Goal 1-Review of Basic Concepts of Power System Calculations

Basic Concepts of Power System Calculations

For circuits operating in sinusoidal-steady-state, Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL) apply to phasor currents and voltages. Thus, the sum of all phasor currents entering any node is zero and the sum of the phasor-voltage drops around any closed path is zero.

Network analysis techniques based on Kirchhoff's laws, including nodal analysis, mesh or loop analysis, superposition, source transformations, and Thévenin's theorem or Norton's theorem, are useful for analyzing such circuits.

Part 1
Electric Network Analysis
(Summary)

MESH ANALYSIS OF A DC NETWORK:
Calculate the current through each of the resistors in the dc circuit of Fig. 1.1 using mesh analysis.

Calculation Procedure
1. Assign Mesh or Loop Currents:
The term *mesh* is used because of the similarity in appearance between the closed loops of the network and a wire mesh fence. One can view the circuit as a "window frame" and the meshes as the "windows." A mesh is a closed pathway with no other closed pathway within it. A loop is also a closed pathway, but a loop may have other closed pathways within it.
Therefore, all meshes are loops, but all loops are not meshes. For example, the loop made by the closed path *BCDAB* (Fig. 1.2) is not a mesh because it contains two closed paths: *BCAB* and *CDAC*.

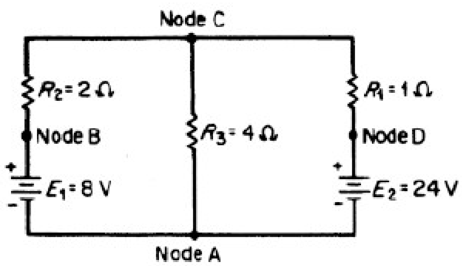


Figure 1.1

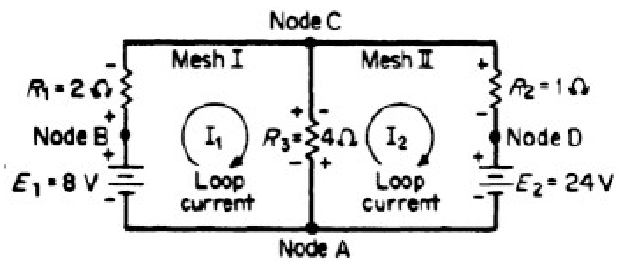


Figure 1.2

Loop currents I1 and I2 are drawn in the clockwise direction in each window (Fig. 1.2). The loop current or mesh current is a fictitious current that enables us to obtain the actual branch currents more easily. The number of loop currents required is always equal to the number of windows of the network. This assures that the resulting equations are all independent. Loop currents may be drawn in any direction, but assigning a clockwise direction to all of them simplifies the process of writing equations.

2. Indicate the Polarities within Each Loop:
Identify polarities to agree with the assumed direction of the loop currents and the passive sign convention. The polarities across *R3* are the opposite for each loop current. The polarities of *E*1 and E2 are unaffected by the direction of the loop currents passing through them.

3. Write KVL around Each Mesh:
Write KVL around each mesh in any direction. It is convenient to follow the same direction as the loop current: mesh I: +8-2I1-4(I1-I2)=0; mesh II: -24-4(I2-I1)-I2=0.

4. Solve the Equations:
Solving the two simultaneous equations gives the following results: I1=-4 A and I2=-8 A. The minus signs indicate that the two loop currents flow in a direction opposite to that assumed; that is, they both flow counterclockwise. Loop I1 is therefore 4 A in the direction of *CBAC.* Loop current I2 is 8 A in the direction *ADCA*. The true direction of loop current I2 through resistor R3 is from C to A. The true direction of loop current I1 through resistor R3 is through resistor R3 is from A to C. Therefore, the current through R3 equals (I2-I1) or 8-4=4 A in the direction of CA.

NODAL ANALYSIS OF A DC NETWORK:
Calculate the current through each of the resistors in the dc circuit of Fig. 1.3 using nodal analysis.

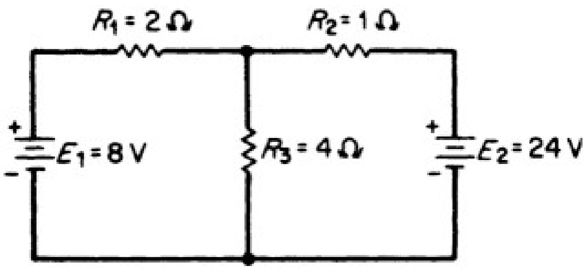


Figure 1.3

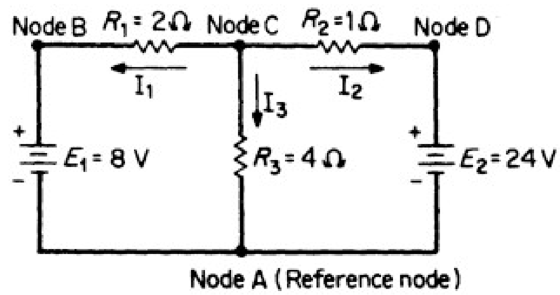


Figure 1.4

Calculation Procedure
1. Label the Circuit:
Label all nodes, as shown in Fig. 1.4. One of the nodes (node *A)* is chosen as the reference node. It can be thought of as a circuit ground, which is at zero voltage or ground potential. Nodes *B* and *D* are already known to be at the potential of the source voltages. The voltage at node *C (VC)* is unknown. Assume that VC>VB and VC>VD. Draw all three currents I1, I2, and I3 away from node C, that is, toward the reference node.

2. Write KCL at Node C:
I1+I2+I3=0.

3. Express Currents in Terms of Circuit Voltages Using Ohm's Law:
Refer to Fig. 1.4: I1=V1/R1=(VC-8)/2, I2=V2/R2=(VC-24)/1, and I3= V3/R3=VC/4.

4. Substitute in KCL Equation of Step 2:
Substituting the current equations obtained in Step 3 into KCL of Step 2, we find I1+ I2+I3=0 or (VC-8)/2+(VC-24)/1+VC/4=0. Because the only unknown is *V*C, this simple equation can be solved to obtain *V*C=16 V.

5. Solve for All Currents

I1=(VC-8)/2=(16-8)/2= 4 A (true direction) and I2=(VC-24)/1=(16- 24)/1= -8 A. The negative sign indicates that *I*2 flows toward node *C* instead of in the assumed direction (away from node *C*). *I*3=*VC*/4=16/4= 4 A (true direction).

Part 2

AC Network Analysis

Network Analysis:

Various computer solutions of power system problems are formulated from nodal equations, which can be systematically applied to circuits. The circuit shown in Figure 2.1, which is used here to review nodal analysis, is assumed to be operating in sinusoidal-steady-state; source voltages are represented by phasors E_{s1} ,E_{s2}, and E_{s3}; circuit impedances are specified in ohms.

Nodal equations are written in the following three steps:

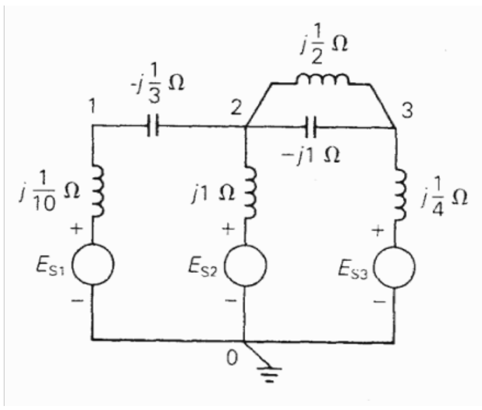


Figure 2-1

STEP 1:

For a circuit with (N + 1) nodes (also called buses), select one bus as the reference bus and define the voltages at the remaining buses with respect to the reference bus. The circuit in Figure 4.1 has four buses - that is, N + 1 = 4 or N = 3. Bus 0 is selected as the reference bus, and bus voltages V10, V20, and V30 are then defined with respect to bus 0**.**

STEP 2:

Transform each voltage source in series with impedance to an equivalent current source in parallel with that impedance. Also, show admittance values instead of impedance values on the circuit diagram. Each current source is equal to the voltage source divided by the source impedance. In Figure 2.2 equivalent current sources I1 , I2, and I3 are shown, and all impedances are converted to corresponding admittances.

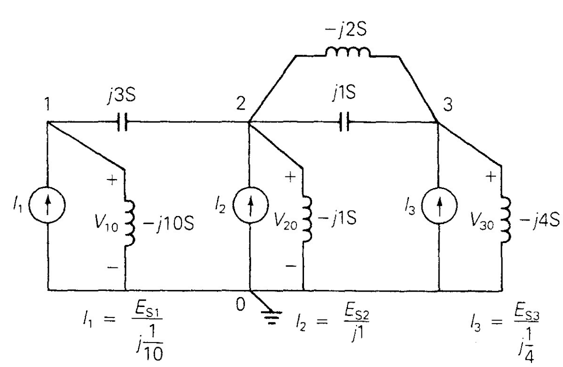


Figure 2-2

STEP 3:

Write nodal equations in matrix format as follows:

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & Y_{23} & \cdots & Y_{2N} \\ Y_{31} & Y_{32} & Y_{33} & \cdots & Y_{3N} \\ \vdots & \vdots & \vdots & & \vdots \\ Y_{N1} & Y_{N2} & Y_{N3} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \\ \vdots \\ V_{N0} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_N \end{bmatrix} \qquad \text{Equation 1}$$

Using matrix notation, equation1 becomes,

$$\boxed{YV = I} \qquad \text{Equation 2}$$

Where Y is the (N x N) bus admittance matrix; V is the column vector of N bus voltages; and I is the column vector of N current sources. The elements Ykn of the bus admittance matrix Y are formed as follows:

Diagonal elements: Y_{kk} = sum of admittances connected to bus k (k= 1,2,... ,N)

Equation 3

Off-diagonal elements: Ykn = — (sum of admittances connected between buses k and n) (k ≠ n)

Equation 4

The diagonal element Ykk is called self-admittance, or the driving-point admittance of bus k, and the off-diagonal element Ykn for k ≠ n is called the mutual admittance or the transfer admittance between the buses k and n. Since Ykn = Ynk, the matrix Y is symmetric.

For the circuit shown in Figure 2-2, equation 2 becomes:

$$\begin{bmatrix} (j3 - j10) & -(j3) & 0 \\ -(j3) & (j3 - j1 + j1 - j2) & -(j1 - j2) \\ 0 & -(j1 - j2) & (j1 - j2 - j4) \end{bmatrix} \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$j \begin{bmatrix} -7 & -3 & 0 \\ -3 & 1 & 1 \\ 0 & 1 & -5 \end{bmatrix} \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

The advantage of this method of writing nodal equations is that a digital computer can be used both to generate the admittance matrix Y and to solve Equation 2 for the unknown bus voltage vector V.
Once a circuit is specified with the reference bus and other buses identified, the circuit admittances and their bus connections become computer input data for calculating the elements Ykn via Equations 3 & 4.

After Y is calculated and the current source vector I is given as input, standard computer programs for solving simultaneous linear equations can then be used to determine the bus voltage vector V.

When double subscripts are used to denote a voltage in this course, the voltage shall be that at the node identified by the first subscript with respect to the node identified by the second subscript. For example, the voltage V10 in Figure 2 is the voltage at node 1 with respect to node 0. Also, a current Iab shall indicate the current from node a to node b. Voltage polarity marks (+/-) and current reference arrows (→) are not required when double subscript notation is employed. The polarity marks in Figure 2 for V10, V20, and V30, although not required, are shown for clarity. The reference arrows for sources I1, I2, and I3 in Figure 2 are required, however, since single subscripts are used for these currents. Matrices and vectors shall be indicated in this text by boldface type (for example, Y or V).

Part 3

Complex Power Calculations

Phasors:

A sinusoidal voltage or current at constant frequency is characterized by two parameters: A maximum value and a phase angle. A voltage v(t) that is described as follows:

$$v(t) = V_{max} \cos(\omega t + \delta) \tag{2.1.1}$$

has a maximum value V_{max} and a phase angle δ when referenced to $\cos(\omega t)$. The root-mean-square (rms) value, also called *effective value*, of the sinusoidal voltage is

$$V = \frac{V_{max}}{\sqrt{2}} \tag{2.1.2}$$

Euler's identity, $e^{j\phi} = \cos\phi + j\sin\phi$, can be used to express a sinusoid in terms of a phasor. For the above voltage,

$$\begin{aligned} v(t) &= \mathrm{Re}[V_{max} e^{j(\omega t + \delta)}] \\ &= \mathrm{Re}[\sqrt{2}(Ve^{j\delta})e^{j\omega t}] \end{aligned} \tag{2.1.3}$$

where $j = \sqrt{-1}$ and Re denotes "real part of." The rms phasor representation of the voltage is given in three forms—exponential, polar, and rectangular:

$$V = \underbrace{Ve^{j\delta}}_{\text{exponential}} = \underbrace{V\underline{/\delta}}_{\text{polar}} = \underbrace{V\cos\delta + jV\sin\delta}_{\text{rectangular}} \tag{2.1.4}$$

A phasor can be easily converted from one form to another. Conversion from polar to rectangular is shown in the phasor diagram of Figur 3-1 Euler's identity can be used to convert from exponential to rectangular form. As an example, the voltage

$$v(t) = 169.7\cos(\omega t + 60^\circ) \quad \text{volts} \tag{2.1.5}$$

has a maximum value $V_{max} = 169.7$ volts, a phase angle $\delta = 60^\circ$ when referenced to $\cos(\omega t)$, and an rms phasor representation in polar form of

$$V = 120\underline{/60^\circ} \quad \text{volts} \tag{2.1.6}$$

Also, the current

$$i(t) = 100\cos(\omega t + 45^\circ) \quad \text{A} \tag{2.1.7}$$

has a maximum value $I_{max} = 100$ A, an rms value $I = 100/\sqrt{2} = 70.7$ A, a phase angle of 45°, and a phasor representation

$$I = 70.7\underline{/45^\circ} = 70.7e^{j45} = 50 + j50 \quad \text{A} \tag{2.1.8}$$

The relationships between the voltage and current phasors for the three passive elements—resistor, inductor, and capacitor—are summarized in Fig-

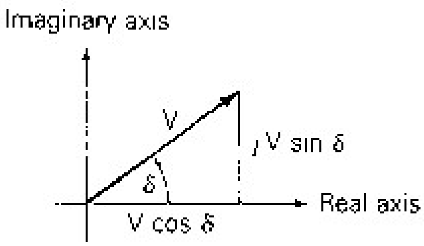


Figure 3-1

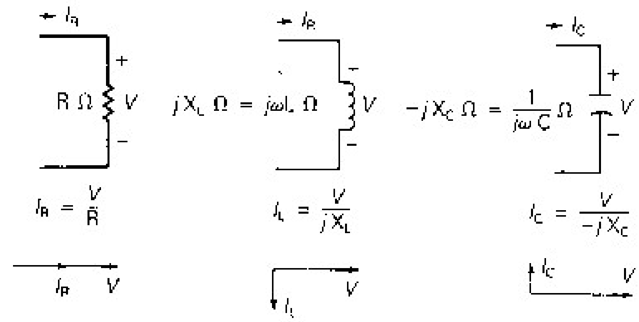


Figure 3-2

3-2 , where sinusoidal-steady-state excitation and constant values of *R*, *L*, and *C* are assumed.

When voltages and currents are discussed in this text, lowercase letters such as $v(t)$ and $i(t)$ indicate instantaneous values, uppercase letters such as V and I indicate rms values, and uppercase letters in italics such as *V* and *I* indicate rms phasors. When voltage or current values are specified, they shall be rms values unless otherwise indicated.

INSTANTANEOUS POWER IN SINGLE-PHASE AC CIRCUITS

Power is the rate of change of energy with respect to time. The unit of power is a watt, which is a joule per second. Instead of saying that a load absorbs energy at a rate given by the power, it is common practice to say that a load absorbs power. The instantaneous power in watts absorbed by an electrical load is the product of the instantaneous voltage across the load in volts and the instantaneous current into the load in amperes. Assume that the load voltage is

$$v(t) = V_{max} \cos(\omega t + \delta) \quad \text{volts} \tag{2.2.1}$$

We now investigate the instantaneous power absorbed by purely resistive, purely inductive, purely capacitive, and general RLC loads. We also introduce the concepts of real power, power factor, and reactive power. The physical significance of real and reactive power is also discussed.

PURELY RESISTIVE LOAD

For a purely resistive load, the current into the load is in phase with the load voltage, $I = V/R$, and the current into the resistive load is

$$i_R(t) = I_{Rmax}\cos(\omega t + \delta) \quad A \tag{2.2.2}$$

where $I_{Rmax} = V_{max}/R$. The instantaneous power absorbed by the resistor is

$$\begin{aligned} p_R(t) &= v(t)i_R(t) = V_{max}I_{Rmax}\cos^2(\omega t + \delta) \\ &= \tfrac{1}{2}V_{max}I_{Rmax}\{1 + \cos[2(\omega t + \delta)]\} \\ &= VI_R\{1 + \cos[2(\omega t + \delta)]\} \quad W \end{aligned} \tag{2.2.3}$$

As indicated by (2.2.3), the instantaneous power absorbed by the resistor has an average value

$$P_R = VI_R = \frac{V^2}{R} = I_R^2 R \quad W \tag{2.2.4}$$

plus a double-frequency term $VI_R \cos[2(\omega t + \delta)]$.

PURELY INDUCTIVE LOAD

For a purely inductive load, the current lags the voltage by 90°, $I_L = V/(jX_L)$, and

$$i_L(t) = I_{Lmax}\cos(\omega t + \delta - 90°) \quad A \tag{2.2.5}$$

where $I_{Lmax} = V_{max}/X_L$, and $X_L = \omega L$ is the inductive reactance. The instantaneous power absorbed by the inductor is*

$$\begin{aligned} p_L(t) &= v(t)i_L(t) = V_{max}I_{Lmax}\cos(\omega t + \delta)\cos(\omega t + \delta - 90°) \\ &= \tfrac{1}{2}V_{max}I_{Lmax}\cos[2(\omega t + \delta) - 90°] \\ &= VI_L \sin[2(\omega t + \delta)] \quad W \end{aligned} \tag{2.2.6}$$

As indicated by (2.2.6), the instantaneous power absorbed by the inductor is a double-frequency sinusoid with *zero* average value.

PURELY CAPACITIVE LOAD

For a purely capacitive load, the current leads the voltage by 90°, $I_C = V/(-jX_C)$, and

$$i_C(t) = I_{Cmax}\cos(\omega t + \delta + 90°) \quad A \tag{2.2.7}$$

where $I_{Cmax} = V_{max}/X_C$ and $X_C = 1/(\omega C)$ is the capacitive reactance. The instantaneous power absorbed by the capacitor is

$$
\begin{aligned}
p_C(t) = v(t)i_C(t) &= V_{max} I_{Cmax} \cos(\omega t + \delta)\cos(\omega t + \delta + 90^\circ) \\
&= \tfrac{1}{2} V_{max} I_{Cmax} \cos[2(\omega t + \delta) + 90^\circ)] \\
&= -VI_C \sin[2(\omega t + \delta)] \quad \text{W} \qquad (2.2.8)
\end{aligned}
$$

The instantaneous power absorbed by a capacitor is also a double-frequency sinusoid with *zero* average value

GENERAL RLC LOAD

For a general load composed of RLC elements under sinusoidal-steady-state excitation, the load current is of the form

$$
i(t) = I_{max} \cos(\omega t + \beta) \quad \text{A} \qquad (2.2.9)
$$

The instantaneous power absorbed by the load is then*

$$
\begin{aligned}
p(t) = v(t)i(t) &= V_{max} I_{max} \cos(\omega t + \delta)\cos(\omega t + \beta) \\
&= \tfrac{1}{2} V_{max} I_{max} \{\cos(\delta - \beta) + \cos[2(\omega t + \delta) - (\delta - \beta)]\} \\
&= VI\cos(\delta - \beta) + VI\cos(\delta - \beta)\cos[2(\omega t + \delta)] \\
&\quad + VI\sin(\delta - \beta)\sin[2(\omega t + \delta)]
\end{aligned}
$$

$$
p(t) = VI\cos(\delta - \beta)\{1 + \cos[2(\omega t + \delta)]\} + VI\sin(\delta - \beta)\sin[2(\omega t + \delta)]
$$

Letting $I\cos(\delta - \beta) = I_R$ and $I\sin(\delta - \beta) = I_X$ gives

$$
p(t) = \underbrace{VI_R\{1 + \cos[2(\omega t + \delta)]\}}_{p_R(t)} + \underbrace{VI_X \sin[2(\omega t + \delta)]}_{p_X(t)} \qquad (2.2.10)
$$

As indicated by (2.2.10), the instantaneous power absorbed by the load has two components: One can be associated with the power $p_R(t)$ absorbed by the resistive component of the load and the other can be associated with the power $p_X(t)$ absorbed by the reactive (inductive or capacitive) component of the load. The first component $p_R(t)$ in (2.2.10) is identical to (2.2.3), where $I_R = I\cos(\delta - \beta)$ is the component of the load current in phase with the load voltage. The phase angle $(\delta - \beta)$ represents the angle between the voltage and current. The second component $p_X(t)$ in (2.2.10) is identical to (2.2.6) or (2.2.8), where $I_X = I\sin(\delta - \beta)$ is the component of load current 90° out of phase with the voltage.

REAL POWER

Equation (2.2.10) shows that the instantaneous power $p_R(t)$ absorbed by the resistive component of the load is a double-frequency sinusoid with average value P given by

$$P = VI_R = VI\cos(\delta - \beta) \quad W \tag{2.2.11}$$

The *average power* P is also called *real power* or *active power*. All three terms indicate the same quantity P given by (2.2.11).

REACTIVE POWER

The instantaneous power absorbed by the reactive part of the load, given by the component $p_X(t)$ in (2.2.10), is a double-frequency sinusoid with zero average value and with amplitude Q given by

$$Q = VI_X = VI\sin(\delta - \beta) \quad \text{var} \tag{2.2.12}$$

The term Q is given the name *reactive power*. Although it has the same units as real power, the usual practice is to define units of reactive power as volt-amperes reactive, or var.

COMPLEX POWER:

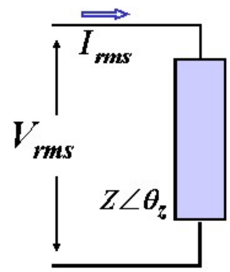
Complex power is an important element in power analysis because it contains all the information related to the power used by a load. *Complex power S* (in *VA*) is the product of the rms voltage phasor and the complex conjugate of the rms current phasor.

Consider the circuit shown in the following figure; the complex power is defined to be:

$$S = V_{rms} I_{rms}^{*}$$

$$S = V_{rms}\angle\theta_v{}^\circ \times \left[I_{rms}\angle\theta_i{}^\circ\right]^{*}$$

$$S = V_{rms} I_{rms} \angle \theta_v - \theta_i$$



$$S = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{Active\ Power} + j\underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{Reactive\ Power}$$

$$Active\ \ Power\ \ (P) = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \quad (\ Watt\ )$$

$$Reactive\ \ Power\ \ (Q) = V_{rms} I_{rms} \sin(\theta_v - \theta_i) \quad (\ VAR\ )$$

Or, the complex power is:

$$S = P + jQ \quad (\ VA\ )$$

Using the load impedance,

$$Z = R + jX$$

$$S = V_{rms} I_{rms}^{*} \quad (\ VA\ )$$

$$V_{rms} = Z I_{rms}$$

$$S = (Z I_{rms}) I_{rms}^{*}$$

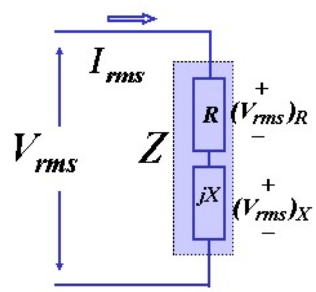
$$S = Z\,|I_{rms}|^2$$

$$S = (R + jX)\,|I_{rms}|^2$$

$$S = P + jQ$$

$$P = R\,|I_{rms}|^2 \quad (\ Watt\ )$$

$$Q = X\,|I_{rms}|^2 \quad (\ VAR\ )$$



$$|I_{rms}| = \frac{|(V_{rms})_R|}{R}$$

$$|I_{rms}| = \frac{|(V_{rms})_X|}{X}$$

$$P = \frac{|(V_{rms})_R|^2}{R} \quad (\ Watt\ )$$

$$Q = \frac{|(V_{rms})_X|^2}{X} \quad (\ VAR\ )$$

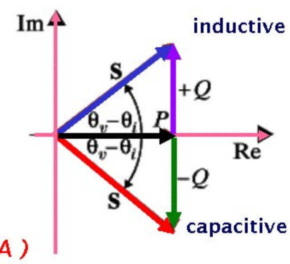
The Complex Power relations could be summarized as:

$$S = V_{rms} I_{rms}^{*} \quad (VA)$$

$$S = |V_{rms}||I_{rms}| \angle(\theta_v - \theta_i) \quad (VA)$$

$$S = |S| \angle(\theta_v - \theta_i) \quad (VA)$$

$$|S| = |V_{rms}||I_{rms}| = Apparent\ \ Power \quad (VA)$$



$$S = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{Active\ \ Power} \pm j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{Reactive\ \ Power} \quad (VA)$$

$$S = P \pm jQ \quad (VA)$$

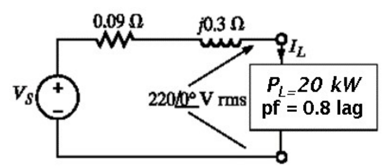
$$P = Re\{S\} = |S| \cos(\theta_v - \theta_i) = R|I_{rms}|^2 \quad (Watt)$$

$$Q = Im\{S\} = |S| \sin(\theta_v - \theta_i) = X|I_{rms}|^2 \quad (VAR)$$

$$pf = \cos(\theta_v - \theta_i) = \cos(\theta_Z)$$

EXAMPLE:

For the network shown, determine the load complex power, voltage and power factor at the source side.



SOLUTION:

$$P = Re\{S\} = |S| \cos(\theta_v - \theta_i) = |S| \times pf$$

$$\therefore \ |S_L| = \frac{P_L}{pf} = \frac{20}{0.8} = 25\ kVA$$

$$pf = \cos(\theta_v - \theta_i) = \cos(\theta_Z) = 0.8$$

$$\theta_Z = 36.87$$

$$Q_L = |S| \sin(\theta_v - \theta_i)$$

$$Q_L = |25| \sin(36.87) = 15kVAR$$

NOTE

p.f = *0.8 lag* and $\theta_v = 0^o$

$\cos(\theta_v - \theta_i) = \cos\theta_z$

$\theta_z = \cos^{-1}(pf) = 36.87^o$

$\theta_v - \theta_i = \theta_z \Rightarrow \theta_v \rangle \theta_i$

$0 - \theta_i = 36.87^o$

$\theta_i = 0 - 36.87 = -36.87^o$

$$S_L = 20 + j15 = 25\angle 36.87^{\circ}$$

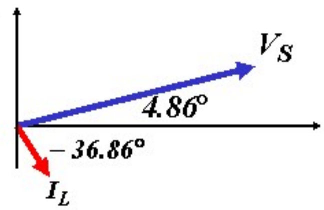
Inductive Load

The complex power is: $S_L = V_L I_L^*$

$$I_L = \left[\frac{S_L}{V_L}\right]^* = \left[\frac{25{,}000\angle 36.87^\circ}{220\angle 0^\circ}\right]^* = 113.64\angle -36.86^\circ (A)$$

$$V_S = (0.09 + j0.3) I_L + 220\angle 0^\circ$$

$$V_S = (0.09 + j0.3)(90.91 - j68.18) + 220 (V)$$



$$V_S = 248.63 + j21.14 = 249.53\angle 4.86^\circ$$

$$pf_{source} = \cos(\theta_v^{source} - \theta_i^{source})$$

$$pf_{source} = \cos(4.86 - (-36.86))$$

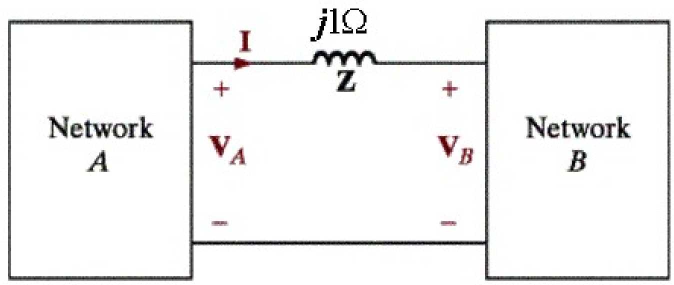
$$pf_{source} = \cos(41.72^\circ) = 0.746$$ **Lag.**

NOTE:

$pf_{source} = 0.746 \ lag$ **And** $pf_{load} = 0.8 \ lag$

EXAMPLE

Compute the average power flow between networks. Determine which network is the source



SOLUTION:

$V_A = 120\angle 30°(V)rms$ $\quad$ $V_B = 120\angle 0°(V)rms$

$$I = \frac{V_A - V_B}{Z} = \frac{120\angle 30° - 120\angle 0°}{j1} = 60 + j16.08 = 62.12\angle 15°(A)rms$$

Using the Passive Sign Convention, the Power received by A is:

$$S_A = V_A(-I)^* = 120\angle 30° \times 62.12\angle -195° = 7{,}454\angle -165° \ \ VA$$

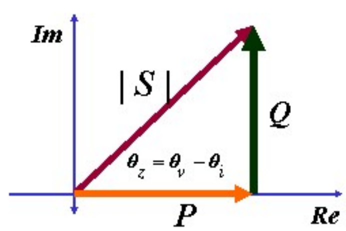
$$P_A = 7{,}454\cos(165°) = -7{,}200\,(W)$$ ***Supplying Power***

$$S_B = V_B(I)^* = 120\angle 0° \times 62.12\angle -15° = 7{,}454\angle -15° \ \ VA$$

$$P_B = 7{,}454\cos(-15°) = 7{,}200(W)$$ ***Absorbing Power***

Network A supplies 7.2kW average power to Network B

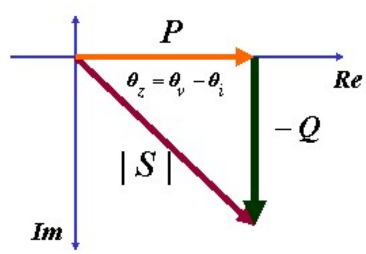
The Power Triangle:



It is a standard practice to represent *S*, *P*, and *Q* in the form of a triangle, known as the *power triangle*.

The *power triangle* has four items, the apparent power, real power, reactive power, and the power factor angle. Given two of these items, the other two can easily be obtained.

For the *power triangle*,



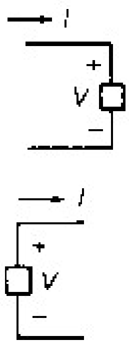
When |*S*| lies in the first quadrant, we have an *inductive load* and a *lagging power factor*.

When |S| lies in the fourth quadrant, we have a *capacitive load* and *leading power factor*.

The *Reactive Power* Q is transferred back and forth between the load and the source. It represents a *lossless interchange* between the load and the source.

$$Q \begin{cases} = 0 & \text{Resistive Load, Unity pf} \\ \langle\ 0 & \text{Capacitive Load, Leading pf} \\ \rangle\ 0 & \text{Inductive Load, Lagging pf} \end{cases}$$

The procedure for determining whether a circuit element absorbs or delivers power is summarized in Figure 2.4 Figure 2.4(a) shows the *load convention*, where the current *enters* the positive terminal of the circuit element. and the complex power *absorbed* by the circuit element is calculated from (2.3.1). This equation shows that, depending on the value of $(\delta - \beta)$, P may have either a positive or negative value. If P is positive, then the circuit element absorbs positive real power. However, if P is negative. the circuit element absorbs negative real power, or alternatively, it delivers positive real power. Similarly, if Q is positive. the circuit element in Figure 2.4(a) absorbs positive reactive power. However, if Q is negative, the circuit element absorbs negative reactive power, or it delivers positive reactive power.



(a) *Load convention* Current *enters* positive terminal of circuit element. If P is positive, then positive real power is *absorbed*. If Q is positive, then positive reactive power is *absorbed*. If P (Q) is negative, then positive real (reactive) power is *delivered*.

(b) *Generator convention* Current *leaves* positive terminal of the circuit element. If P is positive, then positive real power is *delivered*. If Q is positive, then positive reactive power is *delivered*. If P (Q) is negative, then positive real (reactive) power is *absorbed*

Figure 2.4

Figure 2.4(b) shows the *generator convention*. where the current *leaves* the positive terminal of the circuit element. and the complex power *delivered* is calculated from (2.3.1). When P is positive (negative) the circuit element *delivers* positive (negative) real power. Similarly, when Q is positive (negative), the circuit element *delivers* positive (negative) reactive power.

The *load convention* is used for the RLC elements shown in Figure 2.2. Therefore, the complex power *absorbed* by any of these three elements can be calculated as follows. Assume a load voltage $V = V\angle\delta$. Then, from (2.3.1),

$$\text{resistor: } S_R = VI_R^* = [V\angle\delta]\left[\frac{V}{R}\angle-\delta\right] = \frac{V^2}{R} \tag{2.3.3}$$

$$\text{inductor: } S_L = VI_L^* = [V\angle\delta]\left[\frac{V}{-jX_L}\angle-\delta\right] = +j\frac{V^2}{X_L} \tag{2.3.4}$$

$$\text{capacitor: } S_C = VI_C^* = [V\angle\delta]\left[\frac{V}{jX_C}\angle-\delta\right] = -j\frac{V^2}{X_C} \tag{2.3.5}$$

From these complex power expressions, the following can be stated:

A (positive-valued) resistor absorbs (positive) real power, $P_R = V^2/R$ W, and zero reactive power, $Q_R = 0$ var.

An inductor absorbs zero real power, $P_L = 0$ W, and positive reactive power, $Q_L = V^2/X_L$ var.

A capacitor *absorbs* zero real power, $P_C = 0$ W, and *negative* reactive power, $Q_C = -V^2/X_C$ var. Alternatively, a capacitor *delivers positive* reactive power, $+V^2/X_C$.

For a general load composed of RLC elements, complex power S is also calculated from (2.3.1). The real power $P = \text{Re}(S)$ absorbed by a passive load is always positive. The reactive power $Q = \text{Im}(S)$ absorbed by a load may be either positive or negative. When the load is inductive, the current lags the voltage, which means β is less than δ in (2.3.1), and the reactive power absorbed is positive. When the load is capacitive, the current leads the voltage, which means β is greater than δ, and the reactive power absorbed is negative; or, alternatively, the capacitive load delivers positive reactive power.

Complex power can be summarized graphically by use of the power triangle shown in Figure 2.5. As shown, the apparent power S, real power P, and reactive power Q form the three sides of the power triangle. The power factor angle $(\delta - \beta)$ is also shown, and the following expressions can be obtained:

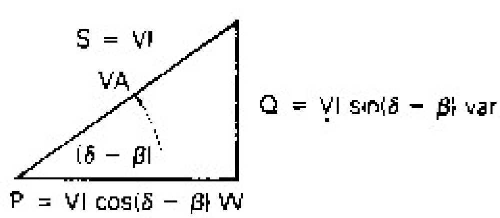
$$S = \sqrt{P^2 + Q^2} \tag{2.3.6}$$

$$(\delta - \beta) = \tan^{-1}(Q/P) \tag{2.3.7}$$

$$Q = P\tan(\delta - \beta) \tag{2.3.8}$$

$$\text{p.f.} = \cos(\delta - \beta) = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}} \tag{2.3.9}$$

Figure 2.5



Part 4

The Per Unit System

Why the Per-Unit System:
Analyzing interconnected three-phase power systems having different voltage levels requires huge transformation of all impedances to a single voltage level. This large mathematical work can be avoided by utilizing the per unit system.

The per-unit system rules:
1- The nominal voltage of lines and equipment is almost always known, as well as the apparent (complex) power in megavolt-amperes, so these two quantities are usually chosen for base value calculation. A minimum of four base quantities are required to complete the per unit system: volt-ampere, voltage, current and impedance. Usually three phase MVA and line-to-line voltages are selected.

2- The entire system under study must have the same base power. All components must use the same base power for their impedance calculation.

3- The impedance of generators & transformers are usually given in percent or per-unit quantities based on their own rating.

4- The impedance of transmission lines are usually expressed by their Ohmmic values.

5- For power system analysis all impedances must be expressed in per-unit on a common system base. Usually, an arbitrary base of S = 100 MVA is selected.

6- The voltage bases are then selected. Once a voltage base has been selected for a point in a system, the remaining voltage bases are determined by the various transformer turn ratios.

Advantages of the Per-Unit System of Calculations:
The per-unit system of calculations is widely used in power systems for the following reasons:

1- It give a better relative sense of the variables under consideration allowing apparatus of widely varying sizes and ratings to be compared with each other in terms of losses, voltage drops, etc.

2- The per unit system permits multiplication and division in addition to addition and subtraction without the requirement of a correction factor (when percentage quantities are multiplied or divided additional factors of 0.01 or 100 must be brought in, which are not in the original equations, to restore the percentage values).

3- The use of per unit values can also be used to avoid problems of transforming impedances across different voltage levels in complex networks involving a large number of voltage steps.

4- The per-unit impedance of equipment of the same general type based upon their own ratings fall in a narrow range regardless of the rating of equipment whereas their impedances in ohms vary greatly with the ratings.

5- The per-unit impedance, voltages, and currents of transformers are the same regardless of whether they are referred to the primary or the secondary side.

6- Different voltage levels disappear across the entire system.

7- The system reduces to a system of simple impedances. The circuit laws are valid in per-unit systems, and the power and voltages equations are simplified since the factors of √3 and 3 are eliminated in the per-unit system

Per-unit values are written with "pu" after the value.

The per unit value of any quantity is a fraction defined by base value per unit value

$$\text{per unit value} = \frac{\text{actual value}}{\text{base value}}$$

Since Voltage, Current, Impedance and Power are related; only two Base or reference quantities can be independently defined. The Base quantities for the other two can be derived there from. Since Power and Voltage are the most often specified, they are usually chosen to define the independent base quantities.

The base values of the other two quantities (current and impedance) are then fixed. In the following sections we will look at how base values are usually selected for three phase p.u. calculations.

For power, voltage, current and impedance, the per-unit quantity may be obtained by dividing by the respective base of that quantity.

$$S_{pu} = \frac{S}{S_{base}} \qquad V_{pu} = \frac{V}{V_{base}} \qquad I_{pu} = \frac{I}{I_{base}} \qquad Z_{pu} = \frac{Z}{Z_{base}}$$

<u>Calculation for Single Phase Systems</u>

If VAbase and Vbase are the selected base quantities of power (complex, active or reactive) and voltage respectively, then

Base Current: $$I_{base} = \frac{V_{base} I_{base}}{V_{base}} = \frac{VA_{base}}{V_{base}}$$

Base Impedance: $$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V^2{}_{base}}{I_{base}V_{base}} = \frac{V^2{}_{base}}{VA_{base}}$$

In a power system, voltages and power are usually expressed in *kV* and *MVA*, thus it is usual to select an *MVAbase* and a *kVbase* and to express them as:

$$I_{base} = \frac{MVA_{base}}{kV_{base}} \quad \text{in } kA$$

$$Z_{base} = \frac{kV^2_{base}}{MVA_{base}} \quad \text{in } \Omega$$

In these expressions, all the quantities are single phase quantities

<u>Calculations for Three Phase Systems</u>

In three phase systems the line voltage and the total power are usually used rather than the single phase quantities. It is thus usual to express base quantities in terms of these. If $VA_{3\varphi base}$ and V_{LLbase} are the base three-phase power and line-to-line voltage respectively,

$$I_{base} = \frac{VA_{base}}{V_{base}} = \frac{3VA_{base}}{3V_{base}} = \frac{VA_{3\phi base}}{\sqrt{3}V_{LLbase}}$$

$$Z_{base} = \frac{V^2{}_{base}}{VA_{base}} = \frac{\left(\sqrt{3}\right)^2 V^2{}_{base}}{3VA_{base}} = \frac{V^2{}_{LLbase}}{VA_{3\phi base}}$$

and in terms of $MVA_{3\varphi base}$ and kV_{LLbase}

$$I_{base} = \frac{MVA_{3\phi base}}{\sqrt{3}kV_{LLbase}}$$

$$Z_{base} = \frac{kV_{LLbase}^2}{MVA_{3\phi base}}$$

It is to be noted that while the base impedance for the three phase can be obtained directly from the $VA_{3\varphi base}$ and V_{LLbase} (or $MVA_{3\varphi base}$ and kV_{LLbase}) without the need of any additional factors, the calculation of base current needs an additional factor of $\sqrt{3}$. However this is not usually a problem as the value of current is rarely required as a final answer in power systems calculations, and intermediate calculations can be done with a variable $\sqrt{3}I_{base}$.

Thus in three phase, the calculations of per unit quantities becomes;

$$S_{pu} = \frac{S_{actual}(MVA)}{MVA_{3\phi base}}$$

$$V_{pu} = \frac{V_{actual}(kV)}{kV_{LLbase}}$$

$$I_{pu} = I_{actual}(kA).\frac{\sqrt{3}kV_{LLbase}}{MVA_{3\phi base}}$$

$$Z_{pu} = Z_{actual}(\Omega).\frac{MVA_{3\phi base}}{kV_{LLbase}^2}$$

P and *Q* have the same base as *S*, so that;

$$P_{pu} = \frac{P_{actual}(MW)}{MVA_{3\phi base}}$$

$$Q_{pu} = \frac{Q_{actual}(M\text{var})}{MVA_{3\phi base}}$$

Similarly, *R* and *X* have the same base as *Z*, so that

$$R_{pu} = R_{actual}(\Omega) \cdot \frac{MVA_{3\phi base}}{kV^2_{LLbase}}$$

$$X_{pu} = X_{actual}(\Omega) \cdot \frac{MVA_{3\phi base}}{kV^2_{LLbase}}$$

The *power factor* remains unchanged in *per unit*.

Conversions from one Base to another

It is usual to give data in per unit to its own rating [ex: The manufacturer of a certain piece of equipment, such as a transformer, would not know the exact rating of the power system in which the equipment is to be used. However, he would know the rating of his equipment]. As different components can have different ratings, and different from the system rating, it is necessary to convert all quantities to a common base to do arithmetic or algebraic operations. Additions, subtractions, multiplications and divisions will give meaningful results only if they are to the same base. This can be done for three phase systems as follows.

$$S_{puNew} = S_{puGiven} \cdot \frac{MVA_{3\phi baseGiven}}{MVA_{3\phi baseNew}}$$

$$V_{puNew} = V_{puGiven} \cdot \frac{kV_{LLbaseGiven}}{kV_{LLbaseNew}}$$

$$Z_{pu} = Z_{puGiven} \cdot \frac{MVA_{3\phi baseNew}}{MVA_{3\phi baseGiven}} \cdot \frac{kV^2_{LLbaseGiven}}{kV^2_{LLbaseNew}}$$

Per Unit Quantities across Transformers

When a transformer is present in a power system, although the power rating on either side of a transformer remains the same, the voltage rating changes, and so does the base voltage across a transformer. [This is like saying that full or 100% (or 1 pu) voltage on the primary of a 220kV/33 kV transformer corresponds to 220 kV while on the secondary it corresponds to 33 kV.] Since the power rating remains unchanged, the impedance and current ratings also change accordingly.

While a common $MVA_{3\varphi base}$ can and must be selected for a power system to do analysis, a common V_{LLbase} must be chosen corresponding to a particular location (or side of transformer) and changes in proportion to the nominal voltage ratio whenever a transformer is encountered. Thus the current base changes inversely as the ratio. Hence the impedance base changes as the square of the ratio. For a transformer with turns ratio N_P:N_S, base quantities change as follows.

Quantity	Primary Base	Secondary Base
Power (S, P and Q)	S_{base}	S_{base}
Voltage (V)	V_{1base}	$V_{1base} . N_S/N_P = V_{2base}$
Current (I)	$S_{base}/\sqrt{3}V_{1base}$	$S_{base}/\sqrt{3}V_{1base} . N_P/N_S = S_{base}/\sqrt{3}V_{2base}$
Impedance (Z, R and X)	V_{1base}^2/S_{base}	$V_{1base}^2/S_{base} . (N_S/N_P)^2 = V_{2base}^2/S_{base}$

Example 4-1

In the single line diagram shown in Figure 4-1, each three phase generator G is rated at 200 MVA, 13.8 kV and has reactances of 0.85 pu and are generating 1.15 pu. Transformer T1 is rated at 500 MVA, 13.5 kV/220 kV and has a reactance of 8%. The transmission line has a reactance of 7.8 Ω. Transformer T2 has a rating of 400 MVA, 220 kV/33 kV and a reactance of 11%. The load is 250 MVA at a power factor of 0.85 lag. Convert all quantities to a common base of 500 MVA, and 220 kV on the line and draw the circuit diagram with values expressed in pu.

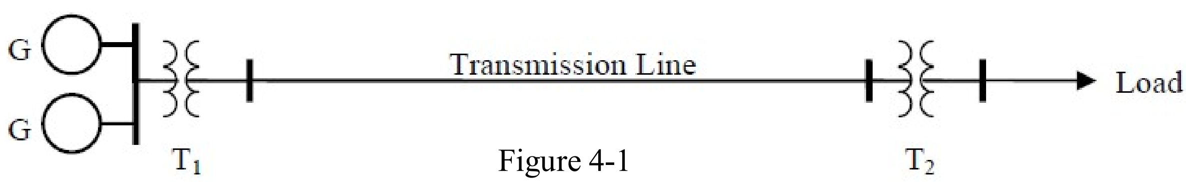


Figure 4-1

Solution:

The base voltage at the generator is (220*13.5/220) 13.5 kV, and on the load side is (220*33/220) 33 kV. [Since we have selected the voltage base as that corresponding to the voltage on that side of the transformer, we automatically get the voltage on the other side of the transformer as the base on that side of the transformer and the above calculation is in fact unnecessary.

Generators G

Reactance of 0.85 pu corresponds 0.355 pu on 500 MVA, 13.5 kV base (see earlier

example)
Generator voltage of 1.15 corresponds to 1.176 on 500 MVA, 13.5 kV base

Transformer T1
Reactance of 8% (or 0.08 pu) remains unchanged as the given base is the same as the new chosen base.

Transmission Line
Reactance of 7.8 Ω corresponds to $7.8 * 500/220^2 = 0.081$ pu

Transformer T2
Reactance of 11% (0.11 pu) corresponds to 0.11 * 500/400 = 0.1375 pu (voltage base is unchanged and does not come into the calculations)

Load
Load of 250 MVA at a power factor of 0.85 corresponds to 250/500 = 0.5 pu at a power factor of 0.85 lag (power factor angle = 31.79°) ∴ resistance of load = 0.5 * 0.85 = 0.425 pu and reactance of load = 0.5 * sin 31.79° = 0.263 pu The circuit may be expressed in per unit as shown in figure 4-2.

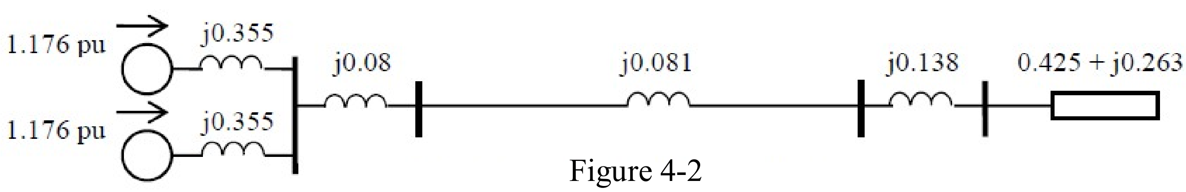


Figure 4-2

Example 4-2

For the system shown in Figure 4.3, draw the electric circuit or reactance diagram, with all reactances marked in per-unit (p.u.) values, and find the generator terminal voltage assuming both motors operating at 12 kV, three-quarters load, and unity power factor.

Generator	Transformers (each)	Motor A	Motor B	Transmission line
13.8 kV	25,000 kVA	15,000 kVA	10,000 kVA	$X = 65\ \Omega$
25,000 kVA	13.2/69 kV	13.0 kV	13.0 kV	
Three-phase	$X_L = 11$ percent	$X'' = 15$ percent	$X'' = 15$ percent	
$X'' = 15$ percent				

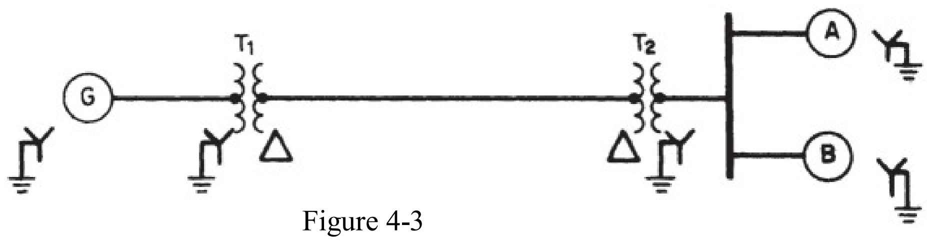


Figure 4-3

Solution:

1. Establish Base Voltage through the System:

By observation of the magnitude of the components in the system, a base value of apparent power *S* is chosen; it should be of the general magnitude of the components, and the choice is arbitrary. In this problem, 25,000 kVA is chosen as the base *S,* and simultaneously, at the generator end 13.8 kV is selected as a base voltage V_{base}. The base voltage of the transmission line is then determined by the turns ratio of the connecting transformer: (13.8 kV)(69 kV/13.2 kV)=72.136 kV. The base voltage of the motors is determined likewise but with the 72.136-kV value: thus, (72.136 kV)(13.2 kV/69 kV)=13.8 kV. The selected base *S* value remains constant throughout the system, but the base voltage is 13.8 kV at the generator and at the motors, and 72.136 kV on the transmission line.

2. Calculate the Generator Reactance:

No calculation is necessary for correcting the value of the generator reactance because it is given as 0.15 p.u.(15 percent), based on 25,000 kVA and 13.8 kV. If a different *S* base were used in this problem, then a correction would be necessary as shown for the transmission line, motors, and transformers.

3. Calculate the Transformer Reactance:

It is necessary to make a correction when the transformer nameplate reactance is used because the calculated operation is at a different voltage, 13.8 kV/72.136 kV instead of 13.2 kV/69 kV.

Use the equation for correction:

$$Z_{pu} = Z_{puGiven} \cdot \frac{MVA_{3\phi baseNew}}{MVA_{3\phi baseGiven}} \cdot \frac{kV^2_{LLbaseGiven}}{kV^2_{LLbaseNew}}$$

per-unit reactance=

$(0.11)(25{,}000/25{,}000)\ (13.2/13.8)^2 = 0.101$ p.u.

This applies to each transformer.

4. Calculate the Transmission-Line Reactance:
Use the equation: *X*per unit=(ohms reactance)(base kVA)/(1000)(base kV)2=(65)(25,000)/
(1000)(72.1)2= 0.313 p.u.

5. Calculate the Reactance of the Motors:
Corrections need to be made in the nameplate ratings of both motors because of differences of ratings in kVA and kV as compared with those selected for calculations in this problem. Use the correcting equation from Step 3, above. For motor A, =(0.15 p.u.)(25,000 kVA/15,000 kVA)(13.0 kV/13.8 kV)^2= 0.222 p.u. For motor B, similarly, =(0.15 p.u.)(25,000 kVA/10,000 kVA)(13.0 kV/13.8 kV)^2= 0.333 p.u.

6. Draw the Reactance Diagram:
The completed reactance diagram is shown in Figure 4-4.

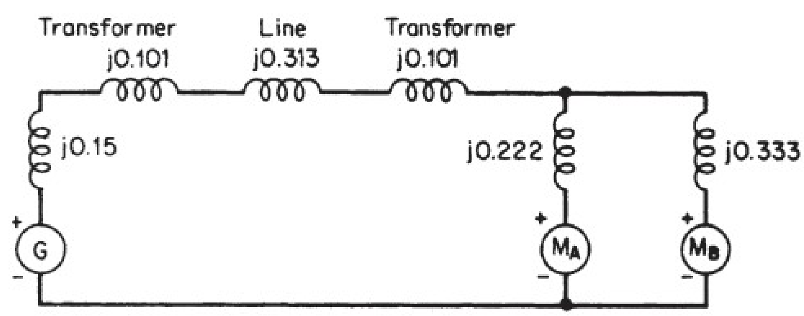


Figure 4-4

7. Calculate Operating Conditions of the Motors:
If the motors are operating at 12 kV, this represents 12 kV/13.8 kV= 0.87 per-unit voltage. At unity power factor, the load is given as three-quarters or 0.75 p.u. Thus, expressed in per unit, the combined motor current is obtained by using the equation I per unit=per-unit power/per unit voltage = 0.75/0.87 = 0.862∟0°p.u.

8. Calculate the Generator Terminal Voltage:

The voltage at the generator terminals, $\mathbf{V}_G = \mathbf{V}_{motor}$ + the voltage drop through transformers and transmission line. This is equal 2:

$$= 0.87\angle 0^\circ + 0.862\angle 0^\circ (j0.101 + j0.313 + j0.101) = 0.87 + j0.444 = 0.977\angle 27.03^\circ \text{ p.u.}$$

In order to obtain the actual voltage, multiply the per-unit voltage by the base voltage at the generator. $13.48\angle 27.03^\circ$ kV. Thus,

Comment:

In the solution of these problems, the selection of base voltage and apparent power are arbitrary. However, the base voltage in each section of the circuit must be related in accordance with transformer turns ratios. The base impedance may be calculated from the equation Z_{base}=(base kV)^2(1000)/(base kVA). For the transmission-line section in this problem, Z_{base}=(72.136)^2(1000)/(25,000)=208.1 Ω; thus the per-unit reactance of the transmission line equals (actual ohms)/(base ohms)=65/208.1=0.313 p.u.

Part 5

Modelling of Power System Components

Introduction:

Before the power system network can be solved and analyzed, it must modeled. The basic fundamentals of power system modeling are:

1. Use of the "per phase" basis to represent a three-phase balanced system.
2. Simple models for generators and transformers to study steady state balanced operation.
4. One-line diagrams to represent a three phase system.
5. The per-unit system and the impedance diagram on a common MVA base.

The main components in any power system are:
Synchronous generators
Transformers and

The model representing each of these components will be developed in the following sections.

1- Development of the Synchronous Generators model:

Large-scale power generation over three-phase lines takes place using synchronous generators, known as ***alternators***. Such generators are driven by steam turbines, hydro-turbines, or gas turbines.
Based on detailed study of the stator, rotor and air-gap magnetic fields present in the machine, a general phasor diagram relating all such magnetic fields with the induced emfs is shown in Figure 5-1.

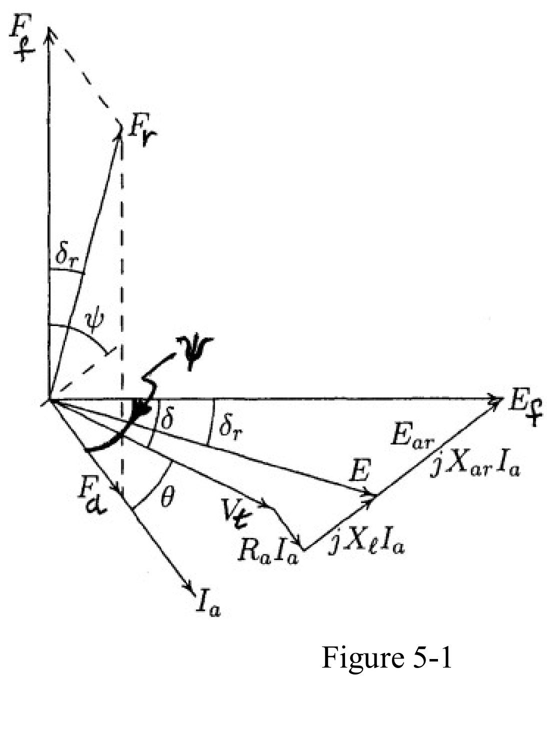


Figure 5-1

The no-load voltage generated in the machine, Ef, is given by:

Ef = Vt + [Ra + j(Xl + Xar)]Ia . . . . (1)

Xs = Xl + Xar, ... (2)

Where,

Ef = Generated voltage in the machine

Vt = Generator terminal voltage

Xl = Armature leakage reactance,

Xar = Armature reaction reactance,

Xs = Synchronous reactance

Also,

Ef = Vt +(Ra + jXs)Ia . . . . (3)

Note that:

Θ is the angle between Ia and Vt

Ψ is the angle between Ia and Ef

δ is that angle between the terminal voltage Vt and the generated on-load voltage Er, and is known as the generator torque or power angle.

Note that in Figure 5-1 the angle between Ef and Er is shown to be δr. The power developed by the machine is proportional to the product of Ff, Fr and δr. The relative positions of these mmfs dictate the action of the synchronous machine, so that:

When Ff is ahead of Fr by an angle δr, the machine is operating as a generator, and

When Ff falls behind Fr, the machine will act as a motor.

Since Ef and Er are proportional to Ff and Fr respectively, the power developed by the machine is proportional to Ef, Er and sin δr. Usually the angle δr is very close to the angle δ and power developed by the machine is proportional to Ef, Er and sin δ.

From equation 3, a simple per-phase model for a cylindrical rotor generator is obtained in Figure 5-2. The armature resistance Ra is generally much smaller than the synchronous reactance and is often neglected. The simplified equivalent circuit of a synchronous machine is shown in Figure 5-3.

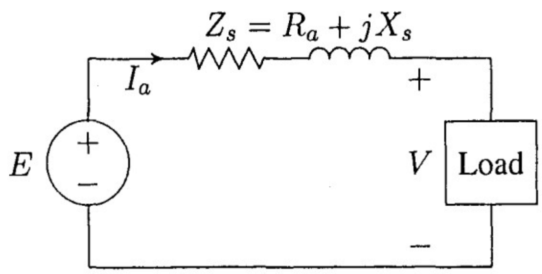


Figure 5-2: Simplified Equivalent Circuit

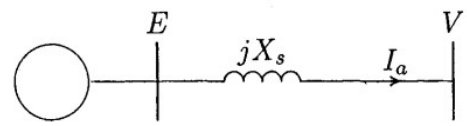


Figure 5-3: Synchronous Machine Equivalent Circuit

Studying the operation of the synchronous generator under different loading conditions can be greatly simplified by using the developed equivalent circuit. The phasor diagram of the machine under the required operating conditions can be obtained from the developed equivalent circuit. Figure 5-4 shows the phasor diagrams of the synchronous generator with terminal voltage as reference for excitations corresponding to lagging, unity and leading power factors.

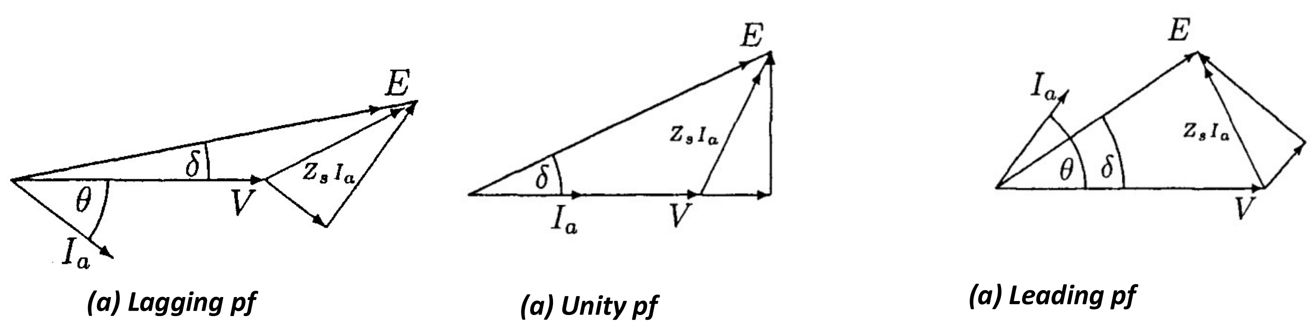


Figure 5-4: Synchronous Generator Phasor Diagrams

Equation 3 and the synchronous machine equivalent circuit shown in Figure 5-3 can be used to study the steady state performance of a synchronous machine, such as the machine power transfer, which is discussed as follows:

Assume that the load connected to the generator is an infinite bus, as shown in Figure 5-5. The generator is operating into a receiving bus large in capacity compared to the sending bus. The receiving bus Is used as the reference, and the angle, δ , will be from the sending bus, which may be a single generator or an entire system. The connecting line impedance includes the generator and receiving bus impedance.

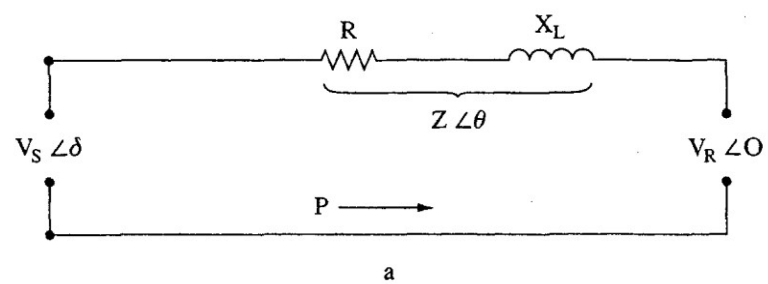


Figure 5-5: A Synchronous Generator Connected to an Infinite

It can be shown from synchronous machine theory that the real power received by the load is given by:

$$P_R = \frac{V_R V_S}{X} \sin\delta$$

A lso, the reactive power received by the load is given by:

$$Q_R = \frac{V_R V_S}{X} \cos\delta$$

When using the above equations, note the following:

a- The maximum occurs at 90° in the approximate equations above. This is approximately true for non-salient pole generator, but for salient pole machines (poles stick out instead of being smooth) the maximum occurs before 90°.

b- A generator operates with a power angle of 20° to 25°,

c- Any angle beyond 90° is unstable because the restoring force on the rotor is lost.

d- The power angle across a long transmission line seldom reaches 10°, and seldom exceeds 40° across a whole system.

From the preceding equations we can see that VAR flow on a line is controlled by adjusting the voltage (V_S — V_R). The power flow between two points in a system must be varied by changing the power angle, δ , between the two points. Phase shifting transformers, both fixed and variable, are available to control power flow by adjusting δ.

2- ***Development of the transformer model:***

To develop the transformer model, first consider an ideal transformer, with no leakage and infinite permeability. Let the transformer have N1 turns on the left side and N2 turns on the right side. An ideal transformer can be represented by the circuit diagram in Figure 5-6. For an ideal transformer, we can show that:

$$\frac{E_1}{E_2} = \frac{I_2}{I_2'} = \frac{N_1}{N_2}$$

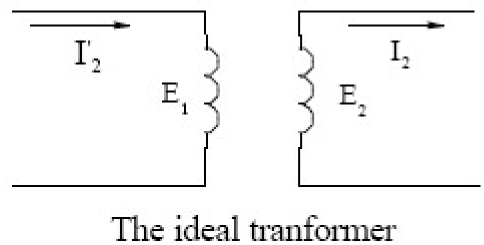


Figure 5-6: Ideal Transformer Model

also,

N1/N2 = α

Where **α** is the turns ratio of the transformer

In a real transformer, there would be some leakage flux, also some losses. There is also a magnetizing current. These are simulated by adding components to the ideal model, thus a real transformer model would be as shown below. This is the T-equivalent circuit and is shown in Figure 5-7.

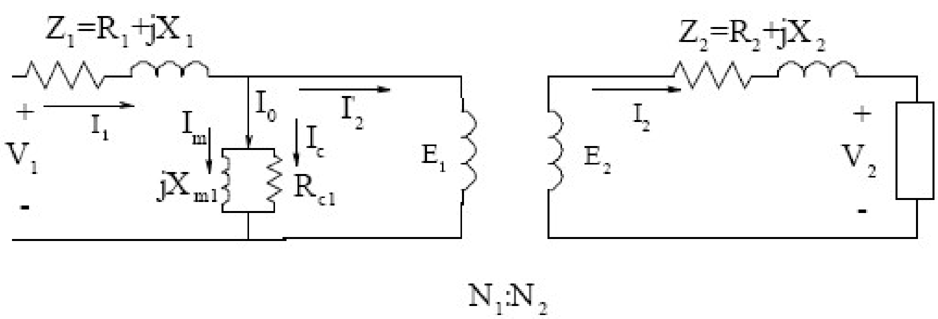


Figure 5-7: The Equivalent Circuit of a “real” Transformer

Assuming the load on an ideal transformer has an impedance Z_2 , then we have:

$$Z_2 = \frac{E_2}{I_2}.$$

The impedance seen at the left of the ideal transformer is

$$Z_1 = \frac{E_1}{I_2'} = \frac{E_2 \frac{N_1}{N_2}}{I_2 \frac{N_2}{N_1}} = \left(\frac{N_1}{N_2}\right)^2 \frac{E_2}{I_2} = \left(\frac{N_1}{N_2}\right)^2 Z_2 .$$

Thus ***impedances are transformed from the secondary side to the primary side as the turns ratio squared*** (***or the inverse turns ratio squared going the other way***).

Let

$$\alpha = \frac{N_1}{N_2}.$$

Then: $E_2 = V_2 + Z_2 I_2$. But, $E_1 = \alpha E_2$ and $I_2 = \alpha I_2'$, therefore:

$$\begin{aligned} E_1 &= \alpha V_2 + \alpha^2 Z_2 I_2' \\ &= V_2' + Z_2' I_2' \end{aligned}$$

Where:

$$Z_2' = R_2' + jX_2' = \alpha^2 R_2 + \alpha^2 X_2 .$$

Using these transformations, the exact model of the transformer, which shows the equivalent circuit referred to the primary side, will be as shown in Figure 5-8.

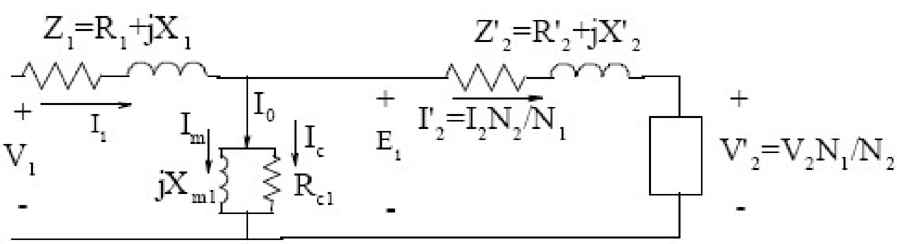


Figure 5-8: Exact Equivalent Circuit of a Transformer (Referred to Primary)

The shunt branch of the exact equivalent circuit can be moved to right of the circuit to make it easier to calculate the no-load current, for example. The same circuit can therefore be redrawn, as shown in Figure 5-9.

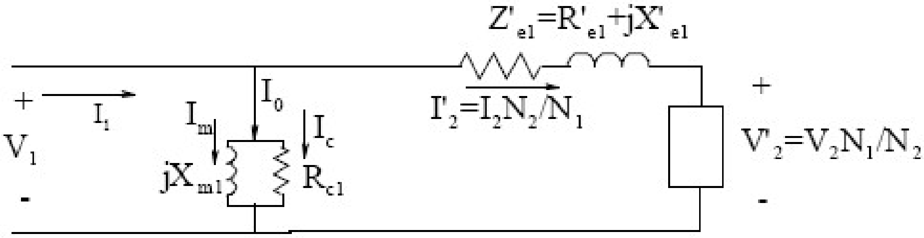


Figure 5-9: Approximate Equivalent Circuit of a Transformer (Referred to Primary)

The same equivalent circuit referred to the secondary can drawn as shown in Figure 5-10. In this circuit, the referred primary voltage V_1' is given by:

$$\mathbf{V_1' = V_2 + (R_{e2} + J\, X_{e2})\, I_2}$$

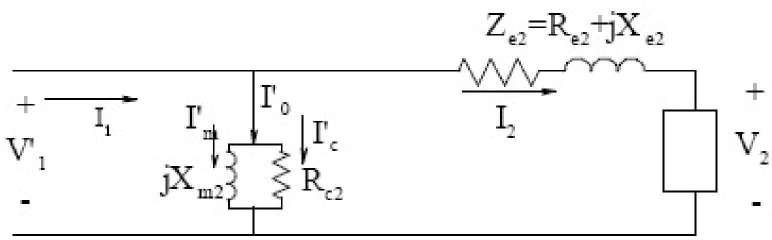


Figure 5-10: Approximate Equivalent Circuit of a Transformer (Referred to

Furthermore, if we neglect the shunt parts (which are relatively of high impedance) the result is only an RL equivalent as shown in Figure 5-11 below. In most applications, only the reactance X is kept since R is so much smaller it is neglected.

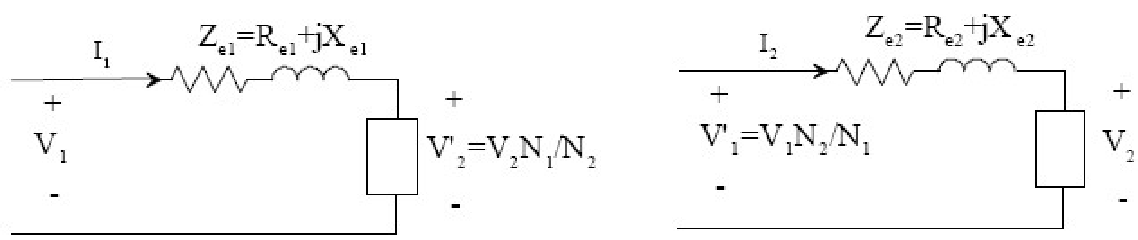


Figure 5-11: Simplified Equivalent Circuit of a Transformer (Referred to Primary or Secondary)