EEL4413 – Power System Analysis

LO 2

Apply Gauss-Seidel and Newton-Raphson techniques to solve load flow problems.

The Need for Load Flow Analysis

Load flow calculations are used to determine the voltage, current, real and reactive power at various points in a power system under normal steady-state conditions.

Load Flow is Used to Determine:

- **P and Q at each load bus bar.**
- **P and δ at each generator bus bar.**
- **P and Q at the slack bus bar.**
- **P and Q losses in each line.**
- **Power Flow (P and Q) in each line.**

The Need for Power (Load) Flow Analysis for Power Systems:

A- Power flow along transmission line

B- Bus voltages and system voltage profile

C- The effect of changes in system configuration and incorporating new circuits on system loading

D- Investigation of the effect of temporary loss of transmission capacity and (or) generation on system loading

E- Economic system operation

F- System loss minimization

G- Transformer tap setting for economic operation

H-Possible improvements to an existing system by change of conductor sizes and system voltages.

The Need for Load Flow Analysis for Evaluating the Following System Changes:

- New generation sites
- Projected load growth
- New transmission line locations
- Interconnections with other systems.
- Various load conditions, such as peak and off peak
- The impact of losing major components (lines, transformers, or generators

Basis of Load Flow Analysis

Consider the general electrical system represented by the circuit in the following Figure

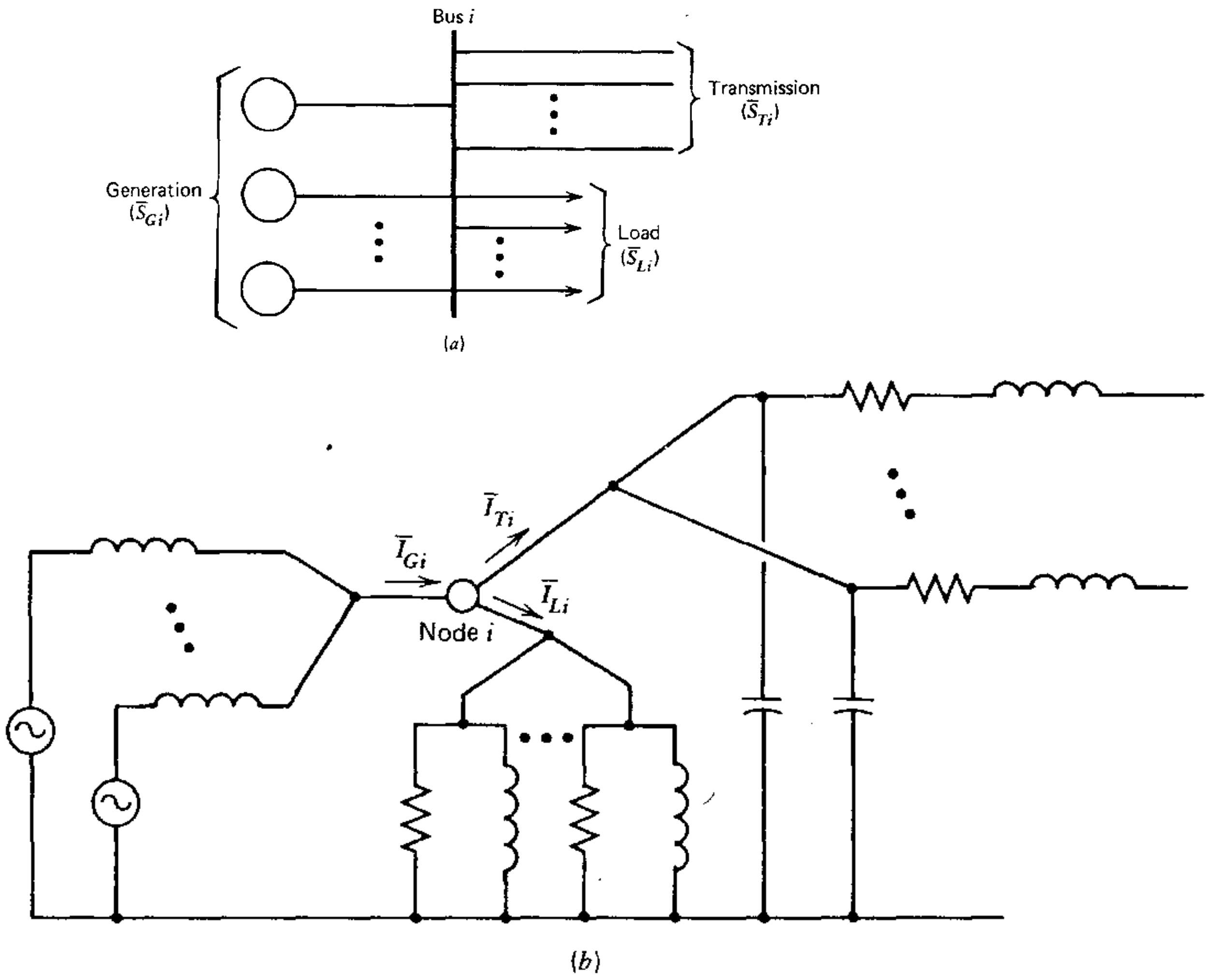


Figure 1. General situation at bus i. (a) Single-line diagram at bus

(b) Positive- sequence-equivalent circuit.

Statement of the Power Flow Problem

The Complex power injected by a source into the ith bus of a power system such as shown in Figure is:

$$S_i = P_i + jQ_i \qquad i = 1, 2, \dots, n$$

But

$$S_i = V_i I_i^*$$

Where Ii is the source current injected into the ith bus

Si can also be written as:

$$S_i^* = V_i^* I_i$$

Statement of the Power Flow Problem

Ii can be written as:

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j$$

Therefore,

$$P_i - jQ_i = V_i^* \sum_{j=1}^{n} Y_{ij} V_j$$

Formulation of the Load Flow Problem

Equating the Real and Imaginary parts, we get:

$$P_i \text{ (real power)} = R_e \left\{ V_i^* \sum_{j=1}^{n} Y_{ij} V_j \right\}$$

And,

$$Q_i \text{ (reactive Power)} = -I_m \left\{ V_i^* \sum_{j=1}^{n} Y_{ij} V_j \right\}$$

Formulation of the Load Flow Problem

Or,

$$P_i = |V_i| \sum_{i=1}^{n} |V_j||Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_i = -|V_i| \sum_{j=1}^{n} |V_j||Y_{ij}| \sin(\theta_{ij} + \delta_k - \delta_i)$$

These are the basic load flow equations

Formulation of the Load Flow Problem

These Equations illustrate the complexity of power flow problem, because:

1. ***They show that the power is a function of the voltage magnitude and phase at all the other buses in the system***
2. ***Two or three variables are multiplied, some are sine or cosine functions***

Basis of Load Flow Analysis

Therefore,

***Four** variables are associated with each bus, k:*

- *Voltage Magnitude, V_k*
- *Phase Angle δ_k,*
- *Net Real Power, P_k, and*
- *Reactive Power, Q_k*

***Two** are specified as Input Data, the other **two** are unknowns, and need to be computed by the load-flow calculations.*

Basis of Load Flow Analysis

The power delivered to bus k is separated into generated and load terms, i.e.

$$P_k = P_{Gk} - P_{Lk}$$
$$Q_k = Q_{Gk} - Q_{Lk}$$

Each bus k is categorized into one of the following three bus types:

Bus Type for Load Flow Studies

a- Load Bus:

Input Data: P_k & Q_k

Data to be Calculated: V_k & δ_k

Load buses are by far the most common, typically comprising more than 80% of all buses.

Bus Type for Load Flow Studies

a- Generator (PV) Bus:

Input Data: P_k & V_k

Data to be Calculated: Q_k & δ_k

This bus type represents the generating stations of the system .

Bus Type for Load Flow Studies

c- Slack Bus:

There is only one such bus in the system.
This have to be a generation bus.

- **Given for this bus: V_1 and $< \delta_1$**
- ***Unknowns are: P_k & Q_k***

The slack bus will provide the necessary power to maintain the power balance in the system.

Bus Type for Load Flow Studies

More on the Slack Bus:

In a network as power flow from the generators to loads through transmission lines power loss occurs due to the losses in the line conductors. These losses when included, we get the power balance relations

$$P_g - P_d - P_L = 0$$

$$Q_g - Q_d - Q_l = 0$$

where

Pg and Qg are the total real and reactive generations,

Pd and Qd are the total real and reactive power demands, and

Pl and Ol are the power losses in the transmission network

Bus Type for Load Flow Studies

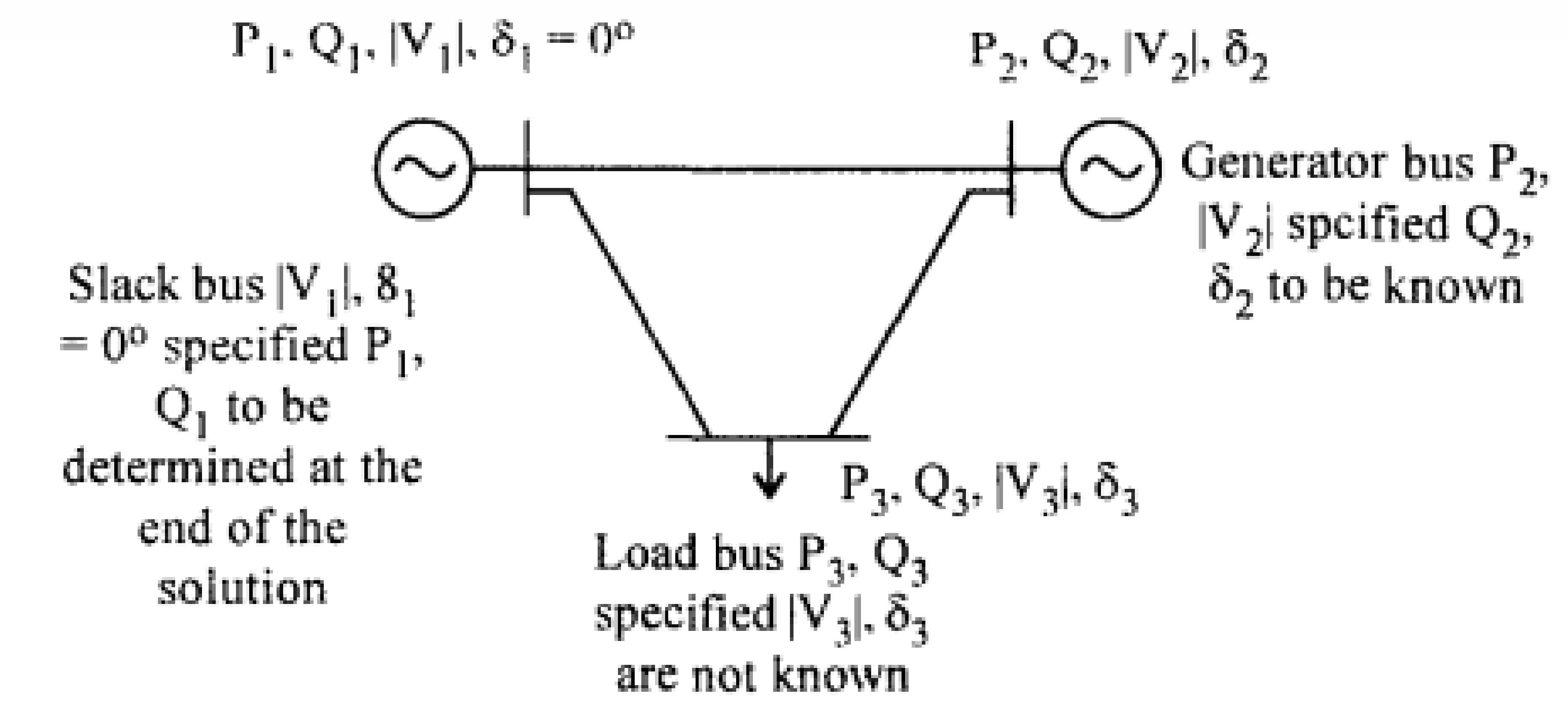
More on the Slack Bus:

The values of Pd and Qd are either known or estimated.

Since the flow of power in the various transmission lines are not known in advance, Pl and Ol remains unknown until the analysis of the network is completed. But, these losses have to be supplied by the generators in the system. For this purpose, one of the generators or generating buses is specified as 'slack bus' or 'swing bus'. At this bus the generation Pg and Qg are not specified, but the voltage magnitude is specified at this bus. Also, the voltage phase angle δ is specified as 0° and becomes the reference. For this reason slack bus is also known as reference bus. All the system losses are supplied by the generation at this bus.

Bus Type for Load Flow Studies

All Three Bus Types are Illustrated in this Figure:



Bus Type for Load Flow Studies

Bus classification is summarized in this table:

Bus	Specified variables	Computed variables
Slack – bus	Voltage magnitude and its phase angle	Real and reactive powers
Generator bus (PV – bus or voltage controlled bus)	Magnitudes of bus voltages and real powers (limit on reactive powers)	Voltage phase angle and reactive power.
Load bus	Real and reactive powers	Magnitude and phase angle of bus voltages

Basis of Load Flow Analysis

Bus Type for Load Flow Studies:

Bus Type	Knowns	Unknowns	Approximate Number
Slack	$V_i = 1.0$	P_{Gi}	1
	$\delta_i = 0$	Q_{Gi}	
Load	P_{Gi}	V_i	$\sim 85\%$
	Q_{Gi}	δ_i	
Generator	P_{Gi}	δ_i	$\sim 15\%$
	V_i	Q_{Gi}	

Solving the power flow problem

- ***The load flow equations are non-linear and can only be solved using numerical iterative techniques.***
- ***Two methods are used:***
  - ***Gauss-Seidel method***
  - ***Newton-Raphson method.***

All commercial power flow programs are based on one or both of these methods.

The Newton-Raphson Method of Analysis

The NR method is based on the Taylor Series, which states that: For a non-linear function, such as:

$$f(x) = 0$$

The expansion for this function is:

$$f(x) = f(x^m) + \frac{1}{1!}\frac{df}{dx}(x^m)(x - x^m) + \frac{1}{2!}\frac{d^2f}{dx^2}(x^m)(x - x^m)^2 + \cdots +$$

If we ignore all the terms after the first derivative as an approximation, we get,

$$f(x) \cong f(x^m) + \frac{df(x^m)}{dx}(x - x^m)$$

The Newton-Raphson Method of Analysis

Therefore:

$$x - x^m \cong 0 - \frac{f(x^m)}{\frac{df}{dx}(x^m)}$$

And

$$\Delta x = \frac{-f(x^m)}{\frac{df}{dx}(x^m)}$$

And use Δx to predict a better root estimate such that:

$$x^{m+1} = x^m + \Delta x$$

The Newton-Raphson Method of Analysis

The NR method requires selecting a starting value x^0, from which Δx is calculated using. $\Delta x > \epsilon$ is calculated. x^1 is calculated from equation and again, Δx is calculated. The process continues until one of three things happens:

a.	***If $\Delta = 0$***	***the solution converged***
b.	***$\Delta x \geq \Delta_{max}$,***	***the solution is said to diverge***
c.	***$m \geq m_{max}$***	***the number of iterations required has reached a maximum value***

The Newton-Raphson Method of Analysis

Example 9-1:

Given the following:

$$x = 2 - \sin x$$

Find a root for this equation by the Newton-Raphson method assuming the following:

$x^{\circ} = 0$, and $\epsilon = 0.00\ 1$

Solution:

$$f(x) = x - 2 + \sin x$$

$$\frac{df(x)}{dx} = 1 + \cos x$$

The Newton-Raphson Method of Analysis

We can use the above equation to Make the following table,

m	x^m	$f(x^m)$	$\frac{df}{dx}(x^m)$	Δx
0	0	-2	2	1
1	1	-0.1585	1.5403	0.1029
2	1.1029	-0.0046	1.4510	0.0031
3	1.1061	-0.0000044	1.4482	0.0000030

After three iterations we find that
$\Delta x < 0.001$

The Newton-Raphson Method of Analysis

2. Solution of Two Equations and Two Unknowns:

$$f(x, y) = 0$$

$$g(x, y) = 0$$

Expanding by Taylor's series,

$$f(x, y) = f(x^m, y^m) + \frac{\partial f}{\partial x}(x^m, y^m)(x - x^m) \; + \frac{1}{1!}\frac{\partial f}{\partial y}(x^m, y^m)(y - y^m) + \cdots$$

$$g(x, y) = g(x^m, y^m) + \frac{1}{1!}\frac{\partial g}{\partial x}(x^m, y^m)(x - x^m) \; + \frac{1}{1!}\frac{\partial g}{\partial y}(x^m, y^m)(y - y^m) + \cdots$$

The Newton-Raphson Method of Analysis

2. Solution of Two Equations and Two Unknowns:

$$\frac{\partial f^m}{\partial x}\Delta x + \frac{\partial f^m}{\partial y}\Delta y = -f^m$$

$$\frac{\partial g^m}{\partial x}\Delta x + \frac{\partial g^m}{\partial y}\Delta y = -g^m$$

Putting these equations in matrix form we get,

$$\begin{bmatrix} \frac{\partial f^m}{\partial x} & \frac{\partial f^m}{\partial y} \\ \frac{\partial g^m}{\partial x} & \frac{\partial g^m}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = -\begin{bmatrix} f^m \\ g^m \end{bmatrix}$$

The Newton-Raphson Method of Analysis

2. Solution of Two Equations and Two Unknowns:

The coefficient matrix is called the Jacobian and symbolized as $[J^m]$, i.e.,

$$[\mathbf{J}^m] = \text{system Jacobian matrix} = \begin{bmatrix} \dfrac{\partial f^m}{\partial x} & \dfrac{\partial f^m}{\partial y} \\[2ex] \dfrac{\partial g^m}{\partial x} & \dfrac{\partial g^m}{\partial y} \end{bmatrix}$$

Therefore,

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = -[\mathbf{J}^m]^{-1} \begin{bmatrix} f^m \\ g^m \end{bmatrix}$$

The Newton-Raphson Method of Analysis

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The Newton-Raphson Method of Analysis

2. Solution of Two Equations and Two Unknowns:

Example:

Solve the following equations for x and y by the Newton-Raphson Method for ϵ_x and ϵ_y = 0.00 1 with starting values $x^0 = 0$ and $y^0 = 3$.

$$2x + y = 4$$

$$2x + y^2 = 6$$

Define,

$$f(x, y) = 2x + y - 4 = 0$$

$$g(x, y) = 2x + y^2 - 6 = 0$$

Solution:

Let's define

The Newton-Raphson Method of Analysis

2. Solution of Two Equations and Two Unknowns:

The Jacobian matrix is

$$[\mathbf{J}] = \begin{bmatrix} 2 & 1 \\ 2 & 2y \end{bmatrix}$$

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = -[\mathbf{J}^m]^{-1} \begin{bmatrix} f^m \\ g^m \end{bmatrix}$$

$$x^{m+1} = x^m + \Delta x$$

$$y^{m+1} = y^m + \Delta y$$

The Newton-Raphson Method of Analysis

2. Solution of Two Equations and Two Unknowns:

Organizing our work in the following table,

m	x^m y^m	$-[J]^{-1}$		f^m g^m	Δx Δy
0	0 3	−0.6 0.2	0.1 −0.2	−1 3	0.9 −0.8
1	0.9 2.2	−0.647 0.294	0.147 −0.294	0 0.64	0.094 −0.188
2	0.994 2.012	−0.665 0.331	0.165 −0.331	0 0.0354	0.00586 −0.0117
3	0.999977 2.000046	−0.667 0.333	0.167 −0.333	0 0.000137	0.000023 −0.000046

The Newton-Raphson Method of Analysis

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3	0.999977 2.000046	−0.667 0.333	0.167 −0.333	0 0.000137	0.000023 −0.000046

As we see, after three iterations,

$$|\Delta x| < 0.001$$

$$|\Delta y| < 0.001$$

The Newton-Raphson Method of Analysis

2. Solution of Two Equations and Two Unknowns:

Organizing our work in the following table,

m	x^m y^m	$-[J]^{-1}$		f^m g^m	Δx Δy
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3	0.999977 2.000046	-0.667 0.333	0.167 -0.333	0 0.000137	0.000023 -0.000046

As we see, after three iterations,

$$|\Delta x| < 0.001$$

$$|\Delta y| < 0.001$$

Thus, x = 1.000 and y = 2.000.

The Newton-Raphson Method of Analysis

3. Solution of 2n Equations and 2n Unknowns:

$$
\begin{aligned}
f_1(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) &= 0 \\
f_2(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) &= 0 \\
\vdots \qquad\qquad\qquad\qquad & \quad \vdots \\
f_n(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) &= 0 \\
g_1(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) &= 0 \\
g_2(x_i, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) &= 0 \\
\vdots \qquad\qquad\qquad\qquad & \quad \vdots \\
g_n(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) &= 0
\end{aligned}
$$

Where

$$
\begin{aligned}
f_i^m &= f_i(\tilde{\mathbf{x}}^m, \tilde{\mathbf{y}}^m) = 0 \\
g_i^m &= g_i(\tilde{\mathbf{x}}^m, \tilde{\mathbf{y}}^m) = 0
\end{aligned}
$$

The Newton-Raphson Method of Analysis

3. Solution of 2n Equations and 2n Unknowns:

Also define the general-system Jacobian matrix to be,

$$[\mathbf{J}^m] = \left[\begin{array}{c:c} \left[\dfrac{\partial f^m}{\partial x}\right] & \left[\dfrac{\partial f^m}{\partial y}\right] \\ \hdashline \left[\dfrac{\partial g^m}{\partial x}\right] & \left[\dfrac{\partial g^m}{\partial x}\right] \end{array}\right] = \left[\begin{array}{c:c} \mathrm{I} & \mathrm{II} \\ \hdashline \mathrm{III} & \mathrm{IV} \end{array}\right] \cdot (2n \times 2n)$$

The four sub-Jacobians are each (n x n) and defined as follows;

The Newton-Raphson Method of Analysis

3. Solution of 2n Equations and 2n Unknowns:

I $\left[\dfrac{\partial f^m}{\partial x}\right]$ with general i, k entry $\dfrac{\partial f_i}{\partial x_k}(\tilde{x}^m, \tilde{y}^m)$

II $\left[\dfrac{\partial f^m}{\partial x}\right]$ with general i, k entry $\dfrac{\partial f_i}{\partial y_k}(\tilde{x}^m, \tilde{y}^m)$

III $\left[\dfrac{\partial g^m}{\partial x}\right]$ with general i, k entry $\dfrac{\partial g_i}{\partial x_k}(\tilde{x}^m, \tilde{y}^m)$

IV $\left[\dfrac{\partial g^m}{\partial y}\right]$ with general i, k entry $\dfrac{\partial g_i}{\partial y_k}(\tilde{x}^m, \tilde{y}^m)$

The Newton-Raphson Method of Analysis

Each entry in the above matrices is a function of all 2n variables, all of which will be evaluated in the mth iteration. We further define,

$$\widetilde{\Delta \mathbf{x}} = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} \quad \text{and} \quad \widetilde{\Delta \mathbf{y}} = \begin{bmatrix} \Delta y_1 \\ \Delta y_2 \\ \vdots \\ \Delta y_n \end{bmatrix}$$

Then,

$$\begin{bmatrix} \widetilde{\Delta \mathbf{x}} \\ \widetilde{\Delta \mathbf{y}} \end{bmatrix} = -\left[\mathbf{J}^m\right]^{-1} \begin{bmatrix} \tilde{\mathbf{f}}^m \\ \tilde{\mathbf{g}}^m \end{bmatrix}$$

All 2n variables are to be upgraded according to,

$$\tilde{\mathbf{x}}^{m+1} = \tilde{\mathbf{x}}^m + \widetilde{\Delta \mathbf{x}}$$

$$\tilde{\mathbf{y}}^{m+1} = \tilde{\mathbf{y}}^m + \widetilde{\Delta \mathbf{y}}$$

Convergence is achieved when,

$$\Delta x_i \leq \varepsilon_x \quad \text{and} \quad \Delta y_i \leq \varepsilon_y$$

The Newton-Raphson Method of Analysis

4. Application of the NR method to the Load Flow equations:

The power equations developed earlier are,

$$P_i = |V_i| \sum_{j=1}^{n} |V_j||Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_i = -|V_i| \sum_{j=1}^{n} |V_j||Y_{ij}| \sin(\theta_{ij} + \delta_k - \delta_i)$$

We will make the following associations with the general Newton-Raphson variables,

$$f_i \to P_i \qquad y_i \to V_i$$

$$g_i \to Q_i \qquad \Delta x_i \to \Delta\delta_i$$

$$x_i \to \delta_i \qquad \Delta y_i \to \Delta V_i$$

The Newton-Raphson Method of Analysis

It follows that the general-system Jacobian for the load flow problem appears as follows:

$$[\mathbf{J}] = \left[\begin{array}{c:c} \left[\dfrac{\partial P}{\partial \delta}\right] & \left[\dfrac{\partial P}{\partial V}\right] \\ \hdashline \left[\dfrac{\partial Q}{\partial \delta}\right] & \left[\dfrac{\partial Q}{\partial V}\right] \end{array}\right] = \left[\begin{array}{c:c} \mathrm{I} & \mathrm{II} \\ \hdashline \mathrm{III} & \mathrm{IV} \end{array}\right]$$

Our first goal is to derive general equations for computing entries in the four quadrants of [J]

The Newton-Raphson Method of Analysis

1. Quadrant I - ∂P/∂δ:

Since P_{Gi} and P_{Li}, do not vary with δ_k or V_k.

$$P_i = |V_i| \sum_{j=1}^{n} |V_j||Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$\frac{\partial P_i}{\partial \delta_i} = -\sum_{\substack{j=1 \\ j \neq i}}^{n} V_i V_j y_{ij} \sin(\delta_i - \delta_j - \gamma_{ij})$$

$$\frac{\partial P_i}{\partial \delta_k} = V_i V_k y_{ik} \sin(\delta_i - \delta_k - \gamma_{ik}) \qquad i \neq k$$

The Newton-Raphson Method of Analysis

4. Application of the NR method to the Load Flow equations:

1. Quadrant I - ∂P/∂δ:

$$\frac{\partial P_i}{\partial \delta_k} = \frac{\partial}{\partial \delta_k}[-P_{Gi} + P_{Li} + P_{Ti}] = -0 + 0 + \frac{\partial P_{Ti}}{\partial \delta_k}$$

Since P_{Gi} and P_{Li}, do not vary with δ_k or V_k.

$$\frac{\partial P_i}{\partial \delta_i} = -\sum_{\substack{j=1 \\ j \neq i}}^{n} V_i V_j y_{ij} \sin(\delta_i - \delta_j - \gamma_{ij})$$

$$\frac{\partial P_i}{\partial \delta_k} = V_i V_k y_{ik} \sin(\delta_i - \delta_k - \gamma_{ik}) \qquad i \neq k$$

The Newton-Raphson Method of Analysis

4. Application of the NR method to the Load Flow equations:

1. Quadrant I - ∂P/∂δ:

$$\frac{\partial P_i}{\partial V_i} = V_i y_{ii} \cos \gamma_{ii} + \sum_{j=1}^{n} V_j y_{ij} \cos(\delta_i - \delta_j - \gamma_{ij})$$

$$\frac{\partial P_i}{\partial V_k} = V_i y_{ik} \cos(\delta_i - \delta_k - \gamma_{ik}) \qquad i \neq k$$