# Fusion of Model-Based and Model-free Approaches to Leakage Diagnosis

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Abstract— A diagnosis scheme for incipient leakage faults is proposed using a combination of two entirely different approaches, namely model-free and model-based ones, ensuring thereby that critical information about the presence or absence of leakage is monitored in the shortest possible time and the complete status regarding the leakage is unfolded in time. Model-free approaches include limit checks & knowledge-based analysis, while model-based approaches include the extended Kalman filter and a parameter identification scheme. The knowledge-based analysis indicates quickly a possible onset of leakage, the Kalman filter detects the presence/absence and finally the identification scheme isolates the detected fault. Further, the whole combined helps in designing an effective preventive maintenance strategy. The proposed scheme is evaluated on a physical fluid system exemplified by a benchmarked two-tank system.

Index Terms — fault diagnosis, leakage fault, Kalman filter, parameter identification, knowledge base.

#### I. INTRODUCTION

The management of leakage faults in fluid systems is becoming increasingly important in recent years from the point of view of economy, potential hazard, pollution, and conservation of scarce resources [1-6]. Leakage in pipes and storage tanks occurs due to faulty joints, aging, excessive loads, holes caused by corrosion and accidents and the like. Model-, neural network-, and statistical inference-based approaches have been proposed. Model-based approaches are becoming increasingly popular in recent years due to their ability to detect incipient leakages.

A model of a process control system is typically highly complex, nonlinear and stochastic, and consequently

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Lahouari Cheded is with the Systems Engineering Department, King Fahd University of Petroleum and Minerals, P. O. Box 116, Dhahran 31261, Saudi Arabia, e-mail: cheded@kfupm.edu.sa. model-based approaches tend to be computationally burdensome and costly. In critical applications such as those involving hazardous leaks, it is important to ensure that a leak is detected quickly and reliably.

In this work, a combination of two entirely different approaches is employed. The tasks of leakage diagnosis are executed in the order of decreasing importance and increasing precision [7] starting with limit checks and followed by a plausibility analysis, then followed by the Kalman filter and finally by a parameter identification scheme. Parameter identification completes the diagnostic picture.

The final stage provides estimates of the parameters of various components of the process control system. This helps to implement a cost-effective conditionbased monitoring procedure.

The mathematical model of a typical process control system consisting of tanks and a network of pipes is derived relating various key variables such as the reference input to the motor driving the pump, the flow rate, and the height of the liquid in the tank. Although the model is nonlinear and since our goal is to detect incipient faults, that is, to estimate small deviations in the states and the parameters about an operating point, a linearized model is therefore employed.

The signal- (or data-) based analysis is employed as it is simpler and faster to implement and does not require a- priori knowledge of the model. The limit value checking and plausibility check are both fast and indicate a possible presence/absence of a fault. However, it cannot detect incipient faults. A leakage fault manifests itself as an abrupt jump in the error signal, flow rate and height data. A knowledge-based scheme is used to locate a change in the slope of the sensor signal.

The Kalman filter is used for the linearized plant model. A statistical decision-theoretic approach, using the likelihood ratio test, is employed to decide between the two hypotheses of a presence or an absence of a fault. Foe this purpose, a Gaussian probability is assumed.

As the Kalman filter is not efficient for fault isolation, a parameter identification scheme is thus employed to remedy this situation and to give a complete diagnostic picture. The identified model of a physical system may generally have a structure different from that of the mathematical model derived from the physical laws due several reasons such as the presence of noise, nonlinearities and inability to capture faster dynamics of the components. To address this model structural mismatch problem, a combination of the Akaike Information criterion (AIC) and the location of the identified poles for different selected model orders is used here.

The proposed scheme is evaluated on a benchmark laboratory-scale process control system using National Instruments LABVIEW. A knowledge-base for leakage diagnosis is deduced from an extensive experimentation and simulation of the model under different leakage scenarios. Model-free and model-based analyses are employed to detect the leakage. The covariance matrices for the Kalman filter were adapted from the measured data so as to find a compromise between fast response and small variance of the noisy residual.

A complete model of the system is finally identified using parameter identification for different types of leakage faults and different types of control strategies including ON/OFF, P and PID schemes.

#### II. TWO-TANK BENCHMARK MODEL

A benchmark model of a cascade connection of a dc motor and pump relating the input to the motor, u, and the flow,  $Q_i$ , is a first order time-delay system

$$\frac{abe^{-ST_d}}{(s+a)(s+b)} \tag{1}$$

Where  $T_d$  is a time-delay, b and a are the model parameters.

It is assumed that leakage  $Q_{\ell}$  occurs in Tank 1 and is given by:

$$Q_{\ell} = C_{d\ell} \sqrt{2gH_1} \tag{2}$$

With the inclusion of the leakage, the liquid level system is modeled by:

$$A_{1}\frac{dH_{1}}{dt} = Q_{i} - C_{db}\sqrt{2g(H_{1} - H_{2})} - C_{d\ell}\sqrt{2gH_{1}} \quad (3)$$

$$A_2 \frac{dH_2}{dt} = C_{db} \sqrt{2g(H_1 - H_2)} - C_{do} \sqrt{2gH_2} \qquad (4)$$

where  $H_1$  is the height of the liquid in tank 1,  $H_2$  is the height of the liquid in tank 2,  $A_1$  and  $A_2$  are the cross-sectional areas of the tanks, g=980 cm/sec<sup>2</sup> is the gravitational constant,  $C_{db}$  and  $C_{do}$  are the discharge coefficient of the valve.



Fig. 1 Two tank fluid system

The model of the two tank (See fig.1) is second order and nonlinear with smooth square root type nonlinearity. As our focus is to detect incipient leakage fault, that is small leakage fault before it develops in to a system failure, a linearized model may be employed. Linearized model becomes:

$$\frac{dh_{1}}{dt} = b_{1}q_{i} - (a_{1} + \alpha)h_{1} + a_{1}h_{2}$$
(5)

$$\frac{dh_2}{dt} = a_2 h_1 - a_2 h_2 \tag{6}$$

where  $h_1$  and  $h_2$  are the increments in  $H_1^0$  and  $H_2^0$ ,

$$b_{1} = \frac{1}{A_{1}}, \quad a_{1} = \frac{C_{db}}{2\sqrt{2g(H_{1}^{0} - H_{2}^{0})}},$$
$$a_{2} = a_{1} + \frac{C_{do}}{2\sqrt{2gH_{2}^{0}}} \quad \alpha = \frac{C_{d\ell}}{2\sqrt{2gH_{1}^{0}}}$$

and parameter  $\alpha$  indicates the amount of leakage.

A PID controller is used to maintain the level of the Tank 1 at the desired reference input. See fig. 2 for the block diagram.

$$\mathbf{u}(s) = \left(\mathbf{k}_{\mathrm{p}} + \frac{k_{I}}{s} + k_{D}s\right) e(s), \quad e(s) = r(s) - h_{\mathrm{I}}(s)$$



Fig.2 block diagram of the overall process control system The state space model is given by:

$$\dot{x} = Ax + Br \tag{7}$$

Where  

$$x = \begin{bmatrix} h_1 \\ h_2 \\ q_i \\ x_4 \\ x_5 \end{bmatrix} A = \begin{bmatrix} -a_1 & a_1 & b_1 & 0 & 0 \\ a_2 & -a_2 & 0 & 0 & 0 \\ 0 & 0 & -b & b & 0 \\ -k_p a - k_D a_1 & -k_D a_2 & -k_D a_1 & -a & k_1 a \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$B = \begin{bmatrix} 0 & 0 & 0 & a & 0 \end{bmatrix}^T$$

#### III. MODEL-FREE AND MODEL BASED

In this work, a fusion of model-free and modelbased approaches is employed. The tasks of leakage diagnosis are executed in the order of decreasing importance and increasing precision [7] starting with limit checks followed by plausibility analysis Kalman filter

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and param- eter identification schemes (See fig.3). The parameter identification provides a complete diagnostic picture.

Form the equation governing the fluid system, it can be deduced that the derivative of the height changes and the time constant with the onset of leakage. This knowledge is employed to detect a possible leakage.



Fig.3 Execution of diagnostics tasks in the order of decreasing importance and increasing precision

## IV. KALMAN FILTER

Kalman filter is designed for the normal leakage free operation. The model of the system for an ideal no leakage case, that is when  $\alpha = 0$ , is given by:

$$x(k+1) = A_0 x(k) + B_0 u(k-d) + w(k)$$
(8)

$$y(k) = C_0 x(k) + \upsilon(k) \tag{9}$$

Where y is the output, e.g., height,  $(A_0, B_0, C_0)$  are obtained from discretized model of (A, B, C) for the ideal case, w(k) and v(k) are zero mean plant white noise and measurement white noise respectively, with covariance

$$Q = E\left[w(k)w^{T}(k)\right]$$
(10)

$$R = E\left[v(k)v^{T}(k)\right]$$
(11)

The plant noise, w(k), is a mathematical artifice introduced to include uncertainty in the *a priori* knowledge of the plant model. Larger the covariance, less accurate is the model  $(A_0, B_0, C_0)$  and smaller the covariance more accurate. The Kalman filter is given by:

$$\hat{x}(k+1) = A_0 \hat{x}(k) + B_0 u(k-d) + K_0 \left( y(k) - C_0 \hat{x}(k) \right)$$
(12)

$$e(k) = y(k) - C_0 \hat{x}(k)$$
 (13)

where d the delay and e(k) is is the residual.

Since, the system model has a pure delay, this delay is incorporated in the Kalman filter formulation. The Kalman filter estimates the states by fusing the information provided by the measurement y(k) and the *a priori* information contained in the model  $(A_0, B_0, C_0)$ . The fusion is based on the a priori information of the plant covariance, Q, and the measurement noise covariance, R. When Q is small, implying that the model is more accurate, estimate is obtained by weighting the plant model more than the measurement model. The Kalman gain,  $K_0$ , will be small. On the other hand when R is small implying that the measurement model is more accurate, estimate is obtained by weighting the measurement model more than the plant model. The Kalman gain,  $K_0$ , will be large.

The larger the gain  $K_0$  faster will be the response of the filter and larger will be the variance of the estimation error. Thus there is a trade-off between the fast response and the smaller covariance of the residual.

It is important to design the Kaman filter for leakage diagnosis so that an acceptable compromise is obtained between a faster detection and an ability to detect a smaller leakage.

Statistical decision theoretic approach was used to decide between two hypotheses:

$$H_o: e(k) = v(k) \tag{14}$$

$$H_1: e(k) = v(k) + c$$
 (15)

Where c is some constant. Likelihood ratio test is used assuming v(k) is white Gaussian noise.

$$\left| e(k) \right| \begin{cases} \geq \sigma & leakage \\ < \sigma & no \ leakage \end{cases}$$

### V. PARAMETRIC IDENTIFICATION BASED

The Kalman filter although detects the faults, it is not efficient in isolating the faults. Hence, a parametric estimation scheme is employed. A complete model of the process control system comprising the motor, pump, the tank and the valves are obtained using a parameter estimation scheme. One can know the status of all devices forming the system. The parameters characterizing devices such as flow sensor, level sensor, valves, and pipes are estimated. These estimates help to monitor their status and plan a preventive maintenance scheme.

A discrete time model of the fluid system relating the height and the input u takes the form:

$$y(k) = \varphi^{T}(k)\theta + v(k) \tag{16}$$

Where  $\varphi$  is a data vector formed of the input and the output, and  $\theta$  is the coefficient of the discrete time transfer function model, and v(k) is the noise. A recursive least-squares method is used to estimate  $\theta$ 

First offline estimate of  $\theta$  denoted  $\hat{\theta}^0$  is obtained when there is no leakage. This estimate is employed in the Kalman filter model. Various system orders are chosen and for each order, the parameter is identified. Akaike Information Criteria (AIC) along with the estimate pole locations are is used to guide the choice of an appropriate order. The pole location can identify the inclusion of noise model artifacts. Using a very large order will tend to model the noise, although the estimation error will be small. This tends to model the noise. Too small order will not capture the system dynamical behaviour.

When the leakage is detected, an estimate of  $\theta$  denoted  $\hat{\theta}^0$  is obtained online.

 $\hat{\theta} - \hat{\theta}^0$  gives a change in the system parameters providing a complete diagnostic picture of the process.

#### VI. EVALUATION ON A PHYSICAL SYSTEM

The physical system is formed of two tanks connected by a pipe. The leakage is simulated in the tank by opening the drain valve. A DC motor driven pump supplies fluid to one of the tanks. A PID controller is used to maintain a specified level.

A single tank configuration is employed by closing the link valve. A step input is applied to the dc motorpump system to fill the tank. The opening of the drainage valve introduces leakage in the tank. Various types of leakage faults are introduced and the liquid height, H and the flow,  $Q_i$  are measured. The National Instruments LABVIEW which is a data acquisition device is employed to collect the data.

Combination of knowledge-base, Kalman & parameter identification approaches was employed for leakage diagnosis.

Various types of leakage faults were introduced by opening the drainage valve and the height profiles were analyzed.



Fig. 4 Input flow rate and the tank height under various degrees of leakage

From Figure 4, one can readily deduce both the onset of the leakage and the amount of leakage from the height profile. The leakage flow has five sections corresponding to the following five degrees of no-leakage, small, medium, large and very large.

# VII. KALMAN FILTER BASED EVALUATION

First the leakage-free model of the system is identified using a recursive least-squares identification scheme. The order of the estimated model was iterated to obtain an acceptable model structure using a combination of the AIC criterion and the identified pole locations. Even though the theoretical model is of a higher order, the fast dynamics of the motor, pump, sensors are dealt with by the AIC criterion. With no leakage, the identified model is essentially an integrator (and thus unstable) with a delay (see Fig. 5).

The model of the entire system relating the input signal u to the dc motor- pump combination, and the height, h is identified. With no leakage, the model is essentially an integrator with a delay as shown below in (17):

$$h(t) - a_h \frac{dh}{dt} = b_h u(t - d) \tag{17}$$

where  $b_h = 0.0033, a_1 = 1, d = 30$ 

The parameter,  $b_h$  characterizes the level sensor.

The model is essentially the first order unstable system with a delay even though the theoretical model is of the third order.

The parameter,  $b_h$  characterizes the level sensor.

![](_page_3_Figure_19.jpeg)

Fig. 5 The flow and the height under no leakage. Note that the height is essentially a ramp

Using the leakage-free model together with the covariance of the measurement noise, R, and the plant noise covariance, Q, the Kalman filter model is finally derived. As it is difficult to obtain an estimate of the plant covariance, Q, a number of experiments were performed under different plant scenarios to tune Kalman gain,  $K_0$ 

$$\hat{x}(k+1) = A_0 \hat{x}(k) + B_0 u(k-d) + K_0 (y(k) - C_0 \hat{x}(k))$$
(18)

$$e(k) = y(k) - C_0 \hat{x}(k)$$
 (19)

where  $A_0 = 1$ ,  $B_0 = 0.0033$ ,  $K_0 = 25$ 

Kalman filter was evaluated under different fault leakage scenarios for on-off controller, P controller, PI controller and PID controller (see Figures 6-9).

# VIII. PARAMETER IDENTIFICATION

The model of the entire system was identified for different leakage magnitudes.

With leakage, the model is essentially a first-order stable system with a delay

$$\frac{h(z)}{u(z)} = \frac{b_h z^{-50}}{\left(1 - a_1 z^{-1}\right)} \tag{20}$$

where  $b_h = 0.0068, a_1 = 0.998$ 

The model is essentially a first order (even though the theoretical model is of third order).

![](_page_4_Figure_6.jpeg)

Fig. 6: Kalman filter results for On-Off Controller: for Flow and Height under various leakage magnitudes

![](_page_4_Figure_8.jpeg)

Fig 7:Kalman filter results for P Controller: for Flow and Height under various leakage magnitudes

![](_page_4_Figure_10.jpeg)

Fig 8:Kalman filter results for PI Controller: for Flow and Height under various leakage magnitudes

![](_page_4_Figure_12.jpeg)

Fig 9:Kalman filter results of PID Controller: for Flow and Height under various leakage magnitudes

The model relating the input, u, to the flow rate,  $q_i$  is given by:

$$\frac{q_i(z)}{u(z)} = \frac{b_q z^{-1}}{(1 - a_z z^{-1})}$$
(21)

The model is essentially a first-order stable system with a delay even though the theoretical model is of a second-order.

The parameter,  $b_q$  characterizes the level sensor.

A discrete-time model of the fluid system relating the height and the input u takes the form

$$y(k) = \varphi^{T}(k)\theta + v(k)$$
(22)

Where  $\theta = \begin{bmatrix} a & b \end{bmatrix}^T$ ,  $\varphi(k) = \begin{bmatrix} y(k-1) & u(k-1) \end{bmatrix}^T$ 

A recursive least-squares estimation of the a and b is

$$\hat{\theta}(k+1) = \hat{\theta}(k) + K(k+1) \left[ y(k+1) - \varphi^T(k) \hat{\theta}(k) \right]$$
(23)

$$K(k+1) = P(k+1)\varphi(k) \left[\varphi^{T}(k)P(k)\varphi(k) + 1\right]^{-1}$$
(24)

$$P(k+1) = \left\lceil I - K(k+1)\varphi^{T}(k) \right\rceil P(k)$$
(25)

where  $\hat{\theta}$  is the estimate of  $\theta$ .

The parameter  $\theta$  gives the complete diagnostic picture of the system. The leakage magnitude is estimated from the estimate  $\hat{a}$  of a.

$$\hat{a} = \begin{cases} 1 & no \ leakage \\ \alpha & leakage : \alpha < 1 \end{cases}$$

The larger  $\alpha$  is , the larger the leakage becomes.

#### IX.CONCLUSION

The proposed incipient leakage fault diagnosis in process control system using various complementary different approaches was evaluated on both simulated and physical systems. The results were encouraging. Leakage diagnosis schemes were robust to measurement noise and system nonlinearity. The Kalman filter and parameter identification schemes based on approximate linearized model were found to be satisfactory. The fusion of these different approaches has the attractive feature of enhancing the incipient fault detection and isolation, thus providing a cost-effective condition-based monitoring scheme.

However, very small leakages were difficult to evaluate as the measurement noise variance was relatively large. Current efforts are under way to address this problem by considering a combination of more powerful model-free techniques, such as wavelets, and an extended Kalman filter scheme to accommodate the effects of the plant nonlinearity.

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