Fault Detection Filter Design for time-delay LTI systems using LKF

Magdi S. Mahmoud and Haris M. Khalid

***Abstract*—In this paper, robust fault detection (FD) problems for time-delay LTI systems with unknown inputs is studied. The paper is proposed to evaluate the robustness as well as sensitivity of residual signals to the unknown inputs as well as to the faults in terms of L_2. First, the weighting matrix are being selected for an appropriate design of filter, then fault detection filter design with Lyapunov-Krasovskii function (LKF) being designed with time delay. The main results include the detail derivation of these steps followed by its implementation on an open-loop time-delay system for chemical reactor example.**

***Contribution:* The main contribution of this paper is the implementation of LKF and the simulation on an open loop chemical reactor system.**

Keyword: Fault detection; filter; Lyapunov-Krasovskii function; time-delay.

I. INTRODUCTION

The research and application of robust fault detection (FD) in automated processes have received considerable attention during last decades and a great number of results have been achieved[1];[2]; [3]; [4]. In the past three decades, many significant results concerning fault detection and isolation (FDI) problems have been developed, see e.g [5]; [6]; [7] and references therein. However, most of the achievements are for delay-free systems.

On the other hand, time delays are frequently encountered in industry and are often the source of performance degradation of a system [[4]. So, this paper focuses on the Fault Detection Filter Design for time-delay LTI systems with unknown inputs. Although, time delay is an inherent characteristic of many physical systems, such as rolling mills, chemical processes, water resources, biological, economic and traffic control systems, only few researches on FDI have been carried out for them [8]; [9]; [10]; [11]. [10] deal with the nominal case fault identification (without considering the influence of model uncertainty and unknown inputs), [5] formulate the fault detection filter (FDF) design problem as a two-objective nonlinear programming problem where no analytic solution can be constructed in general. [11] extends the results to the discrete-time case. The authors of earlier study in [8] has also developed an FDF design approach based on H_∞-filtering, but the most important and difficult issue concerning the selection of a so-called reference residual model has not been successfully solved. An efficient way to tackle the fault detection problem for time-delay systems is as yet to be developed.

The paper is organized as follows: problem formulation and the proposed scheme formulation is presented in Section 2 followed by the main results in section 3, simulation results are followed by in section 4. Finally some concluding remarks are given in Section 5. Figure 1 shows the proposed implementation plan.

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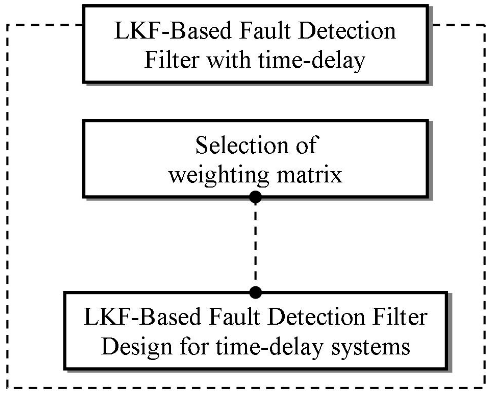

Fig. 1. Fault Detection Filter with time-delay

II. PROBLEM FORMULATION

We now concentrate our attention on the fault detection problems for time-delay LTI systems. The system model under consideration is given by:

$$\begin{aligned}\dot{x} &= Ax(t) + \sum_{i=1}^{N} A_i x(t-d_{xi}(t)) \\ &+Bu + \sum_{i=1}^{L} B_i u(t-d_{ui}) + B_f f + B_d d \qquad (1)\\ x(t) &= 0(t \le 0) \qquad (2)\\ y &= Cx + Du + D_f f + D_d d \qquad (3)\end{aligned}$$

where A_i(i=1,2,3,....,N), B_i(l=1,2,3,....,L) are known matrices with appropriate dimensions. Assume that the time-varying delays satisfy:

$$\begin{aligned}d_{xi}(t) &\le \bar{d}_x < \infty, d_{ul}(t) \le \bar{d}_u < \infty \\ \dot{d}_{xi}(t) &\le \bar{m}_{xi} < 1, \dot{d}_{ul}(t) \le \bar{m}_{ul} < 1 \qquad (4)\end{aligned}$$

We propose to use the following fault detection filter for the purpose of residual generation:

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x}(t) + \sum_{i=1}^{N} A_i \hat{x}(t-d_{xi}(t)) \\ &+Bu + \sum_{i=1}^{L} B_i u(t-d_{ui}) + H(y-\hat{y}) \qquad (5)\end{aligned}$$

$$r = Ce + D_f f + D_d d \tag{6}$$

Denote $e = x - \hat{x}$. The dynamics of the fault detection filter can then be expressed by:

$$\dot{e} = (A - HC)e + \sum_{i=1}^{N} A_i e(t - d_i(t))$$
$$+(B_f - HD_f)f + (B_d - HD_d)d \tag{7}$$
$$r = Ce + D_f f + D_d d \tag{8}$$

For our purpose, we introduce the following system:

$$\dot{e}_f = (A - HC)e_f + \sum_{i=1}^{N} A_i e_f(t - d_i(t))$$
$$+(B_f - HD_f)f \tag{9}$$
$$r_f = Ce_f + D_f f \tag{10}$$

which describes the influence of the faults on the residual signals.

We now formulate the problems of designing a fault detection filter for time-delay LTI system (1-3), analogue to the formulation given in the last sub-section, as follows:

A. Selection of weighting matrix

Given a performance index which describes the sensitivity of the residual signals to the faults, find an observer gain matrix, denoted by H_f, such that r_f, is optimal in the sense of the given performance index. The weighting matrix is then set as:

$$\dot{e}_f = (A - HC)e_f + \sum_{i=1}^{N} A_i e_f(t - d_i(t))$$
$$+(B_f - HD_f)f \tag{11}$$
$$r_f = Ce_f + D_f f \tag{12}$$

B. Design of fault detection filter (FDF) for time-delay LTI system (1-3):

Given constants $\beta(> 0)$ as well as $\gamma\ (\geq ymin)$ and a weighting matrix $W_f(s)$, find an observer gain matrix H such that system (7) and 8 is stable and (16) holds.

C. Design of robust fault detection filter (RFDF) for timedelay LTI system (1-3):

Given a constant $\gamma\ (\geq min)$ and a weighting matrix $W_f(s)$, find an observer gain matrix H and matrix V such that β is minimized under conditions that system (22) and (23) is stable as well as (16) holds, i.e.

$$min_{H,V} \beta \tag{13}$$

such that:
Observer Gain:

$$\int_0^\infty r_e^T r_e dt \leq \beta^2 \int_0^\infty f^T f dt +$$
$$\gamma^2 \int_0^\infty d^T d dt \tag{14}$$

Rate of Change of Error:

$$\dot{e} = (A - HC)e + \sum_{i=1}^{N} A_i e(t - d_i(t))$$
$$+(B_f - HD_f)f + (B_d - HD_d)d \tag{15}$$

Residual:

$$r = Ce + D_f f + D_d d \tag{16}$$

We call system (7)-(8) robust fault detection filter for time-delay LTI system (1-3) if the observer gain matrix solves the above-defined optimization problem.

III. Main Results

In the following of this work, an LMI approach is developed to solve the above-defined robust fault detection filter design problem. To this end, following two subproblems will be solved:

- evaluation of the influence of the faults and selection of weighting matrix and
- solutions of the RFDF problem.

A. Theorem

Given $\varrho > 0$ $\mu > 0$. The system 4.1 with $u(.) = 0$ is delay dependent asymptotically stable with L_2-performance bound γ if there exist symmetric matrices $0 < P$, $0 < W_a$, $0 < W_c$, $0 < Q$, $0 < R$, weighting matrices N_a, N_c, N_s, M_a, M_c, M_s and a scalar $\gamma > 0$ satisfying the following LMI:

$$\begin{bmatrix} \Upsilon_{01} & \Upsilon_{02} & \Upsilon_{03} & \varrho M_a & \varrho N_a & P\Gamma_0 & G_o^T & PA_0^T W \\ * & \Upsilon_{04} & \Upsilon_{05} & \varrho M_c & \varrho N_c & 0 & G_{do}^T & PA_{do}^T W \\ * & * & \Upsilon_{06} & \varrho M_s & \varrho N_s & 0 & 0 & 0 \\ * & * & * & -\varrho W_a & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varrho W_c & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & \Phi_0^T & P\Gamma_0^t W \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -\varrho W \end{bmatrix} \tag{17}$$

where,

$$\Upsilon_{o1} = PA_o + A_o^t P + Q + R + N_a + N_a^t + M_a + M_a^t$$

$$\begin{aligned}
\Upsilon_{o2} &= PA_{do} - 2N_a + N_c^t + M_c^t, W = W_a + W_c \\
\Upsilon_{o3} &= N_a - M_a + N_s^t + M_s^t \\
\Upsilon_{o4} &= -(1-\mu)Q - 2N_c - 2N_c^t \\
\Upsilon_{o5} &= N_c - M_c - 2N_s^t \\
\Upsilon_{o6} &= -R + N_s + N_s^t - M_s - M_s^t
\end{aligned} \quad (18)$$

B. Proof:

In terms of $\xi(t)$= $[e_f{}^t(t) \quad e_f{}^t(t-\tau(t)) \quad e_f{}^t(t-\varrho)]^t$ and using the classical Leibniz rule $e_f(t-\theta)$= $e_f(t) - \int_{t-\theta}^t \dot{e_f}(s)ds$ for any matrices N_a, N_c, N_s, M_a, M_c, M_s of appropriate dimensions, the following equations hold:

$$2\xi^t(t)2N[-\int_{t-\tau(t)}^t \dot{e_f}(s)ds + e_f(t) - e_f(t-\tau)] = 0 \quad (19)$$

$$2\xi^t(t)(M-N)[-\int_{t-\varrho}^t \dot{e_f}(s)ds + e_f(t) - e_f(t-\varrho)] = 0 \quad (20)$$

Expansion of equation (19) and (20) gives:

$$\begin{aligned}
&e_f^t(t)[N_a + N_a^t + M_a + M_a^t]e_f(t) + 2e_f^t(t)[-2N_a + M_c^t \\
&+ N_c^t]e_f(t-\tau(t))) + 2e_f^t(t)[N_a + M_a + N_s^t + M_s^t]e_f(t-\varrho) \\
&+ 2e_f^t(t-\tau(t))[-2N_c - 2N_c^t]e_f(t-\tau(t))2e_f^t(t-\tau(t))[N_c \\
&- 2N_s^t - M_c]e_f(t-\varrho) + 2e_f^t(t-\varrho)[N_s + N_s^t - M_s - M_s^t] \\
&e_f(t-\varrho) - 2\xi^t(t)2N\int_{t-\tau(t)}^t \dot{e_f}(s)ds - 2\xi^t(t)(M-N) \\
&\int_{t-\varrho}^t \dot{x}(s)ds = 0
\end{aligned} \quad (21)$$

Consider now the augmented Lyapunov-Krasoviskii functional (ALKF):

$$\begin{aligned}
V(t) &= V_o(t) + V_a(t) + V_c(t) + V_m(t) \\
V_o(t) &= e_f^t Pe_f(t), \\
V_a(t) &= \int_{-\varrho}^0 \int_{t+s}^t \dot{e_f}{}^t(\alpha)(W_a + W_c)\dot{e_f}(\alpha)d\alpha ds, \\
V_c(t) &= \int_{t-\varrho}^t e_f^t(s)Re_f(s)ds, \\
V_m(t) &= \int_{t-\tau(t)}^t e_f^t(s)Qe_f(s)ds
\end{aligned} \quad (22)$$

where $0 < P = P^t$, $0 < W_a = W_a^t$, $0 < W_c = W_c^t$, $0 < Q = Q^t$, $0 < R = R^t$ are the matrices of appropriate dimensions. The first term in (22) is standard to nominal system without delay. The second and fourth terms correspond to the delay-dependent conditions. The third term is introduced to compensate for the enlarged time interval from $t - \varrho \longrightarrow t$ to $t - \tau \longrightarrow t$. A straightforward computation gives the time-derivative of $V(e_f)$ with $w(t) = 0$ as:

$$\begin{aligned}
\dot{V}_o^t &= 2e_f^t P\dot{e_f}(t) \\
&= 2e_f^t(t)P[A_o e_f(t) + A_{do}e_f(t-\tau)] \\
\dot{V}_a(t) &= \varrho \dot{e_f}{}^t(W_a + W_c)\dot{(e_f)}(t) - \int_{t-\varrho}^t \dot{(e_f)}^t(s)(W_a \\
&\quad + W_c)\dot{e_f}(s)ds \\
\dot{V}_c^t &= e_f^t(t)Re_f(t) - e_f^t(t-\varrho)Re_f(t-\varrho) \\
\dot{V}_m(t) &= e_f^t(t)Qe_f(t) - (1-\dot{\tau})e_f^t(t-\tau(t)) \\
&\quad Qe_f(t-\tau(t)) \\
&\le e_f^t(t)Qe_f(t) - (1-\mu)e_f^t(t-\tau(t)) \\
&\quad Qe_f(t-\tau(t))
\end{aligned} \quad (23)$$

from (22-23)and using (20), we have:

$$\begin{aligned}
&\dot{V}(t) \le e_f^t(t)[PA_o + A_o^t + Q + R + N_a + N_a^t + M_a \\
&+ M_a^t]e_f(t) + 2e_f^t[PA_{do} - 2N_a + M_c^t + N_c^t]e_f(t-\tau) \\
&+ 2e_f^t(t)[N_a - M_a + N_s^t + M_s^t]e_f(t-\varrho) + 2e_f{}^t(t-\tau) \\
&[N_c - 2N_s^t - M_c]e_f(t-\varrho) - e_f{}^t(t-\tau)[(1-\mu)Q \\
&+ 2N_c + 2N_c^t]e_f(t-\tau(t))] + e_f{}^t(t-\varrho)[-R + N_s \\
&+ N_s^t - M_s - M_s^t]e_f(t-\varrho) + \varrho \dot{e_f}{}^t(t)(W_a \\
&+ W_c)\dot{e_f}(t) - \int_{t-\varrho}^t \dot{e_f}{}^t(s) \\
&(W_a + W_c)\dot{e_f}(s)ds - 2\xi^t(t)2N\int_{t-\tau(t)}^t \dot{e_f}(s)ds \\
&- 2\xi^t(t)(-N)\int_{t-\varrho}^t \dot{e_f}(s)ds - 2\xi^t(t)M\int_{t-\varrho}^t \dot{e}_f(s)ds
\end{aligned} \quad (24)$$

To manipulate the terms in 24, first consider:

$$\begin{aligned}
&= -2\xi^t(t)2N\int_{t-\tau(t)}^t \dot{e}_f(s)ds + 2\xi^t(t)N\int_{t-\varrho}^t \dot{e}_f(s)ds \\
&= -2\xi^t(t)N\int_{t-\tau(t)}^t \dot{e}_f(s)ds + 2\xi^t(t)N\int_{t-\varrho}^{t-\tau(t)} \dot{e}_f(s)ds
\end{aligned} \quad (25)$$

and then consider:

$$\begin{aligned}
&\int_{t-\varrho}^t \dot{e}_f^t(s)(W_a + W_c)\dot{e}_f(s)ds \\
&= \int_{t-\varrho}^t \dot{e}_f^t(s)(W_a\dot{e}_f(s)ds + \int_{t-\varrho}^t \dot{e}_f^t(s)(W_c)\dot{e}_f(s)ds \\
&= \int_{t-\varrho}^t \dot{e}_f^t(s)(W_a\dot{e}_f(s)ds + \int_{t-\tau(t)}^t \dot{e}_f^t(s)(W_c)\dot{e}_f(s)ds \\
&+ \int_{t-\varrho}^{t-\tau(t)} \dot{e}_f^t(s)W_c\dot{e}_f(s)ds
\end{aligned} \quad (26)$$

Then $\dot{V}$(t) becomes:

$$
\begin{aligned}
\dot{V}(t) \leq\ & \xi^t \Upsilon_o \xi(t) - \int_{t-\varrho}^{t} \dot{e}_f^t(s) W \dot{e}_f(s) ds \\
& + \xi^t(t) \begin{bmatrix} \varrho A_o^t \\ \varrho A_{do}^t \\ 0 \end{bmatrix} W \begin{bmatrix} \varrho A_o^t \\ \varrho A_{do}^t \\ 0 \end{bmatrix}^t \xi(t) \\
& - 2\xi^t(t) N \int_{t-\tau(t)}^{t} \dot{e}_f(s) ds \\
& - 2\xi^t(t)(-N) \int_{t-\varrho}^{t-\tau(t)} \dot{e}_f(s) ds \\
& - 2\xi^t(t) M \int_{t-\varrho}^{t} \dot{e}_f(s) ds
\end{aligned} \tag{27}
$$

Consider adding and subtracting the terms:

$$
\xi^t(t)[\varrho M W_a^{-1} M^t + \varrho N W_c^{-1} N^t]\xi \tag{28}
$$

Now consider the following terms:

$$
\begin{aligned}
& \varrho\xi^t(t) M W_a^{-1} M^t \xi(t) + \varrho \xi^t(t) N W_c^{-1} N^t \xi(t) \\
& \varrho\xi^t(t) M W_a^{-1} M^t \xi(t) - \tau(t) \xi^t(t) N W_c^{-1} N^t \xi(t) \\
& -(\varrho - \tau(t)) \xi^t(t) N W_c^{-1} N^t \xi(t) \\
& -2\xi^t(t) N \int_{t-\tau(t)}^{t} \dot{e}_f(s) ds + 2\xi^t(t) N \int_{t-\varrho}^{t-\tau(t)} \dot{e}_f(s) ds \\
& -2\xi^t(t) M \int_{t-\varrho}^{t} \dot{e}_f(s) ds + \int_{t-\varrho}^{t-\tau(t)} \dot{e}_f^t(s) W_c \dot{e}_f ds \\
& + \int_{t-\tau(t)}^{t} \dot{e}_f^t(s) W_c \dot{e}_f ds + \int_{t-\varrho}^{t} \dot{e}_f^t(s) W_a \dot{e}_f ds
\end{aligned} \tag{29}
$$

The terms in (29) after some manipulation become:

$$
\begin{aligned}
& = \xi^t(t)[\varrho M W_a^{-1} M^t + \varrho N W_c^{-1} N^t]\xi \\
& - \int_{t-\tau(t)}^{t} [\xi^t N + \dot{e}_f^t W_c] W_c^{-1} [\xi^t N + \dot{e}_f^t W_c]^t ds \\
& - \int_{t-\varrho}^{t-\tau(t)} [-\xi^t N + \dot{e}_f^t W_c] W_c^{-1} [-\xi^t N + \dot{e}_f W_c]^t ds \\
& - \int_{t-\varrho}^{t} [\xi^t M + \dot{e}_f^t W_a] W_a^{-1} [-\xi^t M + \dot{e}_f W_A]^t ds
\end{aligned} \tag{30}
$$

Further Manipulations of (27) result in

$$
\begin{aligned}
\dot{V}(t) \leq\ & \xi^t [\Upsilon_o + \varrho M W_a^{-1} M^t + \tau(t) N W_c^{-1} N^t \\
& + (\varrho - \tau(t)) N W_c^{-1} N^t] \xi(t) + \varrho \dot{x}(t)(W_a + W_c) \dot{e}_f(t) \\
& - \int_{t-\tau(t)}^{t} [\xi^t N + \dot{e}_f W_c] W_c^{-1} [\xi^t N + \dot{e}_f^t W_c]^t ds \\
& - \int_{t-\varrho}^{t-\tau(t)} [-\xi^t N + \dot{e}_f W_c] W_c^{-1} [-\xi^t N + \dot{e}_f^t W_c]^t ds \\
& - \int_{t-\varrho}^{t} [\xi^t M + \dot{e}_f W_a] W_a^{-1} [\xi^t M + \dot{e}_f^t W_a]^t ds \\
& \leq \xi^t(t) [\Upsilon_o + \varrho M W_a^{-1} M^t + \varrho N W_c^{-1} N^t] \xi(t) \\
& + \varrho \dot{e}_f^t (W_a + W_c) \dot{e}_f(t)
\end{aligned} \tag{31}
$$

In view of 17 with $G_o = 0, G_d = 0, \Gamma_o = 0$, and Schur's compliments, it follows from (31) that $\dot{V}(t) < 0$ which establishes the internal asymptotic stability.

Consider the performance measure $J = \int_0^\infty (z^t(s)z(s) - \gamma^2 w^t(s) w(s)) ds$. For any $w(t)\ \epsilon\ L_2(0,\infty) \neq 0$ and zero initial condition $x(0) = 0$, we have:

$$
\begin{aligned}
J &= \int_0^\infty (z^t(s)z(s) - \gamma^2 w^t(s)w(s) + \dot{V}(x)) ds - \dot{V}(x) \\
&\leq \int_0^\infty (z^t(s)z(s) - \gamma^2 w^t(s)w(s) + \dot{V}(x)) ds
\end{aligned} \tag{32}
$$

Proceeding as before, we get:

$$
\begin{aligned}
& z^t(s)z(s) - \gamma^2 w^t(s)w(s) + \dot{V}(s) = \bar{\chi}^t(s) \bar{\Upsilon} \bar{\chi}^t(s), \\
& \bar{\chi}(s) = [{e_f}^t(s) \quad {e_f}^t(s-\tau(t)) \quad {e_f}^t(t-\varrho) \quad w(s)]^t
\end{aligned} \tag{33}
$$

where $\bar{\Upsilon}$ corresponds to Υ_o in 17 by Schur's Complements. It is readily seen from 17 that:

$$
z^t(s)z(s) - \gamma^2 w^t(s)w(s) + \dot{V}(s) < 0 \tag{34}
$$

for arbitrary $s\epsilon[t,\ \infty)$, which implies for any $w(t)\epsilon L_2(0,\infty) \neq 0$ that $J < 0$ leading to $\parallel z(t) \parallel_2 < \gamma \parallel w(t) \parallel_2$.

IV. Simulation

The simulation for the scheme are being made on an open-loop stable time-delay system for chemical reactor is considered here state-feedback design [12]. In the reactor, raw materials A and B take part in three chemical reactions that produce a product P along with some byproducts. By linearization and time scaling, the state variables are the deviations from their nominal values in the weight composition of intermediate product C and in the weight composition of reactant P. The control variables are relative deviations in the feed rates. Using typical values [12], the model matrices are:

$$
A_0 = \begin{bmatrix} -57.83 & 11.21 & -93.6 & -233.4 \\ 14 & -25.5 & 83.2 & 212.3 \\ 6.4 & 0.347 & -32.5 & -1.04 \\ 0 & 0.833 & 11.0 & -3.96 \end{bmatrix},
$$

$$
A_{do} = \begin{bmatrix} 2.92 & 0 & 0 & 0 \\ 0 & 2.92 & 0 & 0 \\ 0 & 0 & 2.87 & 0 \\ 0 & 0 & 0 & 2.724 \end{bmatrix}
$$

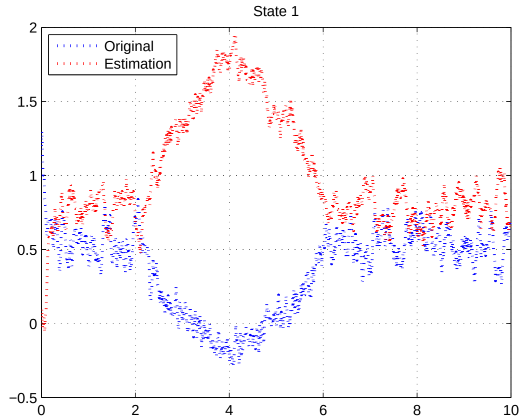

Fig. 2. Fault Detection filter using LKF: State1

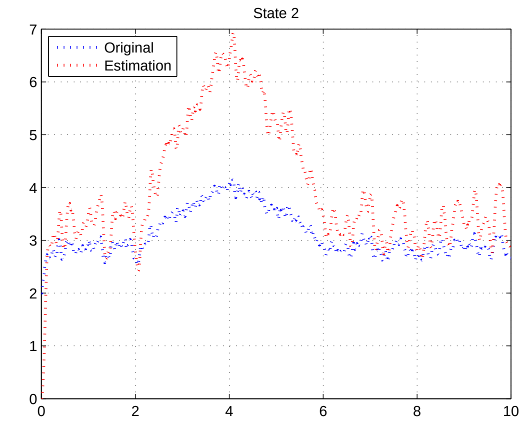

Fig. 3. Fault Detection filter using LKF: State2

$$\Gamma_0 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$$

$$\Phi = 0.1, G_o = \begin{bmatrix} 0.01 & 0 & 0.01 & 0 \end{bmatrix},$$

$$G_{do} = \begin{bmatrix} 0.1 & 0.2 & 0.4 & 0.3 \end{bmatrix}, \qquad (35)$$

Following the main steps for the proposed scheme, the following are the simulation results for the system under observation as can be seen in figure (2-5) and error in figure (6).

V. CONCLUSIONS

In this paper, robust fault detection filter with time-delay has been proposed using Lyapunov-Krasovskii function (LKF). Using an LMI method, the existence conditions and further solution for the optimization problem have been derived and, based on them, an algorithm for the design of the fault detection filters has been proposed.The proposed scheme has been evaluated on an open-loop time-delay system for chemical reactor simulated system thus ensuring the effectiveness of the approach.

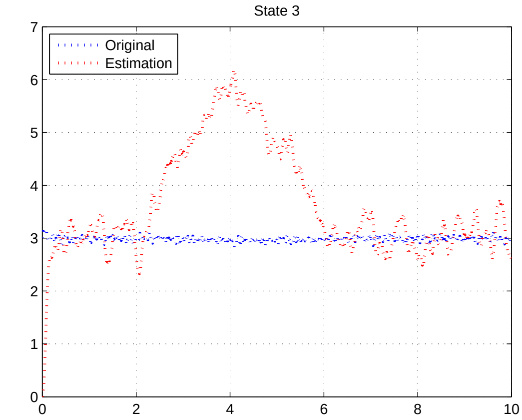

Fig. 4. Fault Detection filter using LKF: State3

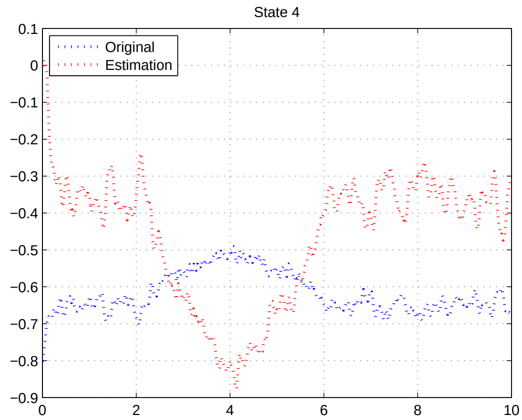

Fig. 5. Fault Detection filter using LKF: State4

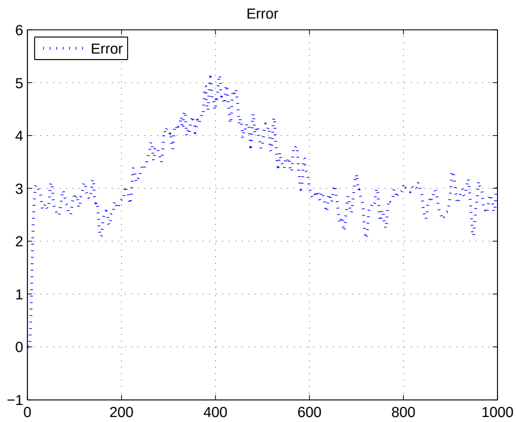

Fig. 6. Fault Detection filter using LKF: Error

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