A Novel Scheme to Estimate the Model Order of Physical Systems

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***Abstract*— A method to estimate the model order of physical systems is proposed. It is assumed that the region where the poles of a physical system are located is known. This *a priori* knowledge is generally obtained from the physical laws governing the physical system. Various model orders are chosen using the conventional model selection criteria. For each order, a model of the physical system is identified. If the poles of the identified model are not located in this region, this model is assumed to include extraneous poles and is rejected. This process is continued till the estimated model does not include extraneous poles. The proposed scheme is extensively evaluated on physical systems and the results are compared with those of the conventional schemes such as Akaike Information Criterion (AIC) .**

Index Terms — **model order selection criteria, Akaike Information Criterion, overfitting, noise artifacts.**

I. INTRODUCTION

In identifying a model of a physical system, the structure of the model may not be identical to that of the mathematical model derived from the physical laws due to various factors including the presence of noise and fast dynamics [1]. A simple approach is to choose the measure of goodness of fit between the outputs of the system and its mathematical model to be the sum of the squares of the residual. The selection of the model order based on this measure may be unsatisfactory. Increasing the order of the system and hence the number of estimated parameters improves the goodness of fit. However, choosing a higher order model may result in overfitting the data and consequently will include noise artifacts. To overcome this problem of overfitting, a number of selection criteria have been proposed. These criteria are based on penalizing not only the sum of the squares of the residuals but also the model order. Commonly used criteria include Akaike Information Crite rion (AIC) and Beysian Information Criterion (BIC), Minimum Description Length (MDL) [2].

These are based on statistical decision theory requiring *a priori* knowledge of the probability distribution function (pdf). In practical systems it may not be possible estimate the pdf and hence Gaussian pdf is generally assumed [3-9]. In many cases, the application of these criteria assuming Gaussian pdf may not always give good result as the estimated model may still contain noise artifacts.

In this work a different approach is proposed. It directly verifies the presence of noise artifacts in the estimated model. It is a two stage process. First the conventional model structure selection criterion such as AIC, BIC and MDL is employed to select an order. In the next stage presence of noise artifacts in the selected model is verified. If the noise artifacts are present, the selected order is discarded and a lower order is chosen and this process in continued till there are no noise artifacts.

The question arises as to how one can verify the presence or absence of noise artifacts. The presence of noise artifacts manifests by the presence of *extraneous* poles in the identified model. The next question is how to identify extraneous poles. It is assumed that the region where the poles of the model of physical system lie is known *a priori*. This knowledge is generally obtained from the physical laws governing the physical systems. If any one of the poles of the identified model is located in different region, then this pole is considered extraneous.

II. THE POLES OF IDENTIFIED MODELS

Models of most physical systems are continuous while the identified models are discrete. The poles of the discrete-time model are related to the continuous time model by an exponential function.

$$\lambda_d = e^{\lambda_c T} \tag{1}$$

Where λ_c a pole of the continuous-time is model and λ_d is the corresponding discrete-time pole. It can be deduced that the poles of the discrete-time model obtained from the continuous time systems are located in the right-half of the z-plane.

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If the identified model contains a pole in the left-half of the z-plane, this pole has not come from the continuous time model of the physical system. It is an extraneous pole due to various factors such as noise artifacts.

III. MODEL SELECTION CRITERIA

In identifying a physical system, the structure of the model may not be identical to that of the mathematical model derived from the physical law due to factors such as the presence of noise and fast dynamics. A simple choice for measure of goodness of fit is the sum of squares of the residual, *J* given by:

$$J = \frac{1}{N}\sum_{i=1}^{N} e^2(i) \qquad (2)$$

Where *e* is the residual.

The figure below gives the sum of the squares of the residual as a loss function of the model order for On/Off Controller. (See Fig 1).

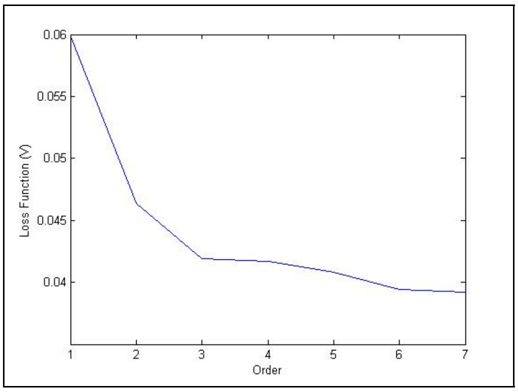

Fig 1. Sum of squares of residual: On/Off Controller

Increasing the order of the system and hence the number of estimated parameters n_P may improve the goodness of fit as given by *J*. *J* may reduce with the increase in the order of the system. Hence one may be tempted to choose a very high order. However, this will result in overfitting the data. The model thus selected will include noise artifacts Commonly used criteria which includes Akaike Information Criterion(AIC) and Beysian Information Criterion(BIC) penalizes not only the sum of the squares of the residuals but also the model order.

Akaike's Information Criterion (AIC) is a measure of the goodness of fit expressed as a function of sum of the squares of residuals and the order of the system. There are

$$AIC = \log J + \frac{2n_P}{N} \qquad (3)$$

Where $J = \frac{1}{N}\sum_{i=1}^{N} e^2(i)$,

n_P is the number of parameters that are estimated in the model and *N* is the number of data points.

As the other measures, namely BIC and MDL, gives results similar to AIC when the pdf is assumed to be Gaussian, in this work only AIC is used in conjunction with analyzing the poles of the estimated model with a view to identify the presence of noise artifacts. If the region where the poles of the system based on the physical laws is different from the region where the estimated poles lie, then these poles can be considered to be generated from the noise artifact. The selected order should be such that the estimated poles should be located in the same region as that dictated by the physical laws.

For example, the poles of the physical system are located on the right half of the z-plane. The poles which are located in the left half of the z-plane are identified as poles resulting from the noise artifacts. The poles on the left half plane contribute oscillatory response.

The lowest order which does not include noise artifact and low value (not necessarily the minimum value) of the AIC is selected as the model order. Information Criterion (AIC) is used to perform a relative comparison.

IV. EVALUATION ON A PHYSICAL SYSTEM

The proposed method was evaluated on a physical process control system formed of two tanks. The model relating the height of the liquid and the input voltage applied to the motor was estimated for various selected order. The figures below give the plot of AIC measures for different controller configurations such as on-off, proportional, proportional integral and proportional integral and derivative controllers. (See fig. 2-5)

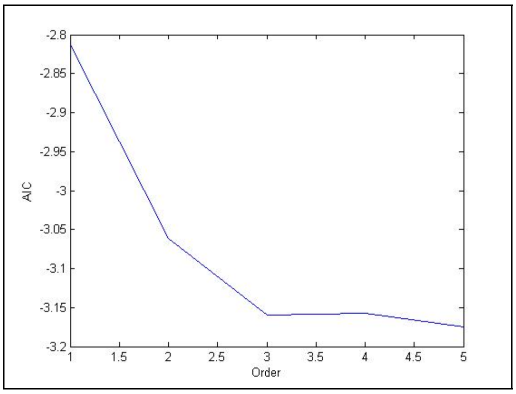

Fig. 2 AIC: On/Off Controller

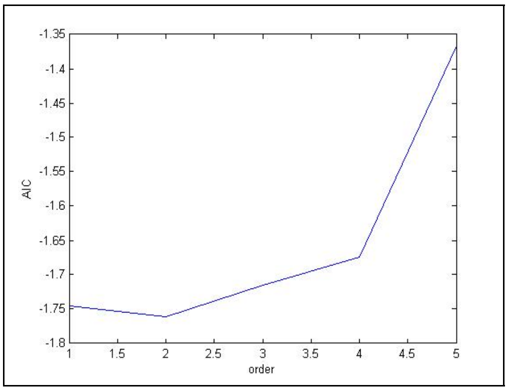

Fig. 3 AIC: P Controller

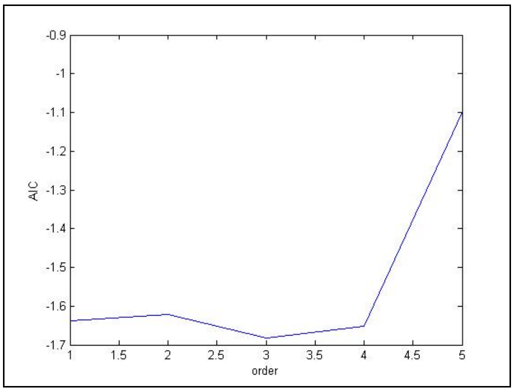

Fig. 4 AIC: PI Controller

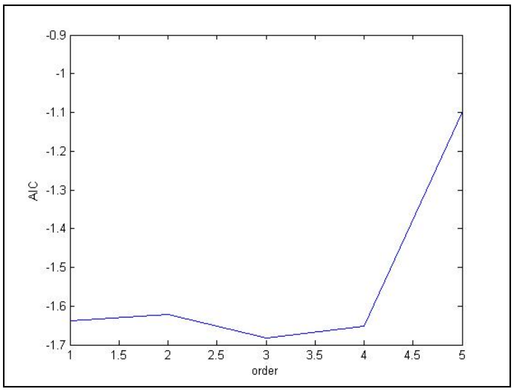

Fig. 5 AIC: PID Controller

Table below gives the table containing the AIC and the pole locations for On/Off Controller, P Controller, PI Controller and PID Controller. (See tables 1-4)

Order	Pole 1	Pole 2	Pole 3	Pole 4	Pole 5
1	1.000				
2	1.000	-0.4966			
3	1.000	-0.3208-0.4392i	-0.3208+0.4392i		
4	1.000	-0.3445-0.4479i	-0.3445+0.4479i	0.0398	
5	1.000	-0.5478-0.4832i	-0.5478+0.4832i	0.2200-0.5042i	0.2200+0.5042i

Table 1. Pole Location: On/Off Controller

Order	Pole 1	Pole 2	Pole 3	Pole 4	Pole 5
1	0.9931				
2	0.9932	-0.1582			
3	0.9930	-0.5141	0.4013		
4	0.9928	0.7209	-0.4573-0.4446i	-0.4573+0.4446i	
5	0.9928	0.7490	-0.4039-0.4272i	-0.4039 + 0.4272i	0.1387

Table 2. Pole Location: P Controller

Order	Pole 1	Pole 2	Pole 3	Pole 4	Pole 5
1	0.9918				
2	0.9917	0.0625			
3	-0.5938	0.9914	0.6280		
4	0.9908	0.8409	-0.4752 -0.4692i	-0.4752 +0.469i	
5	0.9900	0.8954	-0.2551 - 0.5846i	-0.2551 + 0.5846i	-0.5793

Table 3. Pole Location: PI Controller

Order	Pole 1	Pole 2	Pole 3	Pole 4	Pole 5
1	0.9931				
2	0.9929	-0.1202			
3	-0.5548	0.9927	0.4489		
4	-0.4850 +0.490i	-0.4907i -0.4850	0.9923	0.7654	
5	0.9920	0.7793	-0.3608 - 0.4789i	-0.3608+ 0.4789i	-0.3127

Table 4. Pole Location: PID Controller

Note: The highlighted ones "-" denote extraneous poles.

The tables above show the behavior of the poles upto 5th order of the Controller. The orders of the system have been determined in a way to judge the perfect way which suits for the particular controller. Four Controllers have been demonstrated namely on/off, P, PI and PID controllers.

The figure below shows the step responses of On/Off Controller, P Controller, PI Controller, and PID Controller for different choices of the model orders. (See Fig. 6-9).

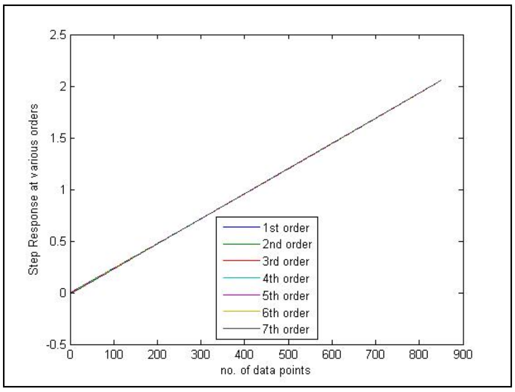

Fig. 6 Step response at various orders: On/Off Controller

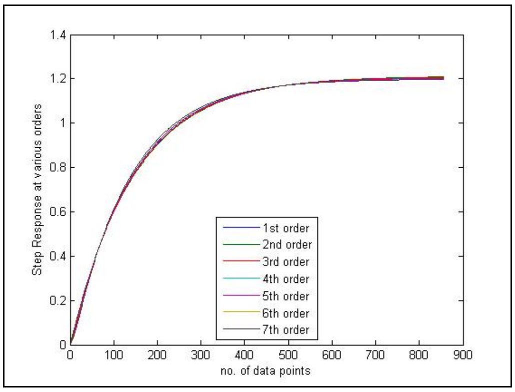

Fig. 7 Step response at various orders: P Controller

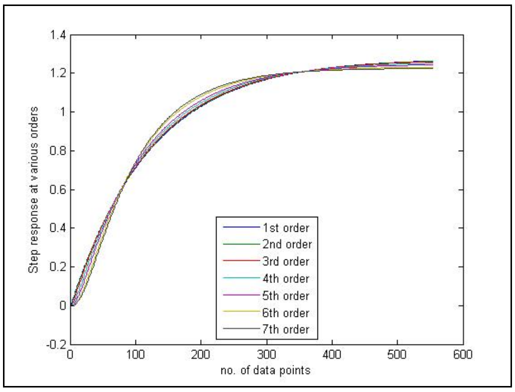

Fig. 8 Step response at various orders: PI Controller

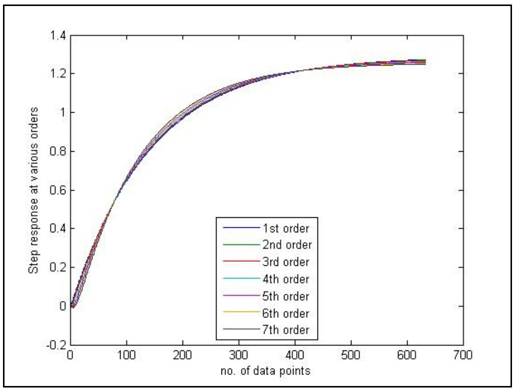

Fig. 9 Step response at various orders: PID Controller

V. CONCLUSION

The proposed model order selection criteria based on verifying the presence of noise artifacts is promising. This criterion ensures that the model captures only the dynamical behavior of the system and not other artifacts. A Kalman filter based fault diagnosis scheme designed using the identified model using on the proposed order selection criterion gave good performance.

Further research into the origin of the extraneous poles, and their role in the performance of the control system and the fault diagnosis scheme using Kalman filter is in order when the identified model with extraneous poles is used.

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