Non-Linear Constrained Optimal Control Problem: A Hybrid PSO-GA-Based Discrete Augmented Lagrangian Approach

Amar Khoukhi
Systems Engg. Department
K.F.U.P.M
Dhahran, Saudi Arabia
Email: amar@kfupm.edu.sa

Fouad M. Al-Sunni
Systems Engg. Department
K.F.U.P.M
Dhahran, Saudi Arabia
Email: alsunni@kfupm.edu.sa

Haris M. Khalid
Systems Engg. Department
K.F.U.P.M
Dhahran, Saudi Arabia
Email: mhariskh@kfupm.edu.sa

S.Z. Rizvi
Electrical Engg. Department
K.F.U.P.M
Dhahran, Saudi Arabia
Email: srizvi@kfupm.edu.sa

Abstract - **This work deals with the optimal control problem which has been proposed to solve using the discrete augmented lagrangian based non-linear programming approach. It is shown that this technique guarantee a satisfactory performance in the face of both optimality by minimizing the energy and maximizing the output. Later on, the optimization has been more effective by using PSO-GA-Based Optimization to achieve the optimal value of Lagrange Multipliers and required dynamic parameters and optimally controlling the dynamics. The designed scheme has been successfully tested through extensive simulation. The successful use of the proposed scheme encourages their extension to other physical systems. The proposed scheme is evaluated extensively on a two-tank process used in industry exemplified by a benchmarked laboratory scale coupled-tank system.**

Keywords - **optimal control, augmented lagrangian, non-linear programming, genetic algorithm, control of two-tank system.**

I. Introduction

Many processes in chemical and biotechnological industries are described by multiple sets of differential and algebraic equations. As such they are difficult to control and optimize in transient regimes if switching between the sets is to be taken into account. The switches can involve different regimes of operation (occurrence of flooding in distillation columns, explosive areas in mixtures of gasses, etc) or external actions (addition of second reactor when production increases, etc).

There are several approaches to solution of such as dynamic optimization problem. If the process to be optimized can be described accurately enough by piece-wise linear and logic formulation, powerful algorithms in the area of explicit model predictive control exists [1]. If fully nonlinear processes are concerned, original dynamic optimization problem has to be approximated by some simplified formulation. The usual approaches are complete discretisation of state and control variables – orthogonal collocation. Such formulation can be found in [2]. Other possibility is to leave the states intact and approximate only the control variables as piece-wise constant, or with some higher order approximations. This approach is known as control vector parameterization. Here different formulations can be found, depending on how gradients of the resulting nonlinear programming problem (NLP) are calculated [3]. Another possibility, which is pursued in this work, is to calculate the gradients of NLP via optimal control theory using the so-called co-state, or adjoint equations [4]. The advantage is that the number of differential equations is not proportional to number of optimized parameters, but to number of constraints.

The classical Lagrange-Newton method [5], one of the most efficient numerical methods of solving optimization problems, was developed for problems with equality-type constraints. In the case of inequality-type constraints, the first-order optimality system cannot be expressed as an equation. However, it can be expressed as an inclusion, or the so-called generalized equation [6]. It was shown by S.M. Robinson [6] that a Newtontype procedure applied to this general equation is locally quadratically convergent to the solution, provided that a property called strong regularity is satisfied. This approach has been successfully applied to a class of nonlinear cone-constrained optimization problems in infinite dimensional spaces [7][8][9] and optimal control problems subject to control and/or state constraints[10]. Hybridizing the classical optimization technique with evolutionary algorithms can be a good proposed option.

GAs have been implemented for a wide variety of problems, both real-world (e.g. Fault diagnosis, fault tolerant) and abstract (e.g. solving NP-complete problems [11]). The bulk of the GA literature is concerned with practical applications. For a very complete bibliography, see [12], which contains a comprehensive survey. A recent evolutionary optimization technique, called particle swarm optimization (PSO) was introduced in [13]and has attracted much attention among researchers in various filelds. It has certainly enriched the computational arsenal encompassed in the class of Genetic Algorithms (GA) and has successfully been used to solve complex optimization problems with wide applications in different fields

Khoukhi et al. has also done considerable work in optimal dynamic modeling and optimal time-energy off-line programming [14-16].

The main aim of this work is to show a detailed derivation of the dynamic Augmented Lagrangian Based optimization based on the optimal control theory and implementation of PSO-GA based optimal control.. The proposed scheme has been evaluated on a laboratory scaled based two-tank system. It is the most used prototype applied in the wastewater treatment plant, the petro-chemical plant, and the oil/gas systems.

The paper is organized as follows: Optimal Control problem statement is considered in Section II. Section III gives the system model and description, Section IV discusses the implementation and simulation results for all the techniques implemented. Finally some conclusions are given Section V.

II. Optimal Control Problem Statement

Complex optimal control problems require a serious attention for surviving smartly in the process industries. Hybrid control addresses the problem of integrating the minimization and maximization for decision making in this context. The early optimal control of mission critical systems becomes highly crucial for preventing failure of equipment, loss of productivity and profits, management of assets, reduction of shutdowns. The proposed scheme has been evaluated on the above- cited process control system.

A. System Description

A Benchmark laboratory-scale two-tank process control system has been used to collect data at a sampling rate of 50 milliseconds. The system is considered as a multi-input single-output (MISO) process with hydraulic height and liquid output flow-rate of the second tank being the two inputs while leakages fault level on a discrete scale of 1 to 4 being the output. The objective of the benchmark dual-tank system is to reach a reference height of 200ml in the second tank. To achieve this objective, a Proportional Integral (PI) controller works in a closed loop configuration. Data is collected by introducing leakage fault in the closed loop system. This is done through the pipe clogs of the system using drainage valve between the two tanks. The PI controller tends to treat the introduced fault as a disturbance and acts to suppress it. The closed-loop nature of the experiment also tends to suppress the faults introduced in the system, thereby making it more difficult to detect these faults.

B. Model of the Coupled Tank System

The physical system under evaluation is formed of two tanks connected by a pipe. The leakage is simulated in the tank by opening the drain valve. A DC motor-driven pump supplies the fluid to the first tank and a PI controller is used to control the fluid level in the second tank by maintaining the level at a specified level, as shown in Fig 2.

A step input is applied to the dc motor- pump system to fill the first tank. The opening of the drainage valve introduces a leakage in the tank. Various types of leakage faults are introduced and the liquid height in the second tank, H_2, and the inflow rate, Q_i, are both measured. The National Instruments LABVIEW package is employed to collect these data. (See Fig. 1)

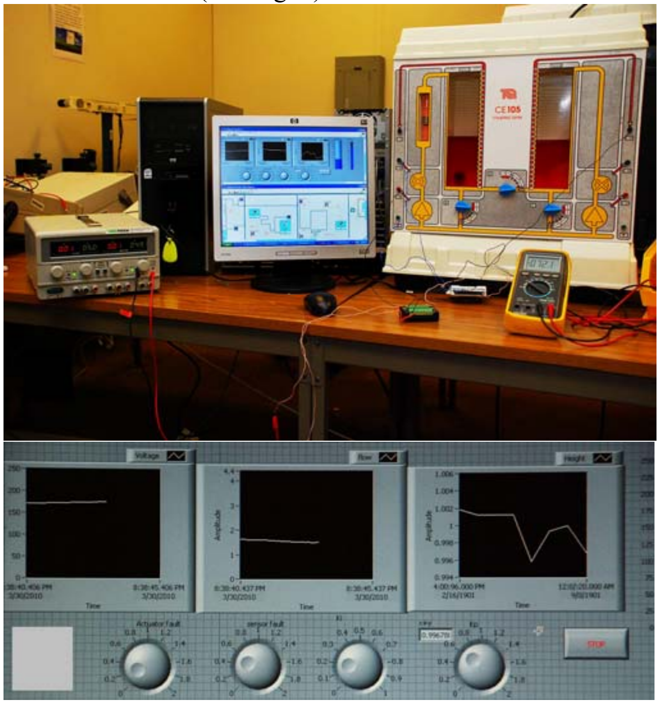


Figure 1 A – The two tank system interfaced with the Labview through a DAQ and the amplifier for the magnified voltage , B – The labview setup of the apparatus including the ciruit window and the block diagram of the experiment.

A benchmark model of a cascade connection of a dc motor and a pump relating the input to the motor, *u,* and the flow, Q_i, is a first-order system:

$$\dot{Q}_i = -a_m Q_i + b_m \phi(u) \quad (1)$$

where a_m and b_m are the parameters of the motor-pump system and $\phi(u)$ is a dead-band and saturation type of nonlinearity. It is assumed that the leakage Q_ℓ occurs in tank 1 and is given by:

$$Q_\ell = C_{d\ell}\sqrt{2gH_1} \quad (2)$$

With the inclusion of the leakage, the liquid level system is modeled by:

$$A_1 \frac{dH_1}{dt} = Q_i - C_{12}\varphi\left(H_1 - H_2\right) - C_\ell \varphi\left(H_1\right) \quad (3)$$

$$A_2 \frac{dH_2}{dt} = C_{12}\varphi\left(H_1 - H_2\right) - C_0 \varphi\left(H_2\right) \quad (4)$$

where $\varphi(.) = sign(.)\sqrt{2g(.)}$, $Q_\ell = C_\ell \varphi\left(H_1\right)$ is the leakage flow rate, $Q_0 = C_0 \varphi\left(H_2\right)$ is the output flow rate, H_1 is the height of the liquid in tank 1, H_2 is the height of the liquid in tank 2, A_1 and A_2 are the cross-sectional areas of the 2 tanks, g=980 $cm/\sec^2$ is the gravitational constant, C_{12} and

C_o are the discharge coefficient of the inter-tank and output valves, respectively.

The model of the two-tank fluid control system, shown above in Fig. 2, is of a second order and is nonlinear with a smooth square-root type of nonlinearity. For design purposes, a linearized model of the fluid system is required and is given below in (5) and (6):

$$\frac{dh_1}{dt} = b_1 q_i - (a_1 + \alpha) h_1 + a_1 h_2 \quad (5)$$

$$\frac{dh_2}{dt} = a_2 h_1 - (a_2 - \beta) h_2 \quad (6)$$

where h_1 and h_2 are the increments in the nominal (leakage-free) heights H_1^0 and H_2^0:

$$b_1 = \frac{1}{A_1}, \quad a_1 = \frac{C_{db}}{2\sqrt{2g(H_1^0 - H_2^0)}}, \quad \beta = \frac{C_0}{2\sqrt{2gH_2^0}},$$

$$a_2 = a_1 + \frac{C_{do}}{2\sqrt{2gH_2^0}} \quad \alpha = \frac{C_{d\ell}}{2\sqrt{2gH_1^0}}$$

and the parameter α indicates the amount of leakage.

A PI controller, with gains k_p and k_I, is used to maintain the level of the Tank 2 at the desired reference input r as:

$$\dot{x}_3 = e = r - h_2$$
$$u = k_p e + k_I x_3 \quad (7)$$

The linearized model of the entire system formed by the motor, pump, and the tanks is given by:

$$\dot{x} = Ax + Br \quad y = Cx \quad (8)$$

where

$$x = \begin{bmatrix} h_1 \\ h_2 \\ x_3 \\ q_i \end{bmatrix}, \quad A = \begin{bmatrix} -a_1 - \alpha & a_1 & 0 & b_1 \\ a_2 & -a_2 - \beta & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -b_m k_p & 0 & b_m k_I & -a_m \end{bmatrix}, \quad (9)$$

$$B = \begin{bmatrix} 0 & 0 & 1 & b_m k_p \end{bmatrix}^T, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

where q_i, q_ℓ, q_0, h_1 and h_2 are the increments in Q_i, Q_ℓ, Q_o, H_1^0 and H_2^0, respectively, the parameters a_1 and a_2 are associated with linearization whereas the parameters α and β are respectively associated with the leakage and the output flow rate, i.e. $q_\ell = \alpha h_1$, $q_o = \beta h_2$.

Proposition: During the implementation process, sign(.) can be approximated with arc tangent. A relationship for approximation can be as follows:

$$sign(x) = \arctan\left(\frac{x}{\sqrt{1 - x * x}}\right), \textit{ where } x < 1$$

Likewise, after approximation, the profiles can be as follows: (See Fig. 3) where (a) representing the *sign(.)* profile and (b) representing the *arctangent* profile for a certain set of numbers.

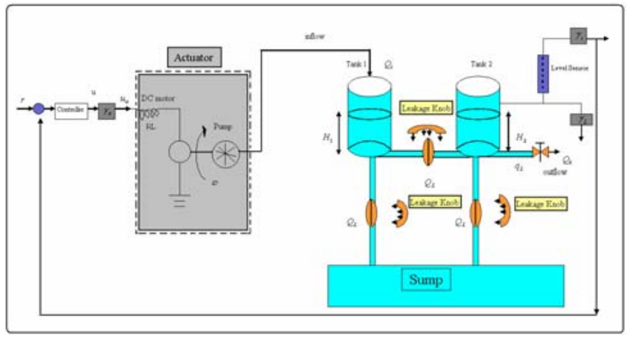
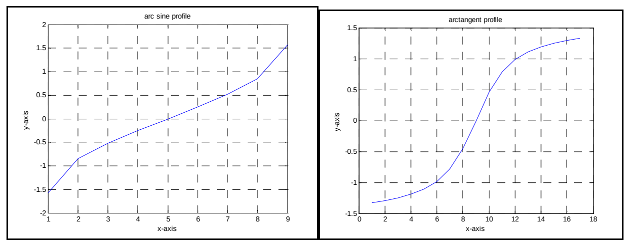


Fig. 2 Process control system: A Lab-scale two-tank system



(a) (b)

Fig. 3 (a) sign(.) profile and (b) Arctangent profile for a certain set of numbers

III. Implementation and Simulation Results

The tasks of our optimal control scheme, are executed with an increasing precision accompanied with a more detailed optimality picture. Firstly, the data collected from the plant has been initialized and the parameters are being optimized which comprises of the pre-processing and normalization of the data and applying the Kalman filter to obtain the feasible solution. Then, the estimation of states based on updated Lagrangian multipliers, and the penalty parameter estimate updates are being done followed by the iterative aggregated updates. To have a cross-optimal solution, the functions have been optimized using Genetic algorithm and Particle Swarm Optimization. The flow chart of the proposed scheme can be seen in the APPENDIX A.

A. Kalman Filter-Based Feasible solution for Optimal Control

Using the leakage-free model together with the covariance of the measurement noise, R, and the plant noise covariance, Q, the Kalman filter model is finally derived. As it is difficult to obtain an estimate of the plant covariance, Q, a number of experiments were performed under different plant scenarios to tune the Kalman gain, K_0 obtained by: $\hat{x}(k+1) = A_0\hat{x}(k) + B_0 u(k-d) + K_0\left(y(k) - C_0\hat{x}(k)\right)$ and $e(k) = y(k) - C_0\hat{x}(k)$. The initial feasible solution for the Kalman filter filter can be shown in Figure 4 where height profile is shown in the upper window and the kalman filter residual analysis is shown in the lower window.

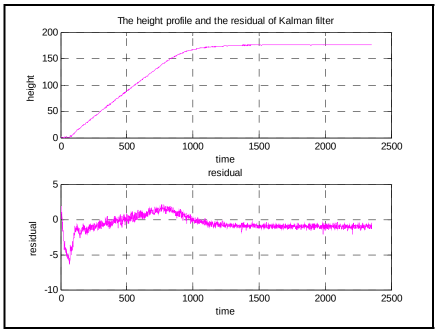


Figure 4. Kalman filter residual analysis considered as feasible solution

B. Cost Function of the System

$$L_p = -\frac{1}{2}H_1^{\,2}(t) - \frac{1}{2}H_2^{\,2}(t) + \frac{1}{2}u^2 \qquad (10)$$

Constraints:

$$H_1(t+1) = H_1(t) + [\frac{1}{A_1}(Q_i - C_{12}\arctan(H_1 - H_2)\sqrt{2g(H_1 - H_2)} \qquad (11)$$
$$-C_l \arctan(H_1)\sqrt{2gH_1})]*T_s$$

$$H_2(t+1) = H_2(t) + [\frac{1}{A_2}(C_{12}\arctan(H_1 - H_2) \qquad (12)$$
$$\sqrt{2g(H_1 - H_2)} - C_0 \arctan(H_2)\sqrt{2gH_2})]*T_s$$

$$Q_i(t+1) = Q_i(t) + [b_m \arctan(u)\sqrt{2gu(t)} - a_m Q_i(t)]*T_s \qquad (13)$$

$$Q_L - C_{dl}\sqrt{2gH_1} = 0 \qquad (14)$$

$$e - r + H_2 = 0 \qquad (15)$$

$$u - k_p e - k_I x_3 = 0 \qquad (16)$$

C. Augmented Lagrangian Implementation in Optimal Control

The augmented lagrangian implementation can be made by the implementation while considering L_p as the main function and α as Lagrange multipliers and r_k as augmented Lagrangian multiplier as shown in equation 17. In order to cater for the constraints in this minimization, we need to allocate them the Lagrange multipliers α and $\mathrm{r_k}$ as augmented Lagrangian multiplier, where $\alpha_i \geq 0 \quad \forall_i$ and $r_k \geq 0$ respectively:

Where $\alpha_1 \to \alpha_6$ are the lagrange multipliers and $h_{n_1} \to h_{n_{14}}$ are the penalty coefficients.

Now minimizing L_p with respect to *H₁*, *H₂*, *Qᵢ*, *Q₀*, *aₘ*, *bₘ*, *u(t)*, *H₁(t+1)*, *H₂(t+1)*, *Qᵢ(t+1)* and setting the derivatives to zero.

$$L_p = -\frac{1}{2}H_1(t)^2 - \frac{1}{2}H_2(t)^2 + \frac{1}{2}u(t)^2 - a_1(t)*[H_1(t+1) - H_1(t) - [\frac{1}{A_1}(Q_i$$
$$-C_{12}\arctan(H_1 - H_2)\sqrt{2g(H_1 - H_2)} - C_l \arctan(H_1)\sqrt{2gH_1})]*T_s] - r_k *$$
$$[*[H_1(t+1) - H_1(t) - [\frac{1}{A_1}(Q_i - C_{12}\arctan(H_1 - H_2)\sqrt{2g(H_1 - H_2)} -$$
$$C_l \arctan(H_1)\sqrt{2gH_1})]*T_s]^2 - a_2(k)*[H_2(t+1) - H_2(t) - [\frac{1}{A_2}.........$$
$$(C_{12}\arctan(H_1 - H_2)\sqrt{2g(H_1 - H_2)} - C_0 \arctan(H_2)\sqrt{2gH_2})]*T_s]$$
$$-r_k*[H_2(t+1) - H_2(t) - [\frac{1}{A_2}(C_{12}\arctan(H_1 - H_2)\sqrt{2g(H_1 - H_2)} -$$
$$C_0 \arctan(H_2)\sqrt{2gH_2})]*T_s]^2 - a_3(k)*[Q_i(t+1) - Q_i(t) - [b_m \arctan(u)$$
$$\sqrt{2gu(t)} - a_m Q_i(t)]*T_s] - r_k*[Q_i(t+1) - Q_i(t) - [b_m \arctan(u)\sqrt{2gu(t)}$$
$$-a_m Q_i(t)]*T_s]^2 - a_4(k)*[Q_L - C_{dl}\sqrt{2gH_1}] - r_k*[Q_L - C_{dl}\sqrt{2gH_1}]^2 - a_5(k)*$$
$$[e - r + H_2] - r_k*[e - r + H_2]^2 - a_6(k)*[u - k_p e - k_I x_3] - r_k*[u - k_p e - k_I x_3]^2$$
$$-h_{n_1}*[Q_i(t)] - r_k*[Q_i(t)]^2 - h_{n_2}*[200 - Q_i(t)] - r_k*[200 - Q_i(t)]^2 - h_{n_3}*$$
$$[Q_0(t)] - r_k*[Q_0(t)]^2 - h_{n_4}*[200 - Q_0(t)] - r_k*[200 - Q_0(t)]^2 - h_{n_5}*[H_1(t)] -$$
$$r_k*[H_1(t)]^2 - h_{n_6}*[200 - H_1(t)] - r_k*[200 - H_1(t)]^2 - h_{n_7}*[H_2(t)] - r_k*[H_2(t)]^2$$
$$-h_{n_8}*[200 - H_2(t)] - r_k*[200 - H_2(t)]^2 - h_{n_9}*[u(t)] - r_k*[u(t)]^2 -$$
$$h_{n_{10}}*[18 - u(t)] - r_k*[18 - u(t)]^2 - h_{n_{11}}*[a_m] - r_k*[a_m]^2 - h_{n_{12}}*[18 - a_m] -$$
$$r_k*[18 - a_m]^2 - h_{n_{13}}*[b_m] - r_k*[b_m]^2 - h_{n_{14}}*[18 - b_m] - r_k*[18 - b_m]^2 \quad (17)$$

Minimizing L_p with respect to *H₁* gives:

$$\frac{\partial L_p}{\partial H_1} \Rightarrow -H_1(t) - \alpha_1(-1 - [\frac{1}{A_1}(\frac{-C_{12}}{1 + ((H_1(t) - H_2(t))\sqrt{2g(H_1(t) - H_2(t)}^2}.$$
$$(\sqrt{2g(H_1(t) - H_2(t))} + \frac{((H_1(t) - H_2(t)).g}{\sqrt{2g(H_1(t) - H_2(t))}}) - (\frac{-C_L}{1 + (H_1(t)\sqrt{2g(H_1(t))}^2}.$$
$$(\sqrt{2g(H_1(t))} + \frac{(H_1(t).g}{\sqrt{2g(H_1(t))}}) - 2r_k[(H_1(t+1) - H_1(t) - [\frac{1}{A_1}(Q_i - C_{12}\arctan(H_1 - H_2)$$
$$\sqrt{2g(H_1 - H_2)} - C_l \arctan(H_1)\sqrt{2gH_1})]*T_s]$$
$$+\frac{\alpha_2}{A_2}(\frac{C_{12}}{1 + ((H_1(t) - (H_2(t))\sqrt{2g(H_1(t) - H_2(t))}^2}).(\sqrt{2g(H_1(t) - H_2(t))}$$
$$-\frac{((H_1(t) - H_2(t)).g}{\sqrt{2g(H_1(t) - H_2(t))}})) - 2r_k(H_2(t+1) - H_2(t) - [\frac{1}{A_2}(C_{12}\arctan(H_1 - H_2)$$
$$\sqrt{2g(H_1 - H_2)} - C_0 \arctan(H_2)\sqrt{2gH_2})]*T_s)] + \alpha_4(\frac{C_{dl}}{\sqrt{2gH_1}}) - 2r_k[(Q_L - C_{dl}\sqrt{2gH_1})]$$
$$-h_{n_5}(1) - 2r_k[(H_1)] - h_{n_6}(1) - 2r_k[(200 - H_1)] = 0 \quad (18)$$

Minimizing L_p with respect to *H₂* gives:

$$\frac{\partial L_p}{\partial H_2} \Rightarrow -H_2(t) - \frac{\alpha_1}{A_1}(\frac{-C_{12}}{1 + ((H_1(t) - H_2(t))\sqrt{2g(H_1(t) - H_2(t)}^2}.(-\sqrt{2g(H_1(t) - H_2(t))}$$
$$-\frac{((H_1(t) - H_2(t)).g}{\sqrt{2g(H_1(t) - H_2(t))}}) - 2r_k[(H_1(t+1) - H_1(t) - [\frac{1}{A_1}(Q_i - C_{12}\arctan(H_1 - H_2)$$
$$\sqrt{2g(H_1 - H_2)} - C_l \arctan(H_1)\sqrt{2gH_1})]*T_s] + \frac{\alpha_2}{A_2}(-1$$
$$+\frac{C_{12}}{1 + ((H_1(t) - (H_2(t))\sqrt{2g(H_1(t) - H_2(t))}^2}).(-\sqrt{2g(H_1(t) - H_2(t))}$$
$$-\frac{((H_1(t) - H_2(t)).g}{\sqrt{2g(H_1(t) - H_2(t))}})) - \frac{C_0}{1 + [(H_2(t)\sqrt{2gH_2(t)}]^2}.(\sqrt{2gH_2(t)} + \frac{(H_2(t)).g}{\sqrt{2g(H_2(t))}}) - 2r_k$$
$$(H_2(t+1) - H_2(t) - [\frac{1}{A_2}(C_{12}\arctan(H_1 - H_2)\sqrt{2g(H_1 - H_2)} - C_0 \arctan(H_2)\sqrt{2gH_2})]$$
$$*T_s)] - \alpha_6(1) - 2r_k[(e - r + H_2)] - h_{n_7}(1) - 2r_k[(H_2)] + h_{n_8}(1) - 2r_k[(200 - H_2)] = 0 \quad (19)$$

Minimizing L_p with respect to *Qᵢ* gives:

$$\frac{\partial L_p}{\partial Q_i(t)} \Rightarrow -Q_i(t) + Q_0(t)) + Q_i(t) - \alpha_3(1 + a_m) - 2r_k[(Q_i(t+1)$$
$$-Q_i(t) - [b_m \arctan(u)\sqrt{2gu(t)} - a_m Q_i(t)]*T_s)]$$
$$-h_{n_1}(1) - 2r_k[(Q_i)] + h_{n_2}(1) - 2r_k[(200 - Q_i)] = 0 \qquad (20)$$

Minimizing L_p with respect to *Q₀* gives:

$$\frac{\partial L_p}{\partial Q_0} \Rightarrow (Q_i(t) - Q_0(t)) - h_{n_3} - 2r_k[(Q_0)] + h_{n_4} - 2r_k[(200 - Q_0)] = 0 \qquad (21)$$

Minimizing L_p with respect to a_m gives:

$$\frac{\partial L_p}{\partial a_m} \Rightarrow a_m - \alpha_3(Q_i(t)) - 2r_k[(Q_i(t+1) - Q_i(t) - [b_m \arctan(u)\sqrt{2gu(t)}$$
$$-a_m Q_i(t)] * T_s)] - h_{n_{11}} - 2r_k[(a_m)] + h_{n_{12}} - 2r_k[(18 - a_m)] = 0 \quad (22)$$

Minimizing L_p with respect to b_m gives:

$$\frac{\partial L_p}{\partial b_m} \Rightarrow b_m - \alpha_3(-\arctan u(t)\sqrt{2gu(t)}) - 2r_k(H_2(t+1) -$$
$$H_2(t) - [\frac{1}{A_2}(C_{12}\arctan(H_1 - H_2)\sqrt{2g(H_1 - H_2)} -$$
$$C_0 \arctan(H_2)\sqrt{2gH_2})] * T_s)] - h_{n_{13}} - 2r_k[(b_m)] + h_{n_{14}} - 2r_k[(18 - b_m)] = 0 \quad (23)$$

Minimizing L_p with respect to *u(t)* gives:

$$\frac{\partial L_p}{\partial u(t)} \Rightarrow u(t) - \alpha_3(-\frac{b_m}{1 + (u(t)\sqrt{2gu(t)})^2}.\sqrt{2gu(t)} + \frac{u(t).g}{\sqrt{2gu(t)}}$$
$$-2r_k[(Q_i(t+1) - Q_i(t) - [b_m \arctan(u)\sqrt{2gu(t)} - a_m Q_i(t)] * T_s)]$$
$$-h_{n_9} - 2r_k[(u(t))] + h_{n_{10}} - 2r_k[(18 - u(t))] = 0 \quad (24)$$

D.I. Solving the derivatives:

Considering all the lagrange multipliers *α*, minimizing L_p with respect to the states gives (See-Equation 25-30):

$$Q_0(t) + \alpha_3 - \alpha_3(a_m) - h_{n_1} + h_{n_2} = 0 \quad (25)$$

$$a_m - \alpha_3(Q_i(t)) - h_{n_{11}} + h_{n_{12}} = 0 \quad (26)$$

$$b_m + \alpha_3(\arctan u(t)\sqrt{2gu(t)} - h_{n_{13}} + h_{n_{14}} = 0 \quad (27)$$

$$u + \alpha_3(\frac{b_m}{1 + (u(t)(\sqrt{2gu(t)})^2})(\sqrt{2gu(t)} + \frac{u(t).g}{\sqrt{2gu(t)}}) - h_{n_9} + h_{n_{10}} = 0 \quad (28)$$

$$H_2(t) - \alpha_6 - h_{n_7} + h_{n_8} = 0 \quad (29)$$

$$H_1(t) - h_{n_5} - h_{n_6} = 0 \quad (30)$$

$$\frac{\partial L_p}{\partial H_1(t+1)} \Rightarrow -\alpha_1 = 0 \quad (31)$$

$$\frac{\partial L_p}{\partial H_2(t+1)} \Rightarrow -\alpha_2 = 0 \quad (32)$$

$$\frac{\partial L_p}{\partial Q_i(t+1)} \Rightarrow -\alpha_3 = 0 \quad (33)$$

Substituting, Equation # (21-33) gives a new formulation (Equation 28, which being dependent on α , we need to maximize:

Thus, the implication of lagrangian without implication of augmentation after the insertion of all the constraint derivates is as follows: (See Equation # 34-54).

$$L_p = -\frac{1}{2}(C_0\sqrt{2gH_2(t)})^2 - H_1(t)[\frac{1}{2}H_1(t) + 200] - H_2(t)[\frac{7}{2}$$
$$H_2(t) + e - r - 200] - \frac{1}{2}[Q_i(t) - Q_0^2(t) + \frac{1}{2}Q_i^2(t) - 4Q_i(t)$$
$$Q_0(t) + 200Q_i(t) + 2Q_0^2(t) + 18a_m + 18b_m - u(t)[\frac{1}{2}u(t) +$$
$$k_p e + k_I x_3 - 18] + C_{dl}(\frac{Q_L}{\sqrt{2gH_1}} - \frac{1}{2}C_{dl}) - \alpha_4(\frac{200gC_{dl}}{\sqrt{2gH_1(t)}})$$
$$+\alpha_5(18 - u(t)) + \alpha_6(2H_2(t) - 200) + \alpha_7(Q_i(t) - 200)$$
$$+\alpha_8(-Q_i(t)) + \alpha_9(-200 + Q_0(t)) + \alpha_{10}(-Q_0(t)) + \alpha_{11}(-H_1(t)$$
$$+200) + \alpha_{12}(H_1(t)) + \alpha_{13}(2H_2(t) + e - r - 200) + \alpha_{14}(-2H_2(t)$$
$$-e + r) + \alpha_{15}(-2u(t) + k_p e + k_I x_3 - 18 + u(t)) + \alpha_{16}(u(t) -$$
$$k_p e - k_I x_3 - u(t)) + \alpha_{17}(a_m - 18) + \alpha_{18}(-a_m) + \alpha_{19}(b_m - 18)$$
$$+\alpha_{20}(-b_m) \quad (34)$$

Where,

$$\alpha_1 = H_1(t+1) - H_1(t) - [\frac{1}{A_1}(Q_i - C_{12}\arctan(H_1 - H_2)\sqrt{2g(H_1 - H_2)}$$
$$-C_l \arctan(H_1)\sqrt{2gH_1})] \text{ , } (35)$$

$$\alpha_2 = H_2(t+1) - H_2(t) - [\frac{1}{A_2}(C_{12}\arctan(H_1 - H_2)\sqrt{2g(H_1 - H_2)}$$
$$-C_0 \arctan(H_2)\sqrt{2gH_2}), \ (36)$$

$$\alpha_3 = Q_i(t+1) - Q_i(t) - [b_m \arctan(u)\sqrt{2gu(t)} - a_m Q_i(t)], \ (37)$$

$$\alpha_4 = \frac{-C_{dl}}{\sqrt{2gH_1}}, \quad (38)$$

$$\alpha_5 = \alpha_{15} - \alpha_{16} - u(t), \quad (39)$$

$$\alpha_6 = H_2(t) - \alpha_{13} + \alpha_{14}, \quad (40)$$

$$h_{n_1} = Q_0(t) + h_{n_2}, \quad (41)$$

$$h_{n_2} = h_{n_1} - Q_0(t), \quad (42)$$

$$h_{n_3} = Q_i(t) - Q_0(t) + h_{n_4}, \quad (43)$$

$$h_{n_4} = Q_0(t) - Q_i(t) + h_{n_3}, \quad (44)$$

$$h_{n_5} = H_1(t) - h_{n_6} + \alpha_4(\frac{gC_{dl}}{\sqrt{2gH_1}}) \quad (45)$$

$$h_{n_6} = H_1(t) - h_{n_5} + \alpha_4(\frac{gC_{dl}}{\sqrt{2gH_1}}) \quad (46)$$

$$h_{n_7} = H_2(t) - \alpha_6 + h_{n_8} \quad (47)$$

$$h_{n_8} = -H_2(t) + \alpha_6 + h_{n_7} \quad (48)$$

$$h_{n_9} = u(t) + \alpha_5 + h_{n_{10}} \quad (49)$$

$$h_{n_{10}} = h_{n_9} - \alpha_5 - u(t) \quad (50)$$

$$h_{n_{11}} = a_m + h_{n_{12}} \quad (51)$$

$$h_{n_{12}} = h_{n_{11}} - a_m \quad (52)$$

$$h_{n_{13}} = b_m + h_{n_{14}} \quad (53)$$

$$h_{n_{14}} = h_{n_{13}} - b_m \quad (54)$$

Now, the final formulation, dependent on *α, penalty function* and augmented lagrange multiplier, r_k yields (See Equation # 55):

$$L_p = -\frac{1}{2}(C_0\sqrt{2gH_2(t)})^2 - H_1(t)[\frac{1}{2}H_1(t)+200] - H_2(t)[\frac{7}{2}H_2(t)+e-r-200] - \frac{1}{2}[Q_i(t)$$
$$-Q_0^{\,2}(t) + \frac{1}{2}Q_i^{\,2}(t) - 4Q_i(t)Q_0(t) + 200Q_i(t) + 2Q_0^{\,2}(t) + 18a_m + 18b_m - u(t)[\frac{1}{2}u(t)$$
$$+k_pe + k_Ix_3 - 18] + C_{dl}(\frac{Q_L}{\sqrt{2gH_1}} - \frac{1}{2}C_{dl}) - \alpha H^T - h_nI^T - 2r_kA^T$$
$$where\ \alpha = \begin{bmatrix}\alpha & \alpha_2 & \alpha_3 & \alpha_4 & -\alpha_5 & -\alpha_6\end{bmatrix}$$
$$where\ I = \begin{bmatrix}-h_{n_1} & -h_{n_2} & -h_{n_3} & -h_{n_4} & -h_{n_5} & -h_{n_6} & -h_{n_7} & -h_{n_8} & -h_{n_9} & -h_{n_{10}} & -h_{n_{11}} & -h_{n_{12}} & -h_{n_{13}} & -h_{n_{14}}\end{bmatrix}$$
$$and\ \ H^T = \begin{bmatrix} H_1(t+1) - H_1(t) - [\frac{1}{A_1}(Q_i - C_{12}\arctan(H_1-H_2)\sqrt{2g(H_1-H_2)} - C_l\arctan(H_1)\sqrt{2gH_1})] \\ H_2(t+1) - H_2(t) - [\frac{1}{A_2}(C_{12}\arctan(H_1-H_2)\sqrt{2g(H_1-H_2)} - C_0\arctan(H_2)\sqrt{2gH_2}) \\ Q_i(t+1) - Q_i(t) - [b_m\arctan(u)\sqrt{2gu(t)} - a_mQ_i(t)] \\ -\frac{200gC_{dl}}{\sqrt{2gH_1(t)}} \\ 18-u(t) \\ 2H_2(t)-200 \end{bmatrix}^T$$

$$and\ \ I^T = \begin{bmatrix} Q_i(t)-200 \\ -Q_i(t) \\ -200+Q_0(t) \\ -Q_0(t) \\ -H_1(t)+200 \\ H_1(t) \\ 2H_2(t)+e-r-200 \\ -2H_2(t)-e+r \\ -2u(t)+k_pe+k_Ix_3-18+u(t) \\ u(t)-k_pe-k_Ix_3-u(t) \\ a_m-18 \\ -a_m \\ b_m-18 \\ -b_m \end{bmatrix}$$

$$and\ \ A^T = \begin{bmatrix} Q_L \\ -C_{dl}\sqrt{2gH_1} \\ u(t) \\ -k_pe \\ -k_Ix_3 \\ e \\ -r \\ H_2 \\ 854 \end{bmatrix} \qquad (55)$$

E. Augmented Lagrangian unconstrained presentation co-states calculation for optimal control:

$$L_\mu(k,1) = -\frac{1}{2}H_1(k)^2 - \frac{1}{2}H_2(k)^2 + \frac{1}{2}u(k)^2 - a_1(k)*[H_1(t+1) - H_1(t)$$
$$-[\frac{1}{A_1}(Q_i - C_{12}\arctan(H_1-H_2)\sqrt{2g(H_1-H_2)} - C_l\arctan(H_1)\sqrt{2gH_1})]$$
$$*T_s] - r_k*[*[H_1(t+1) - H_1(t) - [\frac{1}{A_1}(Q_i - C_{12}\arctan(H_1-H_2)$$
$$\sqrt{2g(H_1-H_2)} - C_l\arctan(H_1)\sqrt{2gH_1})]*T_s]^2 - a_2(k)*[H_2(t+1)$$
$$-H_2(t) - [\frac{1}{A_2}(C_{12}\arctan(H_1-H_2)\sqrt{2g(H_1-H_2)} - C_0\arctan(H_2)$$
$$\sqrt{2gH_2})]*T_s] - r_k*[H_2(t+1) - H_2(t) - [\frac{1}{A_2}(C_{12}\arctan(H_1-H_2)$$
$$\sqrt{2g(H_1-H_2)} - C_0\arctan(H_2)\sqrt{2gH_2})]*T_s]^2 - a_3(k)*[Q_i(t+1)$$
$$-Q_i(t) - [b_m\arctan(u)\sqrt{2gu(t)} - a_mQ_i(t)]*T_s] - r_k*[Q_i(t+1)$$
$$-h_{n_1}*[Q_i(k)] - r_k*[Q_i(k)]^2 - h_{n_2}*[200-Q_i(k)] - r_k*[200-Q_i(k)]^2$$
$$-h_{n_3}*[Q_0(k)] - r_k*[Q_0(k)]^2 - h_{n_4}*[200-Q_0(k)] - r_k*[200-Q_0(k)]^2$$
$$-h_{n_5}*[H_1(k)] - r_k*[H_1(k)]^2 - h_{n_6}*[200-H_1(k)] - r_k*[200-H_1(k)]^2$$
$$-h_{n_7}*[H_2(k)] - r_k*[H_2(k)]^2 - h_{n_8}*[200-H_2(k)] - r_k*[200-H_2(k)]^2$$
$$-h_{n_9}*[u(k)] - r_k*[u(k)]^2 - h_{n_{10}}*[18-u(k)] - r_k*[18-u(k)]^2$$
$$-h_{n_{11}}*[a_m] - r_k*[a_m]^2 - h_{n_{12}}*[18-a_m] - r_k*[18-a_m]^2 - h_{n_{13}}*[b_m]$$
$$-r_k*[b_m]^2 - h_{n_{14}}*[18-b_m] - r_k*[18-b_m]^2$$

(56)

The co-states α_k are determined by backward integration of the adjunct state equation yielding:

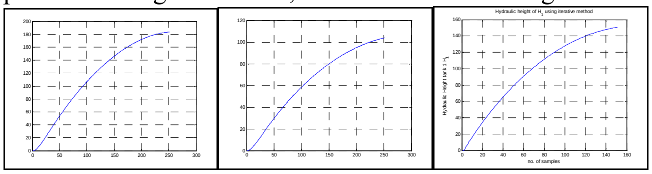
$$\begin{bmatrix}\alpha_1 \\ \alpha_2 \\ \alpha_3\end{bmatrix}_{k-1} = -2h_k\frac{\partial E_d}{\partial x_k} - F_d^T\lambda_k - h_k[\sum_{i=1}^{N}\nabla_{x_k}\Psi_{\mu s}(\sigma_k^j, s_i^{Dl}(x_k))]$$
$$-h_k[\sum_{j=1}^{N}\nabla_{x_k}\phi_{\mu g}(\rho_k^j, g_j^D(x_k, \upsilon_k, h_k))]$$

where

$$x_{k+1} = f_{d_z}^D(x_k, \tau_k, h_k)\ \ , k = 0,......., N-1$$
$$g_j^D(x_k, \upsilon_k, h_k) \le 0,\ j \in \{1,2,......j\}$$
$$F_d^{\ T} = I_d$$
$$E_d = H_1, H_2\ and\ u$$

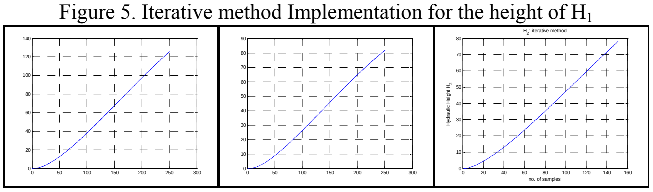
D. Implementation of Augmented Lagrangian Iterative algorithm:

Generating the augmented Lagrangian Iterative Algorithm yields the following states of the system, the lagrangrange multipliers and augmented lagrangians. The profiles of height of tank 1, tank2 can be seen in figure 5-7.



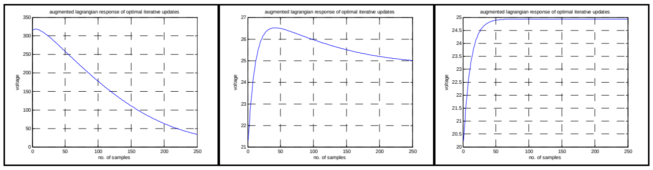
(a) (b) (c)

Figure 5. Iterative method Implementation for the height of H_1



(a) (b) (c)

Figure 6. Iterative method Implementation for the height of H_2



(a) (b) (c)

Figure 7(a-c). Iterative method Implementation for the profile of control u

The profiles of u can be seen in figure 7(a-c). It can be seen that the iteration process is giving updated results in the control u profile.

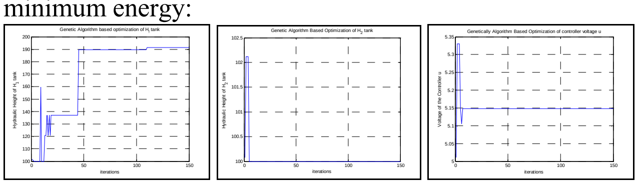
E. Genetic Algorithm Based Implementation of Augmented Lagrangian

Because of highly diversified system, the standard functional technique of implementing the Genetic Algorithm was not successful, Therefore, a generalized genetic algorithm has been designed to capture all the critical sides of implementing the Algorithm. The following are the steps for the Genetic Algorithm.

E.I. Genetic Algorithm Based Implementation Results

E.II. Optimization of the hydraulic Height for H_1 Tank, H_2 Tank and Controller Voltage u for minimum energy:

Figure 8 shows the result of Genetically optimized augmented Lagrangian based controller voltage u for minimum energy:



(a) (b) (c)

Figure 8. Genetic Algorithm Based Optimization for the height of H_1 tank, H_2 tank and controller voltage u.

F. Particle Swarm Optimization Based Implementation of Augmented Lagrangian

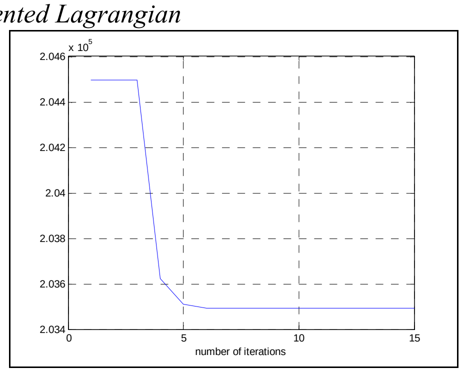
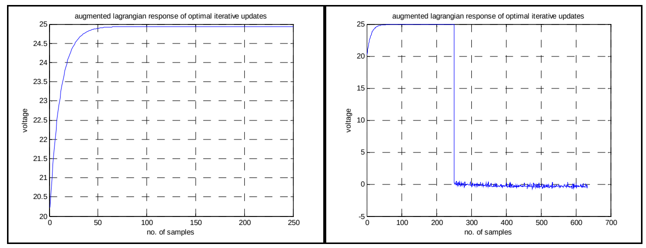


Figure 9. PSO-Based Cost Convergence for controller voltage u

Particle Swarm based Augmented Lagrangian is developed. The developed algorithm is applied on the above defined problem to search for optimal value of control input data clusters. The number of iterations is kept 15, population size is kept 75, cognitive and social parameters c_1 and c_2 are kept equal to 2, and constraints on the radii, as defined above, are observed strictly. The convergence of objective function is shown in figure 9. Cost function convergence to optimal or near optimal solution regardless of initial solution demonstrates the robustness of the algorithm.

IV. Comparison of Results between Iterative Method and Hybrid PSO-GA Algorithm

For the gist of optimal control, the result is being compared for the methods of iteration based algorithm and PSO-GA-based algorithm as can be seen in this section from Figure 10(a-b). It can be seen that both iterative based algorithm and PSO-GA-based algorithm were able to provide a "controlled" u profile in the given number of iterations but the PSO-GA-Based algorithm has an upper edge than the iterative algorithm because it is providing an economical value of u.

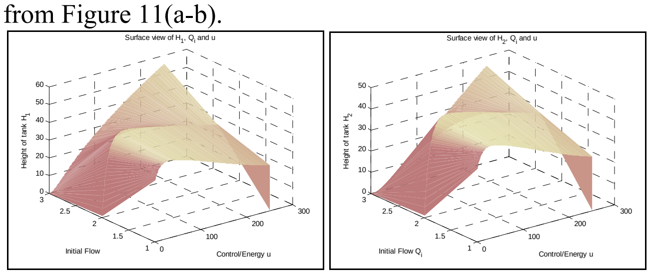


(a) (b)

Figure 10 (a) Iterative algorithm based optimal control and (b) Genetic Algorithm Based Optimal control

V. Surface Plots for Analysis

For the analysis of the effect of control/energy u, on the height of first tank, height of second tank, and the initial flow, the result is being shown as can be seen in this section from Figure 11(a-b).



(a) (b)

Figure 11 Surface view for (a) Height of tank 1, initial flow and control/energy and (b) Height of tank 2, initial flow, and control/energy

VI. Conclusion

In this paper, we presented an optimal control approach to a constrained optimization non-linear problem to the fault diagnosis problem, based on a combination of strategies like augmented lagrangian and PSO-GA-Based algorithm. This optimal control approach ensures the optimal height of water at minimum energy level. As such, this augmented lagrangian based approach can be made an effective part of an overall approach that tackles both optimal control of the system and optimization of the non-linear constraints. For this technique, PSO-GA-Based algorithm has been used. The effectiveness of this scheme has been evaluated on a benchmarked laboratory scaled two tank system.

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REFERENCES

[1] A. Bemporad and M. Morari. Control of systems integrating logic, dynamics, and constraints. Automatica, 35(3):407 – 427, 1999.

[2] M. P. Avraam, N. Shah, and C. C. Pantelides. Modelling and optimisation of general hybrid systems in the continuous time domain. Computers chem. Engineering, S221–S228, 1998.

[3] V. S. Vassiliadis, R. W. H. Sargent, and C. C. Pantelides. Solution of a class of multistage dynamic optimization problems. 1. Problems without path constraints, 2. Problems with path constraints. Ind. Eng. Chem. Res., 33:2111–2122, 2123–2133, 1994.

[4] A. I. Ruban, Sensitivity coefficients for discontinuous dynamic systems. Computer and Systems Sciences International, 36:536–542, 1997.

[5] Stoer J. and Bulirsch R. (1980): Introduction to Numerical Analysis.—New York: Springer.

[6] Robinson S.M. (1980): Strongly regular generalized equations.— Math. Oper. Res., Vol. 5, No. 1, pp. 43–62.

[7] Alt W. (1990a): Lagrange-Newton method for infinite dimensional optimization problems. — Numer. Funct. Anal. Optim., Vol. 11, No. 3/4, pp. 201–224.

[8] Alt W. (1990b): Parametric programming with applications to optimal control and sequential quadratic programming.— Bayreuther Math. Schriften, Vol. 34, No. 1, pp. 1–37.

[9] Alt W. (1990c): Stability of solutions and the Lagrange-Newton method for nonlinear optimization and optimal control *problems*. — (Habilitationsschrift), Universität Bayreuth, Bayreuth.

[10] Alt W. and Malanowski K. (1993): The Lagrange-Newton method for nonlinear optimal control problems. — Comput. Optim. Appl., Vol. 2, No. 1, pp. 77–100.

[11] K.A. De Jong and W.M. Spears, Using genetic algorithms to solve NO-complete problems, in Proceedings of the Third International Conference on Genetic Algorithms, ed. By J. D. Schaffer. Morgan Kaufmann: 1989.

[12] D.E. Goldberg, K. Zakrzewski, B. Sutton, R. Gadient, C. Chang, P. Gallego, B. Miller and E. Cantu-Paz, Genetic Algorithms: A Bibliography, illegal Report no. 97011, Illinois Genetic Algorithms Laboratory, University of Illinois at Urbana-Champaign, 1997.

[13] Kennedy, J. and Eberhart, R. (2001). Academic Press. L.J. De Miguel, L. B. (2005). Fuzzy logic-based decision-making for fault diagnosis in a dc motor. In Engineering Applications of Artificial Intelligence. Volume 18(4), pages 423-450.

[14] Khoukhi A et al, Luc Baron, Marek Balazinski, ”Constrained multi-objective trajectory planning of parallel kinematic machine”, Robotics and Computer Integrated Manufacturing, (2008).

[15] KhoukhiA et al, “Dynamic modeling and optimal time-energy off-line programming for mobile robots, a cybernetic problem. Kybernetes 2002; 31(5–6): 731–5.

[16] KhoukhiA et al, Baron L, Balazinski M., “Constrained multi-objective off-line programming of parallel kinematic machines, Technical Report, E´cole Polytechnique de Montreal, CDT-P2877-05,70Pages; May2007.

APPENDIX A – Flow Chart of the Proposed Scheme