor move to the right at a velocity u (Blu law), thus cutting the lines of flux and generating the emf that gives rise to the current i. On the other hand, the same current causes a force f' to be exerted on the conductor (Bli law) in the direction opposite to the movement of the conductor. Thus, it is necessary that an *externally applied force* f_{ext} exist to cause the conductor to move to the right with a velocity u. The external force must overcome the force f'. This is the basis of electromechanical energy conversion.

An additional observation we must make at this point is that the current i flowing around a closed loop generates a magnetic field, as explained in Section 16.1. Since this additional field is generated by a one-turn coil in our illustration, it is reasonable to assume that it is negligible with respect to the field already present (perhaps established by a permanent magnet). Finally, we must consider that this coil links a certain amount of flux, which changes as the conductor moves from left to right. The area crossed by the moving conductor in time dt is

$$dA = lu\,dt \qquad \textbf{(16.59)}$$

so that if the flux density, B, is uniform, the rate of change of the flux linked by the one-turn coil is

$$\frac{d\phi}{dt} = B\frac{dA}{dt} = Blu \qquad \textbf{(16.60)}$$

In other words, *the rate of change* of the flux linked by the conducting loop is equal to the emf generated in the conductor. The student should realize that this statement simply confirms Faraday's law.

It was briefly mentioned that the Blu and Bli laws indicate that, thanks to the coupling action of the magnetic field, a conversion of mechanical to electrical energy—or the converse—is possible. The simple structures of Figures 16.47 and 16.48 can, again, serve as an illustration of this energy-conversion process, although we have not yet indicated how these idealized structures can be converted into a practical device. In this section we shall begin to introduce some physical considerations. Before we proceed any further, we should try to compute the power—electrical and mechanical—that is generated (or is required) by our ideal transducer. The electrical power is given by

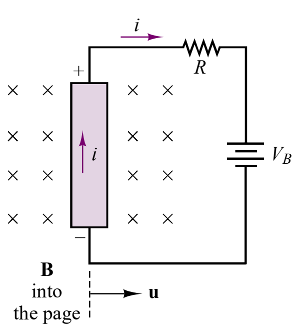


Figure 16.50 Motor and generator action in an ideal transducer

$$P_E = ei = Blui \qquad \text{(W)} \qquad \textbf{(16.61)}$$

while the mechanical power required, say, to move the conductor from left to right is given by the product of force and velocity:

$$P_M - f_{\text{ext}}u = Bliu \qquad \text{(W)} \qquad \textbf{(16.62)}$$

The principle of conservation of energy thus states that in this ideal (lossless) transducer we can convert a given amount of electrical energy into mechanical energy, or vice versa. Once again we can utilize the same structure of Figure 16.47 to illustrate this reversible action. If the closed path containing the moving conductor is now formed from a closed circuit containing a resistance R and a battery, V_B, as shown in Figure 16.50, the externally applied force, f_{ext}, generates a positive current i into the battery provided that the emf is greater than V_B. When $e = Blu > V_B$, the ideal transducer acts as a *generator*. For any given set of values of B, l, R, and V_B, there will exist a velocity u for which the current i is positive. If the velocity is lower than this value—i.e., if $e = Blu < V_B$—then the current i is negative, and the conductor is forced to move to the right. In this case the battery acts as a source of energy and the transducer acts as a *motor* (i.e., electrical energy drives the mechanical motion).

In practical transducers, we must be concerned with the inertia, friction, and elastic forces that are invariably present on the mechanical side of the transducer. Similarly, on the electrical side we must account for the inductance of the circuit, its resistance, and possibly some capacitance. Consider the structure of Figure 16.51. In the figure, the conducting bar has been placed on a surface with coefficient of sliding friction d; it has a mass m and is attached to a fixed structure by means of a spring with spring constant k. The equivalent circuit representing the coil inductance and resistance is also shown.

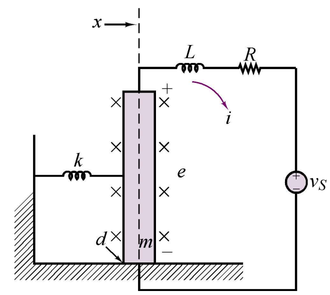


Figure 16.51 A more realistic representation of the transducer of Figure 16.50

If we recognize that $u = dx/dt$ in the figure, we can write the equation of motion for the conductor as:

$$f + m\frac{du}{dt} + du + \frac{l}{k}\int u\,dt = f' = Bli \tag{16.63}$$

where the Bli term represents the driving input that causes the mass to move. The driving input in this case is provided by the electrical energy source, v_S; thus the transducer acts as a motor, and f is the net force acting on the mass of the conductor. On the electrical side, the circuit equation is:

$$v_S - L\frac{di}{dt} - Ri = e = Blu \tag{16.64}$$

Equations 16.63 and 16.64 could then be solved by knowing the excitation voltage, v_S, and the physical parameters of the mechanical and electrical circuits. For example, if the excitation voltage were sinusoidal, with

$$v_S(t) = V_S \cos \omega t \tag{16.65}$$

and the field density were constant:

$$B = B_0$$

we could postulate sinusoidal solutions for the transducer velocity, u, and current, i:

$$u = U\cos(\omega t + \theta_u) \qquad i = I\cos(\omega t + \theta_i) \tag{16.66}$$

and use phasor notation to solve for the unknowns $(U, I, \theta_u, \theta_i)$.

The results obtained in the present section apply directly to transducers that are based on translational (linear) motion. These basic principles of electromechanical energy conversion and the analysis methods developed in the section will be applied to practical transducers in a few examples.

The methods introduced in this section will later be applied in Chapters 17 and 18 to analyze rotating transducers, that is, electric motors and generators.

Seismic Transducer

FIND IT ON THE WEB

The device shown in Figure 16.52 is called a **seismic transducer** and can be used to measure the displacement, velocity, or acceleration of a body. The permanent magnet of mass m is supported on the case by a spring, k, and there is some viscous damping, d, between the magnet and the case; the coil is fixed to the case. You may assume that the coil has length l and resistance and inductance R_{coil} and L_{coil}, respectively; the magnet exerts a magnetic field B. Find the transfer function between the output voltage, v_{out}, and the acceleration of the body, $a(t)$. Note that $x(t)$ is not equal to zero when the system is at rest. We shall ignore this offset displacement.

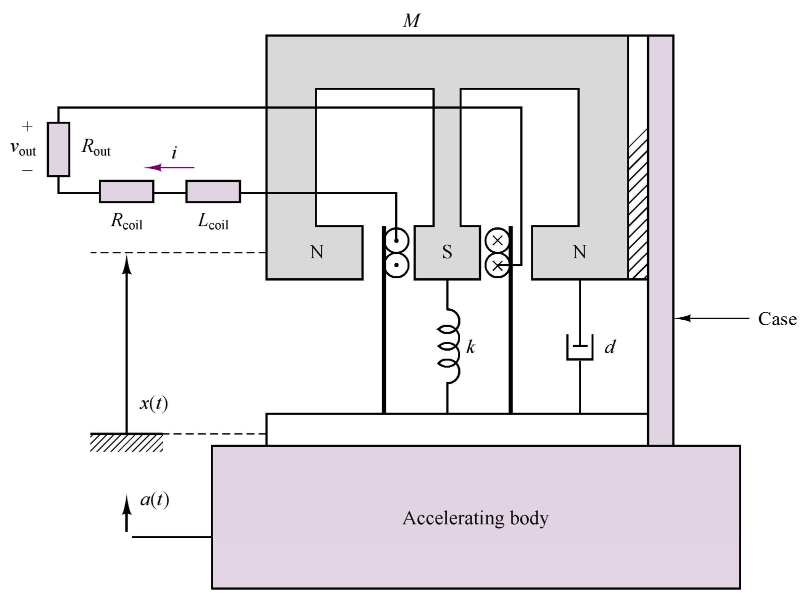


Figure 16.52 An electromagnetomechanical seismic transducer

Solution:

First we apply KVL around the electrical circuit to write the differential equation describing the electrical system:

$$L\frac{di}{dt} + (R_{\text{coil}} + R_{\text{out}})i + Bl\frac{dx}{dt} = 0$$

Also note that $v_{\text{out}} = -R_{\text{out}}i$. Next, we write the differential equation describing the mechanical system. The magnet experiences an inertial force due to the acceleration of the supporting body, $a(t)$, and to its own relative acceleration, d^2x/dt^2; thus, we can sketch a free-body diagram and apply Newton's second law to the permanent magnet, as shown in the sketch.

$$M\left(a + \frac{d^2x}{dt^2}\right) + d\frac{dx}{dt} + kx = Bli$$

Finally, using the Laplace transform, we determine the transfer function from $A(s)$ to $V_{out}(s)$. Let $R = R_{coil} + R_{out}$. Then

$$(Ls + R)I(s) + BlsX(s) = 0$$
$$BlI(s) - (Ms^2 + Ds + K)X(s) = MA(s)$$

Since we need the transfer function from A to V_{out}, we use the expression

$$V_{out}(s) = -R_{out}I(s)$$

and, after some algebra, find that

$$I(s) = \frac{MBlsA(s)}{(Ls + R)(Ms^2 + Ds + K) + B^2l^2s}$$

or

$$\frac{V_{out}(s)}{A(s)} = \frac{-MBsR_{out}}{(Ls + R)(Ms^2 + Ds + K) + B^2l^2s}$$

EXAMPLE 16.13 A Loudspeaker

Problem

A loudspeaker, shown in Figure 16.53, uses a permanent magnet and a moving coil to produce the vibrational motion that generates the pressure waves we perceive as sound. Vibration of the loudspeaker is caused by changes in the input current to a coil; the coil is, in turn, coupled to a magnetic structure that can produce time-varying forces on the speaker diaphragm. A simplified model for the mechanics of the speaker is also shown in Figure 16.53. The force exerted on the coil is also exerted on the mass of the speaker diaphragm, as shown in Figure 16.54, which depicts a free-body diagram of the forces acting on the loudspeaker diaphragm.

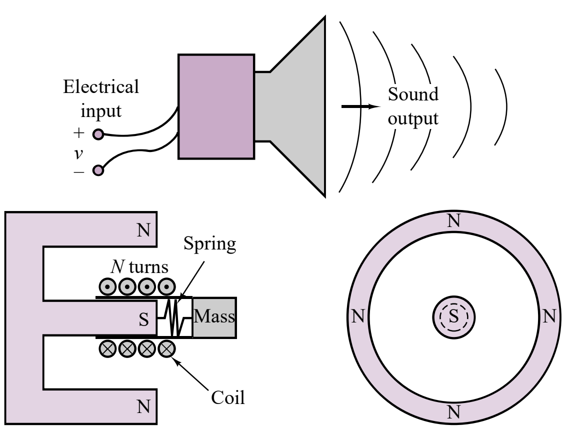


Figure 16.53 Loudspeaker

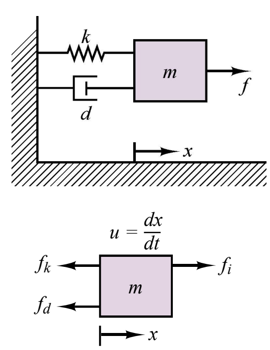


Figure 16.54 Forces acting on loudspeaker diaphragm

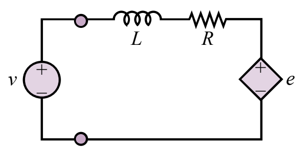


Figure 16.55 Model of transducer electrical side

The force exerted on the mass, f_i, is the magnetic force due to current flow in the coil. The electrical circuit that describes the coil is shown in Figure 16.55, where L represents the inductance of the coil, R represents the resistance of the windlings, and e is the emf induced by the coil moving through the magnetic field.

Determine the frequency response, $U(j\omega)/V(j\omega)$ of the speaker.

Solution

Known Quantities: Circuit and mechanical parameters; magnetic flux density; number of coil turns; coil radius.

Find: Frequency response of loudspeaker, $U(j\omega)/V(j\omega)$.

Schematics, Diagrams, Circuits, and Given Data: Coil radius $= 0.05$ m; $L = 10$ mH; $R = 8\ \Omega$; $m = 0.001$ kg; $d = 22.75\ \text{N}\cdot\text{s}^2/\text{m}$; $k = 5 \times 10^5$ N/m; $N = 47$; $B = 1$ T.

Analysis: To determine the frequency response of the loudspeaker, we write the differential equations that describe the electrical and mechanical subsystems. We apply KVL to the electrical circuit, using the circuit model of Figure 16.55, in which we have represented the Blu term (motional voltage) in the form of a *back electromotive force*, e:

$$v - L\frac{di}{dt} - Ri - e = 0$$

or

$$L\frac{di}{dt} + Ri + Blu = v$$

Next, we apply Newton's second law to the mechanical system, consisting of: a lumped mass representing the mass of the moving diaphragm, m; an elastic (spring) term, which represents the elasticity of the diaphragm, k; and a damping coefficient, d, representing the frictional losses and aerodynamic damping affecting the moving diaphragm.

$$m\frac{du}{dt} = f_i - f_d - f_k = f_i - du - kx$$

where $f_i = Bli$ and therefore

$$-Bli + m\frac{du}{dt} + du + k\int_{-\infty}^{t} u(t')\,dt' = 0$$

Note that the two equations are *coupled*, that is, a mechanical variable appears in the electrical equation (the velocity u in the Blu term), and an electrical variable appears in the mechanical equation (the current i in the Bli term).

To derive the frequency response we Laplace-transform the two equations to obtain:

$$(sL + R)I(s) + BlU(s) = V(s)$$

$$-BlI(s) + \left(sm + d + \frac{k}{s}\right)U(s) = 0$$

We can write the above equations in matrix form and resort to Cramer's rule to solve for $U(s)$ as a function of $V(s)$:

$$\begin{bmatrix} (sL + R) & Bl \\ -Bl & \left(sm + d + \frac{k}{s}\right) \end{bmatrix} \begin{bmatrix} I(s) \\ U(s) \end{bmatrix} = \begin{bmatrix} V(s) \\ 0 \end{bmatrix}$$

with solution

$$U(s) = \frac{\det\begin{bmatrix}(sL+R) & V(s)\\ -Bl & 0\end{bmatrix}}{\det\begin{bmatrix}(sL+R) & Bl\\ -Bl & \left(sm+d+\frac{k}{s}\right)\end{bmatrix}}$$

or

$$\frac{U(s)}{V(s)} = \frac{-Bl}{(sL+R)\left(sm+d+\frac{k}{s}\right)+(Bl)^2}$$
$$= \frac{-Bls}{(Lm)s^3+(Rm+Ld)s^2+(Rd+kL+(Bl)^2)s+(kR)}$$

To determine the frequency response of the loudspeaker, we let $s \to j\omega$ in the above expression:

$$\frac{U(j\omega)}{V(j\omega)} = \frac{-jBl\omega}{(kR)-(Rm+Ld)\omega^2+j[(Rd+kL+(Bl)^2)\omega-(Lm)\omega^3]}$$

where $l = 2\pi Nr$, and substitute the appropriate numerical parameters:

$$\frac{U(j\omega)}{V(j\omega)} = \frac{-j14.8\omega}{(5\times10^5)-(0.008+0.2275)\omega^2+j[(182+5{,}000+218)\omega-(10^{-5})\omega^3]}$$
$$= \frac{-j14.8\omega}{(5\times10^5)-(0.2355)\omega^2+j[(5.4\times10^3)\omega-(10^{-5})\omega^3]}$$

The resulting frequency response is plotted in Figure 16.56.

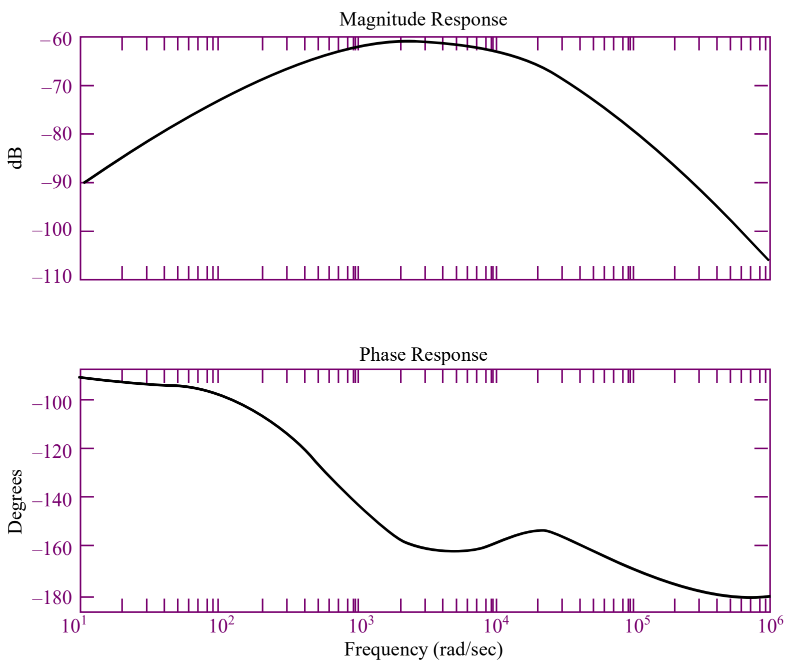


Figure 16.56 Frequency response of loudspeaker

Focus on Computer-Aided Tools: A Matlab m-file containing the frequency response calculations leading to the frequency response (Bode) plot of Figure 16.56 may be found in the accompanying CD-ROM.

Check Your Understanding

16.13 The flux density of the earth's magnetic field is about 50 μT. Estimate the current required in a conductor of length 10 cm and mass 10 g to counteract the force of gravity if the wire is oriented in the optimum direction.

16.14 In Example 16.13, we examined the frequency response of a loudspeaker. However, over a period of time, permanent magnets may become demagnetized. Find the frequency response of the same loudspeaker if the permanent magnet has lost its strength to a point where $B = 0.95$ T.

16.15 In Example 16.10, a solenoid is used to exert force on a spring. Estimate the position of the plunger if the number of turns in the solenoid winding is 1,000 and the current going into the winding is 40 mA.

16.16 For the circuit in Figure 16.47, the conducting bar is moving with a velocity of 6 m/s. The flux density is 0.5 Wb/m^2, and $l = 1.0$ m. Find the magnitude of the resulting induced voltage.

CONCLUSION

- Magnetic fields form a coupling mechanism between electrical and mechanical systems, permitting the conversion of electrical energy to mechanical energy, and vice versa. The basic laws that govern such electromechanical energy conversion are Faraday's law, stating that a changing magnetic field can induce a voltage; and Ampère's law, stating that a current flowing through a conductor generates a magnetic field.
- The two fundamental variables in the analysis of magnetic structures are the magnetomotive force and the magnetic flux; if some simplifying approximations are made, these quantities are linearly related through the reluctance parameter, much in the same way as voltage and current are related through resistance according to Ohm's law. This simplified analysis permits approximate calculations of required forces and currents to be conducted with relative ease in magnetic structures.
- Magnetic materials are characterized by a number of nonideal properties, which should be considered in the detailed analysis of a magnetic structure. The most important phenomena are saturation, eddy currents, and hysteresis.
- Electromechanical transducers, which convert electrical signals to mechanical forces, or mechanical motion to electrical signals, can be analyzed according to the techniques presented in this chapter. Examples of such transducers are electromagnets, position and velocity sensors, relays, solenoids, and loudspeakers.

CHECK YOUR UNDERSTANDING ANSWERS

CYU 16.1	$e = -2.5$ V
CYU 16.2	$I = \pi$ A
CYU 16.3	$W_m = 0.648$ J
CYU 16.5	$\phi = 3.94 \times 10^{-6}$ Wb; $B = 0.0788$ Wb/m^2
CYU 16.6	$\mathcal{R}_{eq} = 1.41 \times 10^6$ A $\cdot$ t/Wb

CYU 16.7	$\mathcal{R}_{eq} = 22 \times 10^6$ A · t/Wb
CYU 16.8	$\mathcal{R}_g = \mathcal{R}_1 = \mathcal{R}_2 = \mathcal{R}_3 = \delta/\mu_0(l+\delta)(w+\delta)$; $\mathcal{F}_1 = Ni_1$; $\mathcal{F}_2 = Ni_2$
CYU 16.9	$\alpha = 5$; $I_1 = I_2/\alpha = 2.4$ A
CYU 16.10	$\alpha = 0.2$
CYU 16.11	$\eta = 98\%$
CYU 16.12	$\alpha = 10$
CYU 16.13	$i = 196 \times 10^2$ A
CYU 16.14	$U(j\omega)/V(j\omega) = 0.056(j\omega/15{,}950)/(1 + j\omega/15{,}950)(1 + j\omega/31{,}347)$
CYU 16.15	$x = 0.5$ cm
CYU 16.16	3 V

HOMEWORK PROBLEMS

Section 1: Electricity and Magnetism

16.1 An iron-core inductor has the following characteristic:

$$i = \lambda + 0.5\lambda^2$$

a. Determine the energy, co-energy, and incremental inductance for $\lambda = 0.5$ V · s.

b. Given that the coil resistance is 1 Ω and that

$$i(t) = 0.625 + 0.01 \sin 400t \text{ A}$$

determine the voltage across the terminals on the inductor.

16.2 For the electromagnet of Figure P16.2:

a. Find the flux density in the core.

b. Sketch the magnetic flux lines and indicate their direction.

c. Indicate the north and south poles of the magnet.

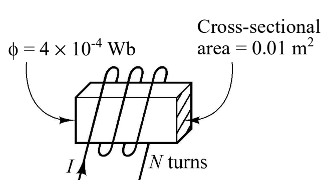


Figure P16.2

16.3 An iron-core inductor has the characteristic shown in Figure P16.3:

a. Determine the energy and the incremental inductance for $i = 1.0$ A.

b. Given that the coil resistance is 2 Ω and that $i(t) = 0.5 \sin 2\pi t$, determine the voltage across the terminals of the inductor.

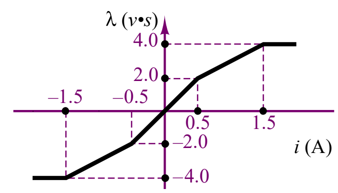


Figure P16.3

16.4 A single loop of wire carrying current I_2 is placed near the end of a solenoid having N turns and carrying current I_1, as shown in Figure P16.4. The solenoid is fastened to a horizontal surface, but the single coil is free to move. With the currents directed as shown, is there a resultant force on the single coil? If so, in what direction? Why?

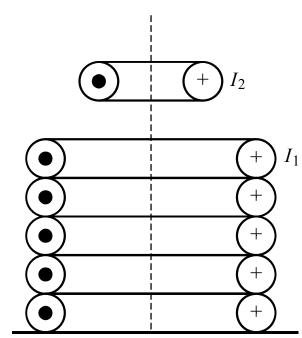


Figure P16.4

16.5 The electromagnet of Figure P16.5 has reluctance given by $\mathcal{R}(x) = 7 \times 10^8(0.002 + x)$ H^{-1}, where x is the length of the variable gap in meters. The coil has 980 turns and 30 Ω resistance. For an applied voltage of 120 VDC, find:

a. The energy stored in the magnetic field for $x = 0.005$ m.

b. The magnetic force for $x = 0.005$ m.

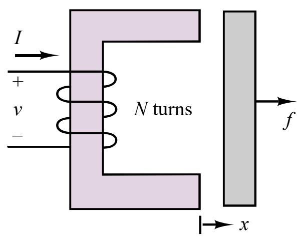


Figure P16.5

16.6 A practical LVDT is typically connected to a resistive load. Derive the LVDT equations in the presence of a resistive load, R_L, connected across the output terminals, using the results of "Focus on Measurements: Linear Variable Differential Transformer."

16.7 On the basis of the equations of "Focus on Measurements: Linear Variable Differential Transformer," and of the results of Problem 16.6, derive the frequency response of the LVDT, and determine the range of frequencies for which the device will have maximum sensitivity for a given excitation. [*Hint:* Compute $dv_{\text{out}}/dv_{\text{ex}}$, and set the derivative equal to zero to determine the maximum sensitivity.]

16.8 A wire of length 20 cm vibrates in one direction in a constant magnetic field with a flux density of 0.1 T; see Figure P16.8. The position of the wire as a function of time is given by $x(t) = 0.1 \sin 10t$ m. Find the induced emf across the length of the wire as a function of time.

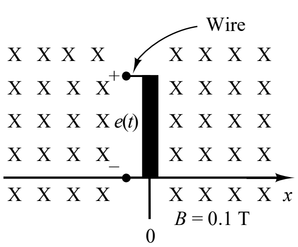


Figure P16.8

16.9 The wire of Problem 16.8 induces a time-varying emf of

$$e_1(t) = 0.02 \cos 10t$$

A second wire is placed in the same magnetic field but has a length of 0.1 m, as shown in Figure P16.9. The position of this wire is given by $x(t) = 1 - 0.1 \sin 10t$. Find the induced emf $e(t)$ defined by the difference in emf's $e_1(t)$ and $e_2(t)$.

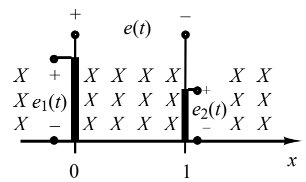


Figure P16.9

16.10 A conducting bar shown in Figure 16.48 in the text, is carrying 4 A of current in the presence of a magnetic field; $B = 0.3$ Wb/m^2. Find the magnitude and direction of the force induced on the conducting bar.

16.11 A wire, shown in Figure P16.11, is moving in the presence of a magnetic field, with $B = 0.4$ Wb/m^2. Find the magnitude and direction of the induced voltage in the wire.

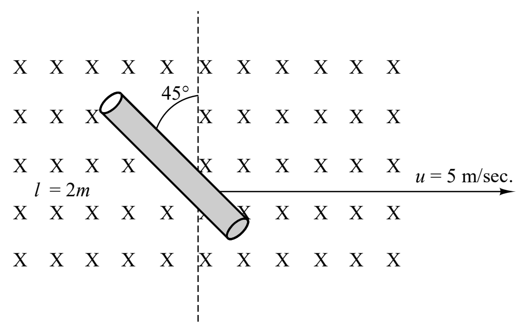


Figure P16.11

Section 2: Magnetic Circuits

16.12

a. Find the reluctance of a magnetic circuit if a magnetic flux $\phi = 4.2 \times 10^{-4}$ Wb is established by an impressed mmf of 400 A · t.

b. Find the magnetizing force, H, in SI units if the magnetic circuit is 6 inches in length.

16.13 For the circuit shown in Figure P16.13:

a. Determine the reluctance values and show the magnetic circuit, assuming that $\mu = 3{,}000\mu_0$.

b. Determine the inductance of the device.

c. The inductance of the device can be modified by cutting an air gap in the magnetic structure. If a gap of 0.1 mm is cut in the arm of length l_3, what is the new value of inductance?

d. As the gap is increased in size (length), what is the limiting value of inductance? Neglect leakage flux and fringing effects.

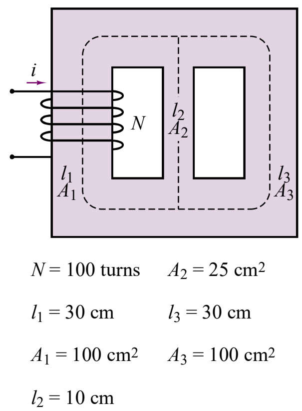


Figure P16.13

16.14 The magnetic circuit shown in Figure P16.14 has two parallel paths. Find the flux and flux density in each of the legs of the magnetic circuit. Neglect fringing at the air gaps and any leakage fields. $N = 1{,}000$ turns, $i = 0.2$ A, $l_{g1} = 0.02$ cm, and $l_{g2} = 0.04$ cm. Assume the reluctance of the magnetic core to be negligible.

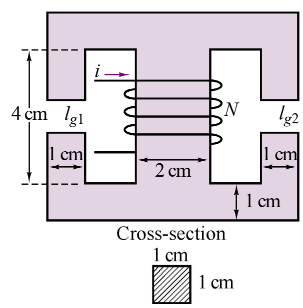


Figure P16.14

16.15 Find the current necessary to establish a flux of $\phi = 3 \times 10^{-4}$ Wb in the series magnetic circuit of Figure P16.15. Here, $l_{iron} = l_{steel} = 0.3$ m, Area (throughout) $= 5 \times 10^{-4}$ m^2, and $N = 100$ turns.

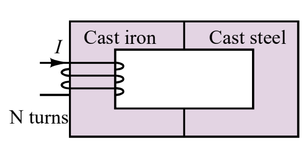


Figure P16.15

16.16

a. Find the current, I, required to establish a flux $\phi = 2.4 \times 10^{-4}$ Wb in the magnetic circuit of Figure P16.16. Here, Area(throughout) $=$ 2×10^{-4} m^2, $l_{ab} = l_{ef} = 0.05$ m, $l_{af} = l_{be} = 0.02$ m, $l_{bc} = l_{dc}$, and the material is sheet steel.

b. Compare the mmf drop across the air gap to that across the rest of the magnetic circuit. Discuss your results using the value of μ for each material.

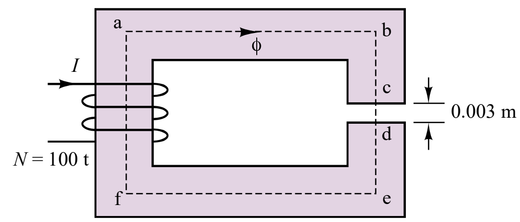


Figure P16.16

16.17 Find the magnetic flux, ϕ, established in the series magnetic circuit of Figure P16.17.

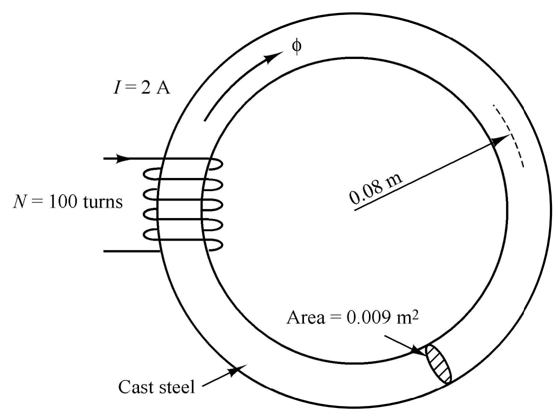


Figure P16.17

16.18 For the series-parallel magnetic circuit of Figure P16.18, find the value of I required to establish a flux in the gap of $\phi = 2 \times 10^{-4}$ Wb. Here, $l_{ab} = l_{bg} = l_{gh} = l_{ha} = 0.2$ m, $l_{bc} = l_{fg} = 0.1$ m, $l_{cd} = l_{ef} = 0.099$ m, and the material is sheet steel.

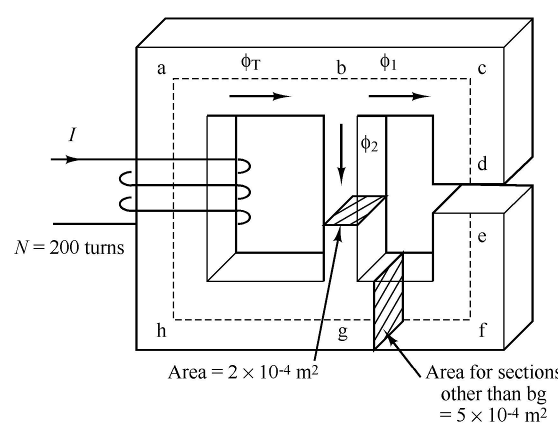


Figure P16.18

16.19 Refer to the actuator of Figure P16.19. The entire device is made of sheet steel. The coil has 2,000 turns. The armature is stationary so that the length of the air gaps, g = 10 mm, is fixed. A direct current passing through the coil produces a flux density of 1.2 T in the gaps. Determine:

a. The coil current.
b. The energy stored in the air gaps.
c. The energy stored in the steel.

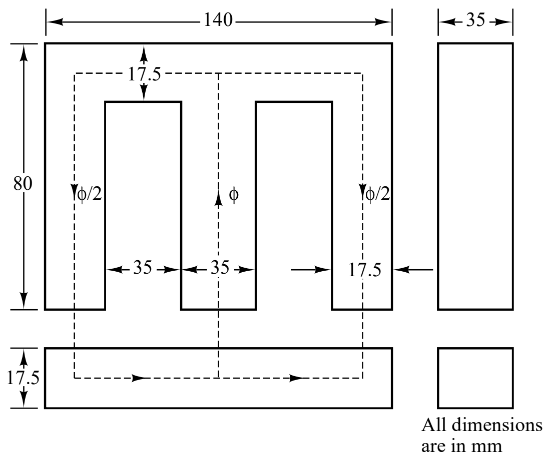


Figure P16.19

16.20 A core is shown in Figure P16.20, with $\mu_r = 2{,}000$ and $N = 100$. Find:

a. The current needed to produce a flux density of 0.4 Wb/m^2 in the center leg.
b. The current needed to produce a flux density of 0.8 Wb/m^2 in the center leg.

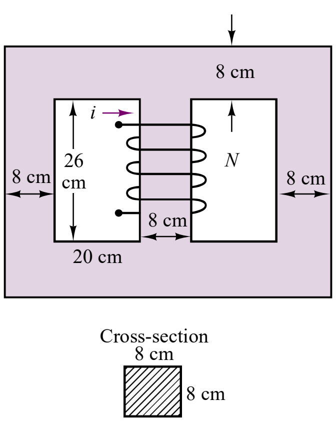


Figure P16.20

Section 3: Transformers

16.21 For the transformer shown in Figure P16.21, $N = 1{,}000$ turns, $l_1 = 16$ cm, $A_1 = 4$ cm^2, $l_2 = 22$ cm, $A_2 = 4$ cm^2, $l_3 = 5$ cm, and $A_3 = 2$ cm^2. The relative permeability of the material is $\mu_r = 1{,}500$.

a. Construct the equivalent magnetic circuit, and find the reluctance associated with each part of the circuit.
b. Determine the self-inductance and mutual inductance for the pair of coils (i.e., L_{11}, L_{22}, and $M = L_{12} = L_{21}$).

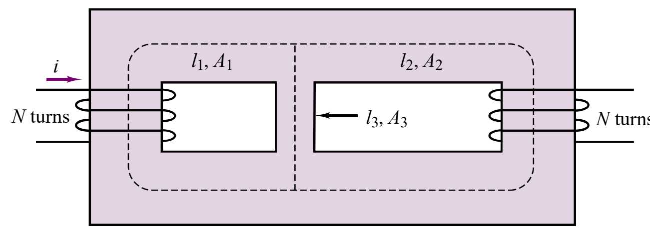


Figure P16.21

16.22 A transformer is delivering power to a 300-Ω resistive load. To achieve the desired power transfer, the turns ratio is chosen so that the resistive load referred to the primary is 7,500 Ω. The parameter values, *referred to the secondary winding,* are:

$r_1 = 20\ \Omega$ $\quad L_1 = 1.0$ mH $\quad L_m = 25$ mH

$r_2 = 20\ \Omega$ $\quad L_2 = 1.0$ mH

Core losses are negligible.

a. Determine the turns ratio.
b. Determine the input voltage, current, and power and the efficiency when this transformer is delivering 12 W to the 300-Ω load at a frequency $f = 10{,}000/2\pi$ Hz.

16.23 A 220/20-V transformer has 50 turns on its low-voltage side. Calculate

a. The number of turns on its high side.
b. The turns ratio α when it is used as a step-down transformer.
c. The turns ratio α when it is used as a step-up transformer.

16.24 The high-voltage side of a transformer has 750 turns, and the low-voltage side 50 turns. When the high side is connected to a rated voltage of 120 V, 60 Hz, a rated load of 40 A is connected to the low side. Calculate

a. The turns ratio.
b. The secondary voltage (assuming no internal transformer impedance voltage drops).
c. The resistance of the load.

16.25 A transformer is to be used to match an 8-Ω loudspeaker to a 500-Ω audio line. What is the turns ratio of the transformer, and what are the voltages at the primary and secondary terminals when 10 W of audio power is delivered to the speaker? Assume that the speaker is a resistive load and the transformer is ideal.

16.26 The high-voltage side of a step-down transformer has 800 turns, and the low-voltage side has 100 turns. A voltage of 240 VAC is applied to the high side, and the load impedance is 3 Ω (low side). Find

a. The secondary voltage and current.
b. The primary current.
c. The primary input impedance from the ratio of primary voltage and current.
d. The primary input impedance.

16.27 Calculate the transformer ratio of the transformer in Problem 16.26 when it is used as a step-up transformer.

16.28 A 2,300/240-V, 60-Hz, 4.6-kVA transformer is designed to have an induced emf of 2.5 V/turn. Assuming an ideal transformer, find

a. The number of high-side turns, N_h, and low-side turns, N_l.
b. The rated current of the high-voltage side, I_h.
c. The transformer ratio when the device is used as a step-up transformer.

Section 4: Electromechanical Transducers

16.29 For the electromagnet of Example 16.9:

a. Calculate the current required to keep the bar in place. (*Hint:* The air gap becomes zero and the iron reluctance cannot be neglected.)
b. If the bar is initially 0.1 m away from the electromagnet, what initial current would be required to lift the magnet?

16.30 With reference to Example 16.10, determine the best combination of current magnitude and wire diameter to reduce the volume of the solenoid coil to a minimum. Will this minimum volume result in the lowest possible resistance? How does the power dissipation of the coil change with the wire gauge and current value? To solve this problem you will need to find a table of wire gauge diameter, resistance, and current ratings. Table 2.2 in this book contains some information. The solution can only be found numerically.

16.31 Derive the same result obtained in Example 16.10 using equation 16.46 and the definition of inductance given in equation 16.30. You will first compute the inductance of the magnetic circuit as a function of the reluctance, then compute the stored magnetic energy, and finally write the expression for the magnetic force given in equation 16.46.

16.32 Derive the same result obtained in Example 16.11 using equation 16.46 and the definition of inductance given in equation 16.30. You will first compute the inductance of the magnetic circuit as a function of the reluctance, then compute the stored magnetic energy, and finally write the expression for the magnetic force given in equation 16.46.

16.33 With reference to Example 16.11, generate a simulation program (e.g., using Simulink™) that accounts for the fact that the solenoid inductance is not constant, but is a function of plunger position. Compare graphically the current and force step responses of the constant-L simplified solenoid model to the step responses obtained in Example 16.11.

16.34 With reference to Example 16.12, calculate the required holding current to keep the relay closed.

16.35 The relay circuit shown in Figure P16.35 has the following parameters: $A_{gap} = 0.001\ \text{m}^2$; $N = 500$ turns; $L = 0.02$ m; $\mu = \mu_0 = 4\pi \times 10^{-7}$ (neglect the iron reluctance); $k = 1000$ N/m, $R = 18\ \Omega$. What is the minimum DC supply voltage, v, for which the relay will make contact when the electrical switch is closed?

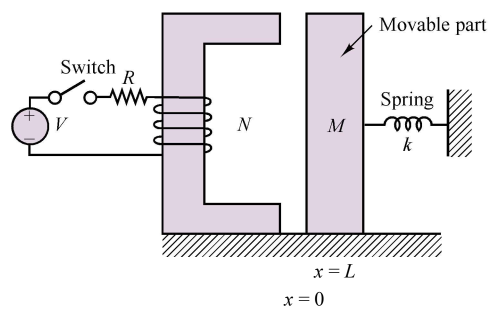


Figure P16.35

16.36 The magnetic circuit shown in Figure P16.36 is a very simplified representation of devices used as *surface roughness sensors*. The stylus is in contact with the surface and causes the plunger to move along with the surface. Assume that the flux ϕ in the gap is given by the expression $\phi = \beta/\mathcal{R}(x)$, where β is a known constant and $\mathcal{R}(x)$ is the reluctance of the gap. The emf e is measured to determine the surface profile. Derive an expression for the displacement x as a function of the various parameters of the magnetic circuit and of the measured emf. (Assume a frictionless contact between the moving plunger and the magnetic structure and that the plunger is restrained to vertical motion only. The cross-sectional area of the plunger is A.)

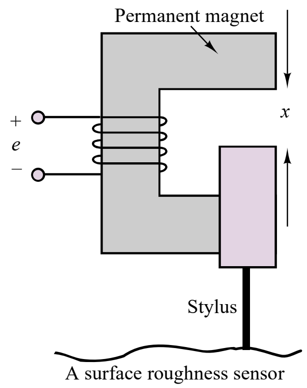


Figure P16.36 A surface roughness sensor

16.37 The electrodynamic shaker shown in Figure P16.37 is commonly used as a vibration tester. A constant current is used to generate a magnetic field in which the armature coil of length l is immersed. The shaker platform with mass m is mounted in the fixed structure by way of a spring with stiffness k. The platform is rigidly attached to the armature coil, which slides on the fixed structure thanks to frictionless bearings.

a. Neglecting iron reluctance, determine the reluctance of the fixed structure, and hence compute the strength of the magnetic flux density, B, in which the armature coil is immersed.
b. Knowing B, determine the dynamic equations of motion of the shaker, assuming that the moving coil has resistance R and inductance L.
c. Derive the transfer function and frequency response function of the shaker mass *velocity* in response to the input voltage V_S.

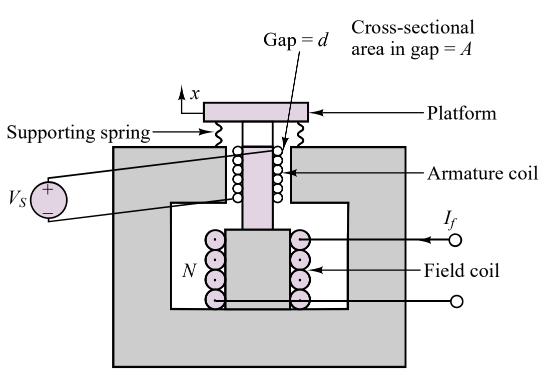


Figure P16.37 Electrodynamic shaker

16.38 A cylindrical solenoid is shown in Figure P16.38. The plunger may move freely along its axis. The air gap between the shell and the plunger is uniform and equal to 1 mm, and the diameter, d, is 25 mm. If the exciting coil carries a current of 7.5 A, find the force acting on the plunger when $x = 2$ mm. Assume $N = 200$ turns, and neglect the reluctance of the steel shell.

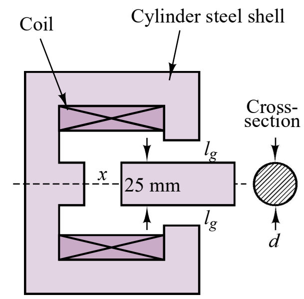


Figure P16.38

16.39 The double-excited electromechanical system shown in Figure P16.39 moves horizontally. Assuming that resistance, magnetic leakage, and fringing are negligible, the permeability of the core is very large, and the cross section of the structure is $w \times w$, find

a. The reluctance of the magnetic circuit.
b. The magnetic energy stored in the air gap.
c. The force on the movable part as a function of its position.

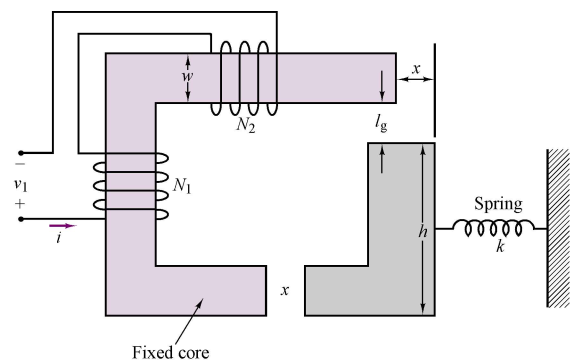


Figure P16.39

16.40 Determine the force, F, between the faces of the poles (stationary coil and plunger) of the solenoid pictured in Figure P16.40 when it is energized. When energized, the plunger is drawn into the coil and comes to rest with only a negligible air gap separating the two. The flux density in the cast steel pathway is 1.1 T. The diameter of the plunger is 10 mm.

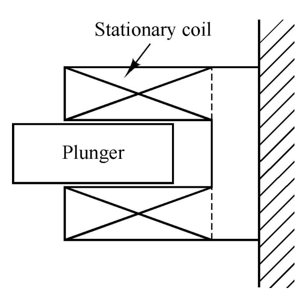


Figure P16.40

16.41 An electromagnet is used to support a solid piece of steel as shown in Example 15.10. A force of 10,000 N is required to support the weight. The cross-sectional area of the magnetic core (the fixed part) is 0.01 m^2. The coil has 1,000 turns. Determine the minimum current that can keep the weight from falling for $x = 1.0$ mm. Assume negligible reluctance for the steel parts and negligible fringing in the air gaps.

16.42 The armature, frame, and core of a 12-VDC control relay are made of sheet steel. The average length of the magnetic circuit is 12 cm when the relay is energized, and the average cross section of the magnetic circuit is 0.60 cm^2. The coil is wound with 250 turns and carries 50 mA. Determine:

a. The flux density, $\mathcal{B}$, in the magnetic circuit of the relay when the coil is energized.
b. The force, $\mathcal{F}$, exerted on the armature to close it when the coil is energized.

16.43 Derive and sketch the frequency response of the loudspeaker of Example 16.13 for (1) $k = 50{,}000$ N/m and (2) $k = 5 \times 10^6$ N/m. Describe qualitatively how the loudspeaker frequency response changes as the spring stiffness, k, increases and decreases. What will the frequency response be in the limit as k approaches zero? What kind of speaker would this condition correspond to?

16.44 A relay is shown in Figure P16.44. Find the differential equations describing the system.

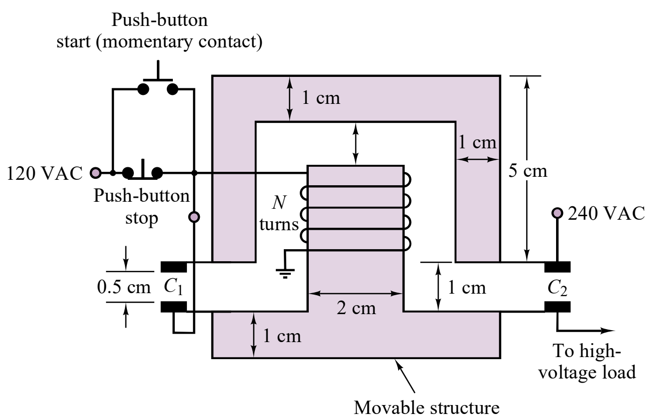


Figure P16.44

16.45 A solenoid having a cross section of 5 cm^2 is shown in Figure P16.45.

a. Calculate the force exerted on the plunger when the distance x is 2 cm and the current in the coil (where $N = 100$ turns) is 5 A. Assume that the fringing and leakage effects are negligible. The relative permeabilities of the magnetic material and the nonmagnetic sleeve are 2,000 and 1.
b. Develop a set of defferential equations governing the behavior of the solenoid.

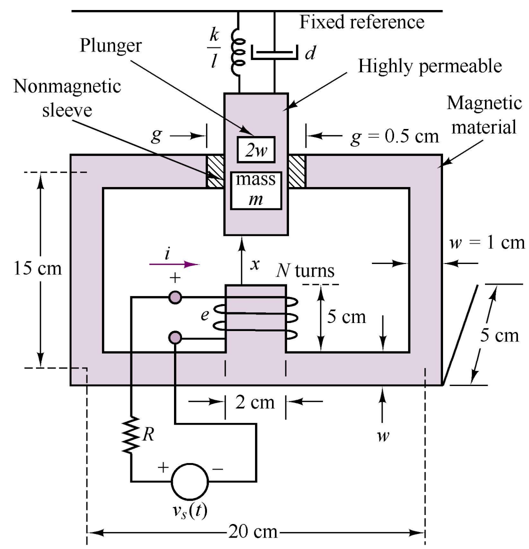


Figure P16.45