Robust control of a closed-loop identified system with parametric/model uncertainties and external disturbances

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Abstract—**This paper deals with the closed-loop identification of a two-tank process used in industry. The identified model is then utilized to develop robust controllers i.e. H_∞ and sliding mode controllers. It is shown that these controllers guarantee a satisfactory performance in the face of both model/parametric uncertainties and external disturbances. The designed controllers have been successfully tested through extensive simulation. In addition, this paper shows that the designed robust controllers far outperform traditional controllers such as P, PI, and PID, in the face of parametric model uncertainties and the effects of external disturbances. The successful use of the designed robust controllers encourages their extension to other physical systems.**

Keywords- Closed loop identification, tuning, H_∞ control, sliding mode control, control of two-tank system.

I. Introduction

Closed -loop identification has always been recognized as being of primary importance and has been the subject of intense research since the late sixties. It is widely recognized that in a large number of practical situations, feedback cannot be removed for various reasons including the following situations: (i) feedback may be intrinsic in the physical mechanism generating the data or (ii) data may come from an industrial plant where feedback loops cannot be opened due to safety or production quality reasons or (iii) the physical mechanism might be too complex and may not be easily manipulated (for example a communication network). Another reason for performing identification in a closed- loop setting comes from the need to design experiments that would reduce the uncertainty in certain key frequency bands to a desired level.

Employing closed-loop in process control has numerous drawbacks, notably, the absence of a quality-check in the midst of the process, which results in a faulty output. For example, in a plant that mass-produces television sets, the process involves assembling CRTs and television bodies. During this assembly, if a body or a CRT gets damaged in the process, or even if it were damaged beforehand, a faulty product will be produced and the loop will be mistakenly acknowledged as having produced an acceptable product and will be allowed to continue to do so, until a subsequent quality-control stage detects the faulty products and alerts the operators that a remedial action is in order.

Identification of systems operating in a closed loop has received considerable attention in the System Identification literature [4], [8], [12], [14], [15].

There are safety and economic reasons for performing identification experiments in a closed loop. Also, it is known that the optimal experiment is usually performed in closed loop [5], [10], [11], [15], [17]. Indeed, recent research has established that, for a general class of systems, and when there is a constraint on the output power, the optimal experiment is necessarily closed loop [3].

Unfortunately, the identification of systems operating under the presence of feedback presents several difficulties [4], [15]. For example, correlation between the input signal and the noise is problematic in the context of several identification techniques. In fact, it is well known that the Prediction Error Method (PEM) provides a non-consistent estimate in the presence of under-modeling of the noise transfer function [4].

Several attempts to overcome this difficulty have been made. In particular, indirect identification is a popular approach to mitigate this difficulty. Traditional indirect identification is a two-step procedure where the identification of a plant object is first obtained and then the open-loop system is unraveled from this preliminary estimate. Here, and in the sequel, we use the term "plant object" to refer to a transfer function that depends on the system. In traditional indirect identification, the plant object to be identified is usually the complementary sensitivity transfer function relating the reference signal to the output [14].

However, several difficulties are known to exist with this approach. For example, it is common that the identified open-loop process is not necessarily stabilized by the controller used in the identification experiment, even though it is known that the real system is stabilized by this controller.

The aim of this paper is to find an open-loop model of a process using closed-loop data that has sufficient excitation exerted from the set-point and then design a robust controller to attain robustness against model/parametric uncertainties and the effect of external disturbances. This paper aims to explore whether normal operating closed-loop data of a real process can be practically used to identify the open-loop process model and use these data to find the tuning parameters automatically. Hence, the objectives are as follows:

1- To be able to use normal operating closed-loop data to model the open-loop process.

2- To identify the tuning parameters using famous tuning techniques.

3- To develop linear and nonlinear robust controllers for the identified model to make the model robust in practice.

An evaluation of the proposed scheme was performed on a benchmark laboratory-scale process control system using National Instruments LABVIEW.

The paper is organized as follows. In Section II, the process control system is described. Section III describes the modeling of the two-tank bench mark model and the leaks. Section IV deals with the evaluation of the proposed schemes on the physical system and Section V contains the conclusion.

II. THE PROPOSED SCHEME

In order to find the model, the following stages were developed:

- Data Pretreatment: remove bias and outliers.
- Use the excitation test and make sure that at least a model of 5th order can be found.
- Find the process delay by iteration of several least square estimations with varying delays.
- Find a process model using the least squares method.
- Find the model prediction errors and verify the accuracy of the model.
- Find the tuning constants based on the reaction curve technique and lambda tuning rules.
- Find a step response of the identified model and strive to obtain a better step response of the system by employing linear and robust controllers.

The proposed scheme has been evaluated on a process control system. The proposed scheme has been evaluated on a Bench-marked laboratory scale two tank apparatus.

III. A TWO TANK BENCH-MARK MODEL

A benchmark model of a cascade connection of a dc motor and a pump relating the input to the motor, *u*, and the flow, Q_i, is a first-order system:

$$\dot{Q}_i = -a_m Q_i + b_m \phi(u) \tag{1}$$

where a_m and b_m are the parameters of the motor-pump system and $\phi(u)$ is a dead-band and saturation type of nonlinearity. It is assumed that the leakage Q_ℓ occurs in tank 1 and is given by:

$$Q_\ell = C_{d\ell}\sqrt{2gH_1} \tag{2}$$

With the inclusion of the leakage, the liquid level system is modeled by:

$$A_1 \frac{dH_1}{dt} = Q_i - C_{12}\varphi(H_1 - H_2) - C_\ell \varphi(H_1) \tag{3}$$

$$A_2 \frac{dH_2}{dt} = C_{12}\varphi(H_1 - H_2) - C_0\varphi(H_2) \tag{4}$$

where $\varphi(.) = sign(.)\sqrt{2g(.)}$, $Q_\ell = C_\ell \varphi(H_1)$ is the leakage flow rate, $Q_0 = C_0\varphi(H_2)$ is the output flow rate, H_1 is the height of the liquid in tank 1, H_2 is the height of the liquid in tank 2, A_1 and A_2 are the cross-sectional areas of the 2 tanks, g=980 $cm/\sec^2$ is the gravitational constant, C_{12} and C_o are the discharge coefficient of the inter-tank and output valves, respectively.

The model of the two-tank fluid control system, shown above in Fig. 1, is of a second order and is nonlinear with a smooth square-root type of nonlinearity. For design purposes, a linearized model of the fluid system is required and is given below in (5) and (6):

$$\frac{dh_1}{dt} = b_1 q_i - (a_1 + \alpha)h_1 + a_1 h_2 \tag{5}$$

$$\frac{dh_2}{dt} = a_2 h_1 - (a_2 - \beta)h_2 \tag{6}$$

where h_1 and h_2 are the increments in the nominal (leakage-free) heights H_1^0 and H_2^0:

$$b_1 = \frac{1}{A_1}, \quad a_1 = \frac{C_{db}}{2\sqrt{2g(H_1^0 - H_2^0)}}, \quad \beta = \frac{C_0}{2\sqrt{2gH_2^0}},$$

$$a_2 = a_1 + \frac{C_{do}}{2\sqrt{2gH_2^0}} \quad \alpha = \frac{C_{d\ell}}{2\sqrt{2gH_1^0}}$$

and the parameter α indicates the amount of leakage.

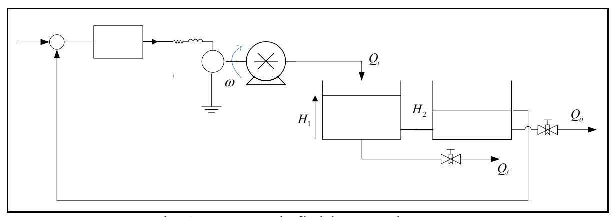

Fig.1: Two-tank fluid control system

A PI controller, with gains k_p and k_I, is used to maintain the level of the Tank 2 at the desired reference input r .

$$\begin{aligned} \dot{x}_3 &= e = r - h_2 \\ u &= k_p e + k_I x_3 \end{aligned} \tag{7}$$

The state space model is given by:

$$\dot{x} = Ax + Br \tag{8}$$

with

$$x = \begin{bmatrix} h_1 \\ h_2 \\ x_3 \\ q_i \end{bmatrix}, \quad A = \begin{bmatrix} -a_1 - \alpha & a_1 & 0 & b_1 \\ a_2 & -a_2 - \beta & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -b_m k_p & 0 & b_m k_I & -a_m \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 1 & b_m k_p \end{bmatrix}^T, \qquad C = [1 \quad 0 \quad 0 \quad 0]$$

where q_i , q_ℓ , q_0 , h_1 and h_2 are the increments in Q_i ,Q_ℓ ,Q_o , H_1^0 and H_2^0, respectively, the parameters a_1 and a_2 are associated with linearization whereas the parameters α and β are respectively associated with the leakage and the output flow rate, i.e. $q_\ell = \alpha h_1$, $q_o = \beta h_2$.

IV. EVALUATION ON A PHYSICAL SYSTEM

The physical system under evaluation is formed of two tanks connected by a pipe. The leakage is simulated in the tank by opening the drain valve. A DC motor-driven pump supplies the fluid to the first tank and a PI controller is used to control the fluid level in the second tank by maintaining the level at a specified level, as shown in Fig. 1.

A step input is applied to the dc motor- pump system to fill the first tank. The opening of the drainage valve introduces a leakage in the tank. Various types of leakage faults are introduced and the liquid height in the second tank, H_2, and the inflow rate, Q_i, are both measured. The National Instruments LABVIEW package is employed to collect these data.

As mentioned earlier, various types of leakages were introduced by opening the drainage valve and the liquid height profiles in the second tank were subsequently analyzed.

A. Data pretreatment: removal of bias and outliers

The input/output data of a flow controller of a real process was collected at a sample rate of 1 second to be used for this project. A graph showing, from top to bottom, the process output *y(t)*, the controller output *u(t)* and the input reference signal *r(t*) respectively as shown in Figure 2(a). A total of 5000 samples were collected for this process.

Flow is measured using differential pressure measurement across an orifice plate with flange tapings. The controller used is a PI feedback controller with a controller gain of 0.15 which is measured by percent of full range of process output measurement over the full scale percentage of the controller output which is 100%. The integral constant Ti was 0.5 minute per repeat.

The data is already filtered at the transmitter device stage using a second-order filter with damping constant of 0.4 to filter out any high frequency noise. Hence, no further noise filtering of the data is necessary.

The data pretreatment process for this project is designed to remove any outliers from the measurements. This is being shown in the following sections.

B. Persistent excitation test

The persistent excitation condition is the minimum requirement imposed on the test signal to guarantee that the estimation algorithms have unique solutions. For a finite impulse response model, the persistency-of- excitation test is found from the correlation matrix as follows:

$$R_{uu} = \begin{bmatrix} r_{uu}(0) & r_{uu}(1) & r_{uu}(2) & \dots & r_{uu}(p-1) \\ r_{uu}(1) & r_{uu}(0) & r_{uu}(1) & \dots & r_{uu}(p-2) \\ r_{uu}(2) & r_{uu}(1) & r_{uu}(0) & \dots & r_{uu}(p-3) \\ \dots & \dots & \dots & \dots & \dots \\ r_{uu}(p-1) & r_{uu}(p-2) & \dots & r_{uu}(0) & \end{bmatrix}$$

Here, our agenda is to find the maximum model order that can be found with reasonable accuracy. The degree of excitation of the input signal is defined as the order of a model that the input is capable of estimating in an unambiguous way. The process will do the following:

1. Find the correlation matrix of different sizes.
2. Determines the maximum matrix size by finding the singular values of the correlation matrices using a threshold of 1e-9.
3. If the technique determines that the maximum order is less than 5, the process will be terminated.

For the data provided, the result came out to be *n=50*. Hence, the process will continue.

C. Finding the process delay

The process delay is found by an estimation which is based on a comparison of ARX models with different delays *nk*.

$$\begin{aligned} & y(t) + a_1 y(t-1) + \dots + a_{na} y(t-na) = b_1 u(t-nk) \\ & + \dots + b_{nb} u(t-nb-nk+1) \end{aligned} \tag{9}$$

The parameters are found using the least-squares method. The delay of the data used was found as 4 samples or 4 seconds. This is very reasonable for process control valves that have the speed of 0.15 inches per second for their stem movement.

D. Finding a process model using least squares method

The least-squares problem solution of a ARX model was found to be $\hat{\theta} = A^{\dagger} Y$.

where $A^{\dagger}$ is the pseudo-inverse of the A given by:

$$A^{\dagger} = \left[A^T A \right]^{-1} A^T$$

Where the minimization problem to be solved was:

$$\min_{\{\theta\}} \left\{ \left(Y(n) - A(n)\theta \right)^T \left(Y(n) - A(n)\theta \right) \right\}$$

The results were as follows:

$$\begin{aligned} & \textit{Discrete time IDPOLY Model}: A(q)y(t) = B(q)u(t) + e(t) \\ & A(q) = 1 - 0.7314q^{-1} + 0.01969q^{-2} - 0.06907q^{-3} - 0.1476q^{-4} \\ & B(q) = 0.1036q^{-4} - 0.0312q^{-5} - 0.03938q^{-6} - 0.02451q^{-7} \\ & \textit{Sampling Interval} = 1 \end{aligned}$$

E. Finding the performance of the identified model

The commonly-used performance measures are unbiasedness, consistency and efficiency. Unbiasedness is guaranteed for ARX models because the noise is uncorrelated with the input data matrix. It measures the average value of the parameters and verifies that it is equal to the actual process parameter average.

The consistency involves the sum of squares of the residuals. One can validate the consistency of the identified model by verifying that:

$$E[res^T res] = (N - M)\sigma_v^2$$

where N and M are the number of rows and columns of matrix A, respectively. Or, validation can be performed by taking the autocorrelation of the residuals, which should show that they are close to zero for all nonzero lags, i.e.

having a delta function-like shape.

The efficiency can be measured by finding the covariance of the model parameters from the real process parameters as $\mathrm{cov}[\hat{\theta}] = \sigma_v^2 (A^T A)^{-1}$. The determinant of the covariance matrix of the prediction errors is the determinant of the noise variance matrix and it is called the Loss function *(LossFcn)*. This value measures the prediction error reasonably well. It also provides the so called **FPE**: Akaike's Final Prediction Error, defined as the *LossFcn* *(1+d/N}/(1-d/N), where d is the number of estimated parameters and N is the length of the data record.

The *LossFcn* was used as a performance measure and it was found that *Loss function=0.000633849 and FPE=0.000635884.*

All performance tests show very good and reliable results for the identified model. Hence, we may proceed to the next step which is finding the tuning parameters of the controller.

Note that, due to the presence of output feedback, the cross correlation between the input and the residuals showed negative values which is expected for feedback control loops as shown in the fig. 2(b).

F. Finding the tuning constants based on the reaction curve technique

The tuning constants can be found by using the reaction curve technique applied on the step response of the identified model found previously in step 6. The results of applying this technique and the comparison of the model approximation with the original step response model are shown in the fig. 3(a) and fig. 3(b) respectively.

G. PID tuning

There are many PID tuning rules around for first-order plus time delay system models. The following tuning table was derived by Ziegler-Nichols to provide a quarter-decay ratio (the ratio of the second peak over the first peak) as shown in Table 1. Controller settings are shown in Table 2. (alpha: time delay, tau: time constant, Kp: gain).

The results were found as follows:

Process gain: 0.120062, Time constant: 1.15871 sec, Time delay: 4 sec.

H. Step response of the identified model using PID

The step response model of the identified model is as shown below in Fig. 4(a).

Table 1: Ziegler Nocholas settings

Controller	Kc	Ti	Td
P	tau/(Kp*alpha)		
PI	0.9*tau/(Kp*alpha)	3.33*alpha	
PID	1.2*tau/(Kp*alpha)	2*alpha	0.5*alpha

The open loop response of the system with leakage faults is as shown below in Fig. 4(b). The closed-loop step response has been simulated for the PID controller and is depicted in the fig. 4(c). Figure 4(c) shows that the process output step response reaches the steady state after approximately 180 seconds or 3 minutes.

Table 2: Controller settings for P, PI and PID

Controller	Kc	Ti	Td
P	2.4127		
PI	2.1715	13.32	
PID	2.8953	8	2

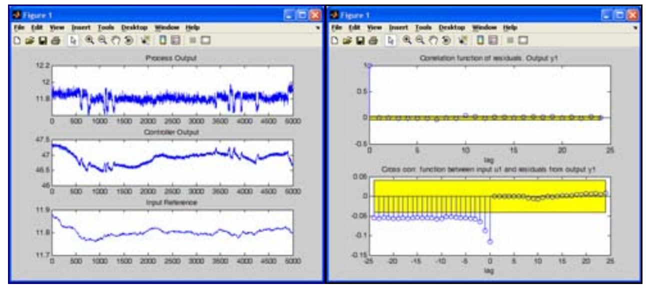

Fig.2: (a) Process Output y(t), the controller output u(t), and Input reference signal r(t), (b) Cross Correlation function between input u1 and output y1.

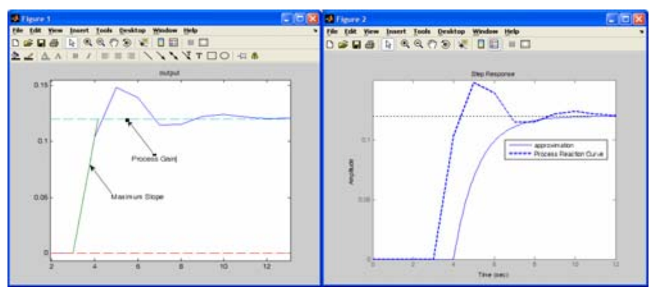

Fig.3: (a) Reaction curve technique, (b) Comparison of model approximation and the original step response model

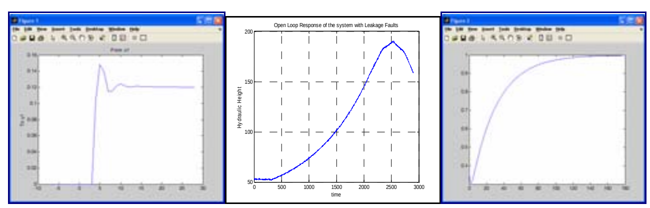

Fig.4: (a) Step response of the identified model, (b) Open loop response of the system with leakage faults, (c) Closed loop response for PID Controller

I. Step response of the identified model using H_∞ control

H_∞ controller has been often used for robust control of dynamical systems [18], [19], [20]. The aim of the H_∞ controller design is to track the reference input given by the operator and achieve the desired position in a minimum time and at given performance level. For this purpose we employed H_∞ controller for which the standard configuration is shown in Fig. 5. The signals involved are: the control variable ‘u’, the measured variables ‘v’, the exogenous inputs 'w' such as disturbances and commands and the exogenous outputs 'z'. The closed-loop transfer function from w to z is given by linear fractional transformation [18].

$$z = F_l(P,K)w \tag{10}$$

Where

$$F_l(P,K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$$

The standard H_∞ optimal control problem is to find all the stabilizing controllers K which minimize the following H_∞ norm:

$$\|F_l(P,K)\|_\infty = \max_\omega \bar{\sigma}(F_l(P,K)(j\omega)) \tag{11}$$

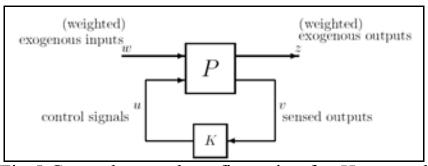

Fig.5 General control configuration for H_∞ control

The H_∞ norm has several interpretations in terms of performance [18] where it minimizes the peak of the maximum singular value of $F_l(P,K)$. The general algorithm used to compute the controller is based on the solution presented in

H_∞ loop shaping [18] is essentially a two-stage design procedure. First, the open-loop plant is augmented by a pre and a post-compensator as shown in Fig. 6 to give a desired shape to the singular values of the open-loop frequency response. Then the resulting shaped plant is robustly stabilized with respect to co-prime factor uncertainty using H_∞ optimization. The employed weights allow us to modify the model dynamics with a view to improve its tracking by increasing the gains at lower frequencies and changing the slope at the cross-over frequency to '1' so as to improve the controller's robustness. The re-shaped model of the identified plant can be utilized for controller synthesis as described in [19]. The design procedure resulted in an H_∞ controller with $\gamma_{\min} = 1.2817$, which confirms that our designed controller based on the identified model is robust against both external disturbances and system uncertainties. However, if we evaluate the performance of the system with the designed controller, we can observe that the system attains the desired level in less that 10 sec. which is much better than the results obtained with a PID controller. The step response in Fig.7 shows that there is an overshoot in the system and some percent of steady state error.

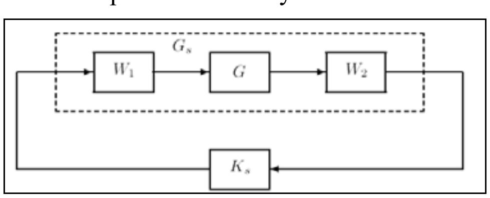

Fig.6 LSDP implementation

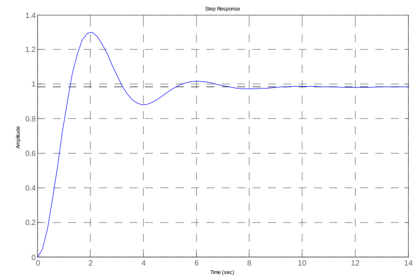

Fig.7 Step response of the identified model using H_∞ control

J. Step response of the identified model using sliding mode control

As an alternate robust controller, a sliding mode controller has been designed for the identified model. In sliding-mode control design, a hyper-plane is defined as a sliding-surface. This design approach comprises of two stages: the first one is the reaching phase and the second one is the sliding phase. In the reaching phase, the system's states are driven to a stable manifold by the help of an appropriate equivalent control law and in the sliding phase, these states slide to an equilibrium point. One advantage of this design approach is that the effect of nonlinear terms, which may be construed as a disturbance or as an uncertainty in the nominal plant, has been completely rejected. Another benefit accruing from this situation is that the system is forced to behave as a reduced order system; this guarantees the absence of overshoot while attempting to regulate the system from an arbitrary initial condition to the designed equilibrium point. The design of a sliding-mode controller for the identified model is carried out by defining the sliding manifold based on its error dynamics defined as:

$$e = x - x_d$$

where x_d is the desired value of the system state at the equilibrium position. For the above-discussed design approach, the sliding manifolds are designed as follows:

$$S = e_1$$

The system error tends to zero if $S = 0$ and the rate of convergence will be governed by the manifold dynamics. The Lyapunov function [20], [21], [22] for the surfaces defined above can be written as:

$$V = \frac{1}{2}s^2$$

which are positive definite functions and their time derivative can be written as:

$$\dot{V} = s\dot{s} \tag{12}$$

The equivalent control u_{eq} on the manifold $s_1 = e_1 = 0$ can be shown to be equal to:

$$u_{eq} = \frac{0.04276}{0.0625}x_1$$

The control input vector *'u'* that will make the system to converge at $S = 0$ can be written as:

$$u = u_{eq} - Ksign(s)$$

This control law will ensure both the system's convergence to a sliding manifold and robustness against the system uncertainties and external disturbances. To avoid high frequency switching, i.e. chattering, implementation of the control laws have been performed by employing a saturation function $sat(S)$ [22] defined as:

$$\begin{aligned} \mathrm{sat(s)} &= \mathrm{sign}(\frac{s}{\varepsilon}); && \textit{if } \mathrm{abs}(\frac{s}{\varepsilon})>1 \\ \mathrm{sat(s)} &= \frac{s}{\varepsilon}; && \textit{if } \mathrm{abs}(\frac{s}{\varepsilon})<1 \end{aligned} \tag{13}$$

The chattering reduction depends on the value of ε in the range '$\varepsilon < 1$' but at the cost of robustness. The higher the value of ε, the lesser the chattering will be and the more reduced the robustness will be too. Now by substituting the defined control laws in equation (12), we get:

$$\dot{V} = -s^2$$

$\dot{V}$ will always be negative definite for non-zero manifolds.

In the first phase of controller validation, only simulations are carried out. But later, the designed controller was tested on real data extracted from a physical two-tank system, and it delivered the desired tracking control of the identified model as shown in Fig. 8 along with sliding manifolds convergence. The convergence of states is in finite time with no overshoot and steady state error. The robust nature of sliding mode control makes the identified systems indifferent to parametric uncertainties and external disturbances. However the performance of the system is best in the previously discussed and simulated controllers i.e. the system reaches the desired level in less than 6 seconds which is almost twice as fast as its linear counter-part.

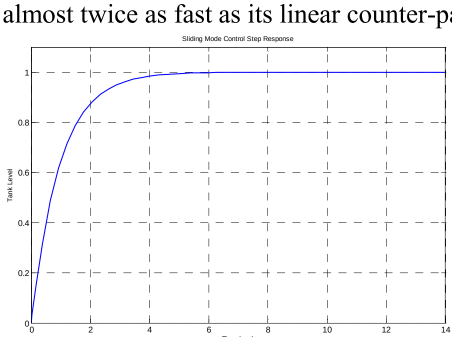

Fig.8 Step response of the identified model using sliding mode control

V. Conclusion

In this paper, a new scheme for Closed Loop Identification has been proposed and successfully tested. It has been shown that it is possible to model a process while the controller is in an open loop model. This work also makes use of the model to develop robust controllers. The designed robust controllers have been shown to outperform the traditional controllers of P, PI and PID and guarantee the desired performance of the system along with robustness against parametric and model uncertainties as well as the effects of external disturbances.

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References

[1] J. C. Ag¨uero. System identification methodologies incorporating constraints. PhD thesis, School of Electrical Engineering and Computer Science, The University of Newcastle, Australia, 2005.

[2] J. C. Ag¨uero and G. C. Goodwin. Virtual closed loop identification: A Subspace Approach. In Proceedings of the 43rd IEEE Conference on Decision and Control, CDC, pages 364–369, Atlantis, Paradise Island, Bahamas, 2004.

[3] J. C. Ag¨uero and G. C. Goodwin. Choosing between open and closed loop experiments in linear system identification. IEEE Trans. Automatic Control, 52(8):1475–1480, 2007.

[4] U. Forssell and L. Ljung. Closed loop identification revisited. Automatica, 35:1215–1241, 1999.

[5] M. Gevers. Identification for control: from the early achievements to the revival of experiment design. European Journal of Control, 11(4- 5):335–352, 2005.

[6] G. C. Goodwin, J. C. Ag¨uero, J. S. Welsh, J. I. Yuz, G. J. Adams, and C. R. Rojas. Robust identification of process models from plant data. Journal of Process Control, To appear, 2008.

[7] G. C. Goodwin, S. F. Graebe, and M. E. Salgado. Control System Design. Prentice Hall, Upper Saddle River, NJ, 2001.

[8] G. C. Goodwin and R. Payne. Dynamic System Identification: Experiment design and data analysis. Academic Press, 1977.

[9] F. Hansen, G. Franklin, and R. Kosut. Closed loop identification via the fractional representation: Experiment design. Proceedings of the American Control Conference, pages 1422–1427, 1989.

[10] H. Hjalmarsson. From experiment design to closed-loop control. Automatica, 41(3):393–438, 2005.

[11] H. Hjalmarsson, M. Gevers, and F. De Bruyne. For model-based control design, closed-loop identification gives better performance. Automatica, 32(12):1659–1673, 1996.

[12] L. Ljung. System Identification: Theory for the user. Prentice Hall, 2nd edition, 1999.

[13] R.J.P. Schrama. An open-loop solution to the approximate closed loop identification problem. In C. Banyasz and L. Keviczky, editors, Identification and System Parameter Estimation 1991, number 3 in IFAC Symposia Series 1992, pages 761–766, Budapest, Hungary, July 1991 1991. Proc. 9th IFAC/IFORS Symp. Identification and Syst. Param. Estimation.

[14] T. S¨oderstr¨om and P. Stoica. System identification. Prentice-Hall International, 1989.

[15] P. M. J. Van den Hof. Closed-loop issues in system identification. Annual Reviews in Control, 22:173–186, 1998.

[16] P. M. J. Van den Hof and R. J. P. Schrama. An indirect method for transfer function estimation from closed loop data. Automatica, 29:1523–1527, 1993.

[17] P.M.J. Van den Hof and R.J.P. Schrama. Identification and control closed loop issues. Automatica, 31(12):1751–1770, 1995.

[18] Skogestad, S. and Postlethwaite, I.(1996) "Multivariable Feedback control ,Analysis and Design" John Wiley & Sons. New York

[19] Doyle, J.Grover, K.Khargonekar, and Francis,B. (1989),"State space solutions to standard H2 and H control problems". IEEE Trans. On automatic control, Vol 34. no 8, pp831-846, August 1989.

[20] Q. Ahmed, A. I. Bhatti, S. Iqbal, Robust Decoupling Control Design for Twin Rotor System using Hadamard Weights, Proc. of CCA, MSC 2009, St. Petersburg, Russia

[21] Slotine, J. J. and Li W. (1991), *Applied Nonlinear Control*, Prentice Hall, ISBN 0-13-040-890-5

[22] Q. Ahmed, A. I. Bhatti, S. Iqbal, Nonlinear Robust Decoupling Control Design for Twin Rotor System, Proc. Asian Control Conference 2009, Hong Kong.