Department of Engineering Technology

HCT-SWC

EEL 2023
Power Generation and Transmission

Course: EEL- 2023 Power Generation and Transmission
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HIGHER COLLEGES OF TECHNOLOGY

LO 4

Develop expressions for Resistance, Inductance and Capacitance of High voltage power Transmission lines and determine the equivalent circuit of a three phase Transmission Lines.

1. List of types used in power transmission lines.
2. Develop the expression for the inductance and capacitance of a simple, single phase, tow wire transmission line composed of solid round conductors.
3. Deduce the expression for the inductance and capacitance of a simple, single-phase composite (stranded) conductor line.
4. Derive the expression for the inductance and capacitance of three-phase lines having symmetrically and asymmetrically spacing and for bundled conductors.
5. Discuss the effect of earth on the capacitance of three-phase transmission lines

A Short Note on 3-phase/1-phase Transmission System

Question: Why 3-phase and not 1-phase transmission system? Why does power transmission use three lines with three different phases? Why not three lines all in the same phase? Does it have to do with the alternators used for generating the power, or is there less loss when the phases of the three lines are all different?

Why not three lines all in the same phase?

- Because then there is no return path.
- Because single phase has no "rotation". Three phase makes it very simple to make a rotating motor with phase sequence determining the direction of rotation. Swap two phases and the direction is reversed.

Is there less loss when the phases of the three lines are all different?

1. Three phase power distribution requires less copper or aluminum for transferring the same amount of power as compared to single phase power.
2. The size of a three phase motor is smaller than that of a single phase motor of the same rating.
3. Three phase motors are self-starting as they can produce a rotating magnetic field. The single phase motor requires a special starting winding as it produces only a pulsating magnetic field.
4. In single phase motors, the power transferred in motors is a function of the instantaneous power which is constantly varying. In three-phase the instantaneous power is constant.
5. Single phase motors are more prone to vibrations. In three phase motors, however, the power transferred is uniform through-out the cycle and hence vibrations are greatly reduced.
6. Three phase motors have better power factor regulation.
7. Three phase enables efficient DC rectification with low ripple.

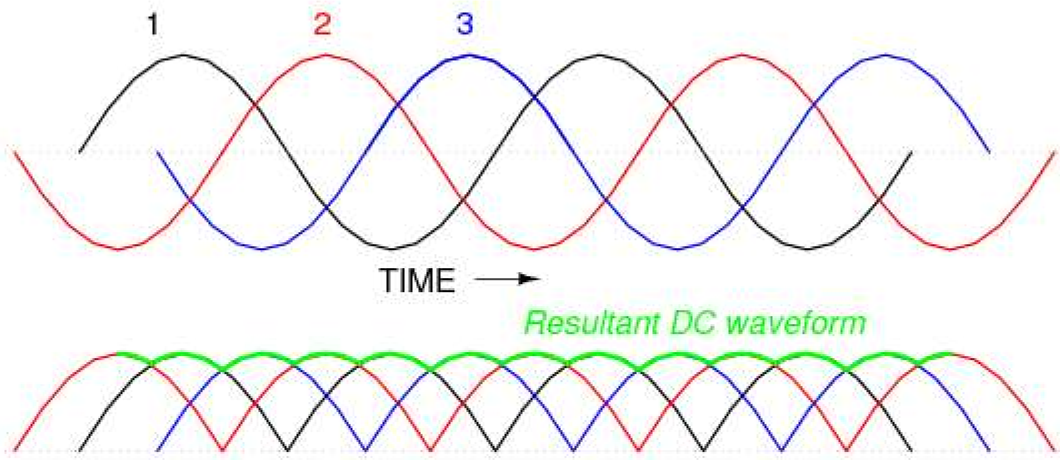
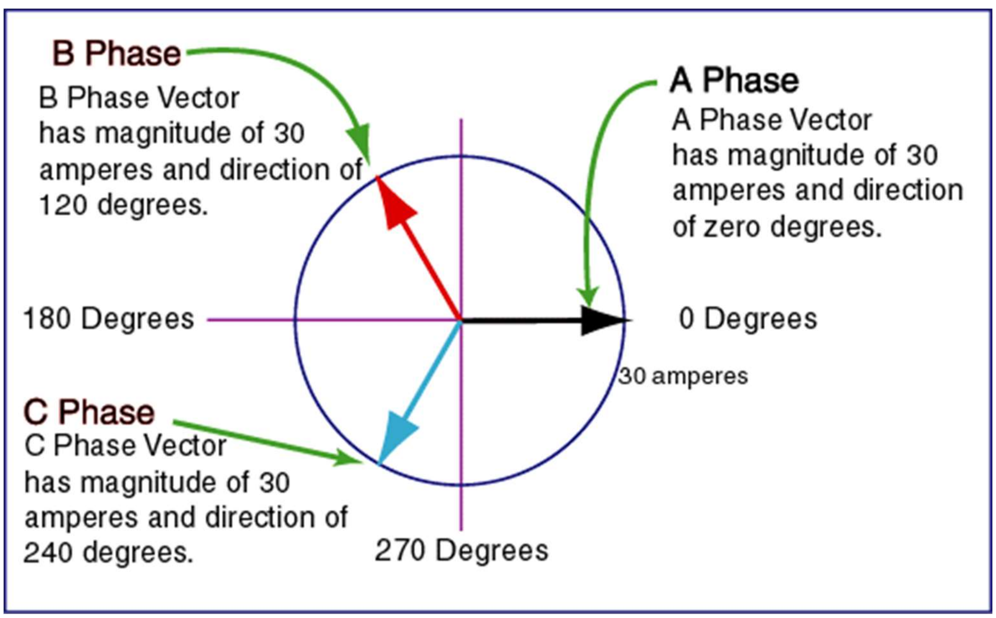


Figure 1. Resultant DC from three-phase rectifier.

8. Generators also benefit by presenting a constant mechanical load through the full revolution, thus maximizing power and also minimizing vibration.



Wildi Industrial Problem 13

(practical level)

Three-phase power lines versus single-phase lines

Three-phase lines that carry power from point A to a point B are much more common than single phase lines. The reason is that three-phase lines can carry more power for a given weight of current-carrying conductors. Furthermore, for a given power to be transmitted, the I^2R line losses are less. Industrial Problems 13 and 14 show why this is so.

Problem 13 examines the properties of a single-phase line. Problem 14 then looks at the properties of a three-phase line having the same length, the same size conductors, and operating at the same line voltage.

The combination of Problems 13 and 14 enables you to see why 3-phase lines are more economical than single phase lines in the transmission of electric power.

Technical data and relevant questions

A single phase source of 600 V is connected to a load by means of two bare copper conductors supported on wooden poles. The load is 150 meters away and the conductors, operating at a temperature of 75 °C, carry a current of 125 A. The conductors have a resistance of 0.8 Ω/km and a weight of 187 kg/km. Calculate:

a) The power delivered by the source [kW]

b) The resistance per conductor [Ω]

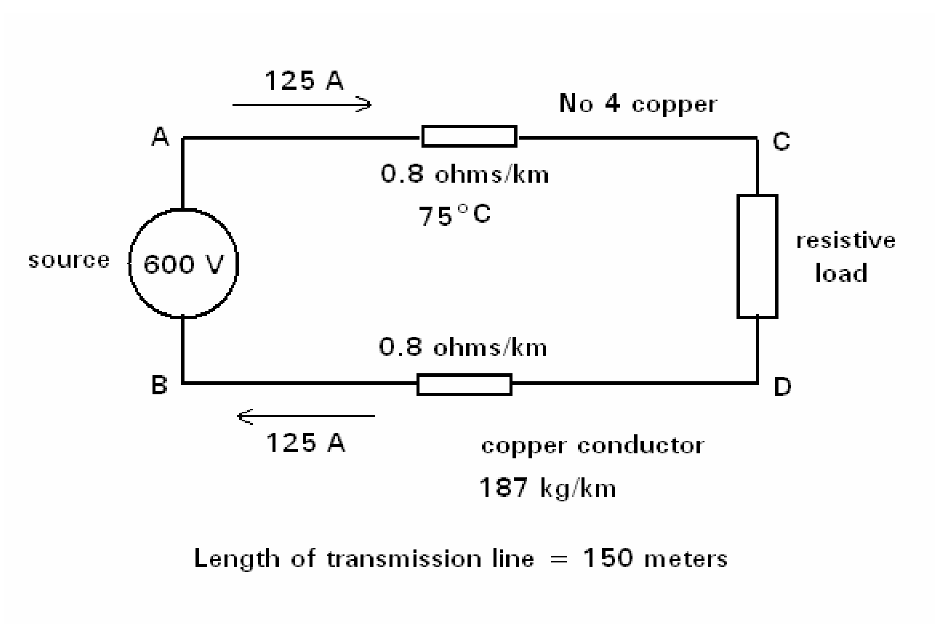


Figure 13

c) The voltage drop per conductor [V]

d) The total power loss in the transmission line [kW]

e) The power loss as a percent of the power delivered by the source [%]

f) The weight of the transmission line [kg]

g) The ratio of power delivered by the source to the weight of transmission line [kW/kg]

h) The voltage at the load [V]

References

Electrical Machines, Drives and Power Systems

Sections 25.2, Table 25D, 25.20, Table AX3

Answers

a) 75 kW b) 0.12 Ω c) 15 V d) 3.75 kW e) 5 % f) 56.1 kg

g) 1.34 kW/kg h) 570 V

Solution to Industrial Problem 13

<u>don't peek</u> until you've done your best to answer the questions by yourself !

Wildi Industrial Problem 14

(practical level)

Three-phase power lines versus single phase lines (continued from Problem 13)

Three-phase lines that carry power from point A to a point B are much more common than single phase lines. The reason is that three-phase lines can carry more power for a given weight of current-carrying conductors. Furthermore, for a given power to be transmitted, the line losses are less. Industrial Problems 13 and 14 show why this is so.

Problem 13 examined the properties of a single-phase line. Problem 14 now looks at the properties of a three-phase line having the same length, the same size conductors, and operating at the same line voltage.

The combination of Problems 13 and 14 enables us to see why 3-phase lines are more economical than single phase lines.

Technical data and relevant questions

A three-phase source of 600 V is connected to a load by means of three bare copper conductors supported on wooden poles. The resistive load is 150 meters away and the conductors, operating at a temperature of 75 °C, carry a current of 125 A. The conductors have a resistance of 0.8 Ω/km and a weight of 187 kg/km. Calculate:

a) The power delivered by the source [kW]

b) The resistance per conductor [Ω]

c) The voltage drop per conductor [V]

d) The total power loss in the transmission line [kW]

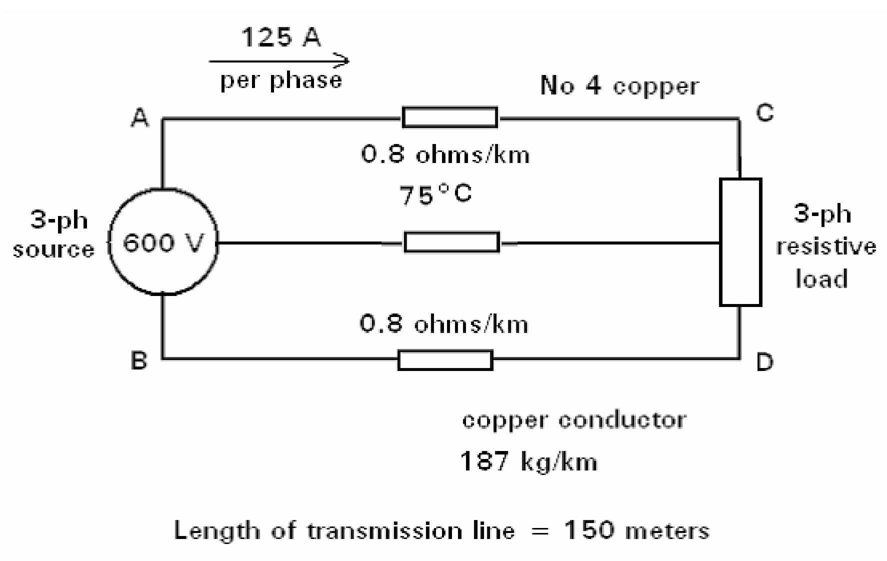


Figure 14

e) The power loss as a percent of the power delivered by the source [%]

f) The weight of the transmission line [kg]

g) The ratio of weight of transmission line/ power delivered [kg/kW]

h) Power delivered to the load [kW]

i) The voltage at the load [V]

References

Electrical Machines, Drives and Power Systems

Sections 8.11, 25.2, 25.20, Table 25D, Table AX3

Answers

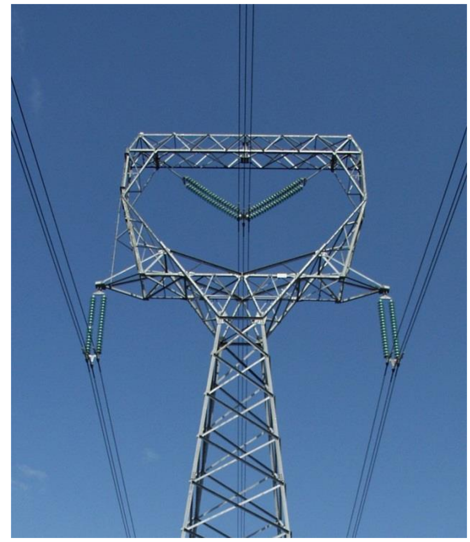
a) 129.9 kW   b) 0.12 Ω   c) 15 V   d) 5.63 kW   e) 4.33 %   f) 84.2 kg

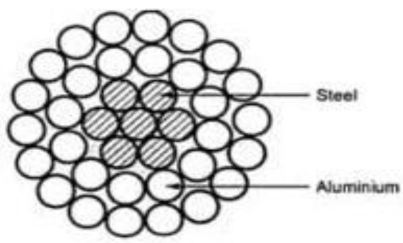
g) 1.54 kW/kg   h) 574 V

Solution to Problem 14

don't peek until you've done your best to answer the questions by yourself !

EEL-2023
LO4- Resistance, Inductance and Capacitance of high-voltage Power Transmission Lines



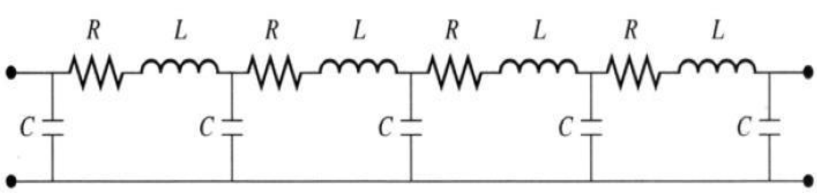


Electrical Characteristics

•Transmission lines are characterized by a **series resistance**, **inductance**, and **shunt capacitance** per unit length.

•These values determine the power-carrying capacity of the transmission line and the voltage drop across it at full load.

The DC resistance of a conductor is expressed in terms of resistively, length and cross sectional area as follows:



$$R_{DC} = \frac{\rho l}{A}$$

Cable resistance

•The resistively increases linearly with temperature over normal range of temperatures.

•If the resistively at one temperature and material temperature constant are known, the resistively at another temperature can be found by

$$\rho_{T2} = \frac{M + T_2}{M + T_1} \rho_{T1}$$

Material	Resistivity at 20°C [Ω·m]	Temperature constant [°C]
Annealed copper	$1.72 \cdot 10^{-8}$	234.5
Hard-drawn copper	$1.77 \cdot 10^{-8}$	241.5
Aluminum	$2.83 \cdot 10^{-8}$	228.1
Iron	$10.00 \cdot 10^{-8}$	180.0
Silver	$1.59 \cdot 10^{-8}$	243.0

Cable Resistance

•AC resistance of a conductor is always higher than its DC resistance due to the skin effect forcing more current flow near the outer surface of the conductor. The higher the frequency of current, the more noticeable skin effect would be.

•Wire manufacturers usually supply tables of resistance per unit length at common frequencies (50 and 60 Hz). Therefore, the resistance can be determined from such tables.

Aluminum Conductor Steel Reinforced

Electrical Properties

	SIZE & STRANDING		RESISTANCE				60 HZ REACTANCE 1 FOOT EQUIVALENT SPACING			
				AC-60-HZ(Ohms/1000 Ft.)				Inductive (Ohms/1000 Ft.)		
CODE WORD	AWG or kcmil	Aluminum/ Steel	DC (Ohms/1000 Ft.) @20°	@25° C	@50° C	@75° C	Capacitive (Megohms-1000 Ft.)	@25° C	@50° C	@75° C

								Inductive (Ohms/1000 Ft.)	GMR (Ft.)
WAXWING	266.8	18/1	0.0644	0.0657	0.0723	0.0788	0.576	0.0934	0.0197
PARTRIDGE	266.8	26/7	0.0637	0.0652	0.0714	0.0778	0.565	0.0881	0.0217
MERLIN	336.4	18/1	0.0510	0.0523	0.0574	0.0625	0.560	0.0877	0.0221
LINNET	336.4	26/7	0.0506	0.0517	0.0568	0.0619	0.549	0.0854	0.0244
ORIOLE	336.4	30/7	0.0502	0.0513	0.0563	0.0614	0.544	0.0843	0.0255
CHICKADEE	397.5	18/1	0.0432	0.0443	0.0487	0.0528	0.544	0.0856	0.0240
IBIS	397.5	26/7	0.0420	0.0430	0.0401	0.0525	0.539	0.0035	0.0265
LARK	397.5	30/7	0.0425	0.0434	0.0477	0.0519	0.533	0.0824	0.0277
PELICAN	477.0	18/1.	0.0360	0.0369	0.0405	0.0441	0.528	0.0835	0.0263
FLICKER	477.0	24/7	0.0358	0.0367	0.0403	0.0439	0.524	0.0818	0.0283
HAWK	477.0	26/7	0.0357	0.0366	0.0402	0.0438	0.522	0.0814	0.0290
HEN	477.0	30/7	0.0354	0.0362	0.0389	0.0434	0.517	0.0803	0.0304
OSPREY	556.5	18/1	0.0309	0.0318	0.0348	0.0379	0.518	0.0818	0.0284
PARAKEET	556.5	24/7	0.0307	0.0314	0.0347	0.0377	0.512	0.0801	0.0306

ACSR Conductor Table Data

TABLE A8.1. BARE ALUMINUM CONDUCTORS, STEEL REINFORCED (ACSR) ELECTRICAL PROPERTIES OF MULTILAYER SIZES *(Cont'd)*

				Resistance					Phase-to-Neutral, 60 Hz Reactance at One ft Spacing	
					ac-60 Hz					
Code Word	Size (kcmil)	Stranding Al./St.	Number of Aluminum Layers	dc 20°C (Ohms/ Mile)	25°C (Ohms/ Mile)	50°C (Ohms/ Mile)	75°C (Ohms/ Mile)	GMR (ft)	Inductive Ohms/ Mile X_a	Capacitive Megohm-Miles X'_a
Flicker	477	24/7	2	0.1889	0.194	0.213	0.232	0.0283	0.432	0.0992
Hawk	477	26/7	2	0.1883	0.193	0.212	0.231	0.0290	0.430	0.0988
Hen	477	30/7	2	0.1869	0.191	0.210	0.229	0.0304	0.424	0.0980
Osprey	556.5	18/1	2	0.1629	0.168	0.184	0.200	0.0284	0.432	0.0981
Parakeet	556.5	24/7	2	0.1620	0.166	0.183	0.199	0.0306	0.423	0.0969
Dove	556.5	26/7	2	0.1613	0.166	0.182	0.198	0.0313	0.420	0.0965
Eagle	556.5	30/7	2	0.1602	0.164	0.180	0.196	0.0328	0.415	0.0957
Peacock	605	24/7	2	0.1490	0.153	0.168	0.183	0.0319	0.418	0.0957
Squab	605	26/7	2	0.1485	0.153	0.167	0.182	0.0327	0.415	0.0953

Inductive

0.415

and Capacitive Reactance for 1-foot Spacing

Layers					
2	0.1889	0.194	0.213	0.232	0
2	0.1883	0.193	0.212	0.231	0
2	0.1869	0.191	0.210	0.229	0
2	0.1629	0.168	0.184	0.200	0
2	0.1620	0.166	0.183	0.199	0
2	0.1613	0.166	0.182	0.198	0
2	0.1602	0.164	0.180	0.196	0
2	0.1490	0.153	0.168	0.183	0
2	0.1485	0.153	0.167	0.182	0

Geometric Mean Radius

Line inductance

$$L = \frac{\lambda}{I}$$

$$\boxed{l = \frac{\mu}{\pi}\left(\frac{1}{4} + \ln\frac{D}{r}\right) \quad [H/m]}$$

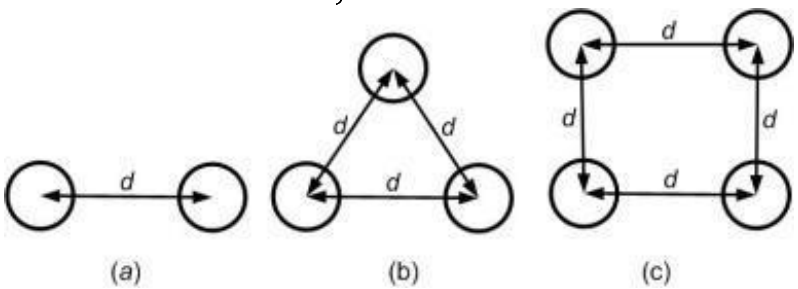
Remarks on line inductance

•The greater the spacing between the phases of a transmission line, the greater the inductance of the line.

–Since the phases of a high-voltage overhead transmission line must be spaced further apart to ensure proper insulation, a high-voltage line will have a higher inductance than a low-voltage line.

–Since the spacing between lines in buried cables is very small, series inductance of cables is much smaller than the inductance of overhead lines

•**The greater the radius of the conductors in a transmission line, the lower the inductance of the line.** In practical transmission lines, instead of using heavy and inflexible conductors of large radii, two and more conductors are bundled together to approximate a large diameter conductor, and reduce corona loss.



Inductance of 3-phase transmission line

Shunt capacitance

•Since a voltage *V* is applied to a pair of conductors separated by a dielectric (air), charges q of equal magnitude but opposite sign will accumulate on the conductors. Capacitance C between the two conductors is defined by $C=q/V$

•The capacitance of a single-phase transmission line is given by (see derivation in the book): ($\varepsilon = 8.85 \times 10^{-12}$ F/m)

Capacitance of 3-phase transmission line

Remarks on line capacitance

1.The greater the spacing between the phases of a transmission line, the lower the capacitance of the line.

–Since the phases of a high-voltage overhead transmission line must be spaced further apart to ensure proper insulation, a high-voltage line will have a lower capacitance than a low-voltage line.

–Since the spacing between lines in buried cables is very small, shunt capacitance of cables is much larger than the capacitance of overhead lines.

2. The greater the radius of the conductors in a transmission line, the higher the capacitance of the line. Therefore, bundling increases the capacitance.

Use of Tables

Inductive reactance (in Ω/mi):

- The first term is defined as X_a: the inductive reactance at 1-foot spacing

- The second term is defined as X_d : the inductive reactance spacing factor
- Both of the above components are already calculated in Table A.3 & A.4

Capacitive reactance (in MΩ.mi):

–The first term is defined as X'_a: the capacitive reactance at 1-foot spacing

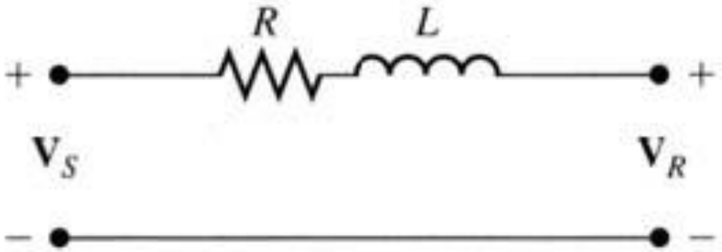
–The second term is defined as X'_d : the capacitive reactance spacing factor

–Both of the above components are already calculated in Table A.3 & A.5

Short line model

•Overhead transmission lines shorter than 50 miles can be modeled as a series resistance and inductance, since the shunt capacitance can be neglected over short distances.

•The total series resistance and series reactance can be calculated as



$$R = rd$$

$$X = xd$$

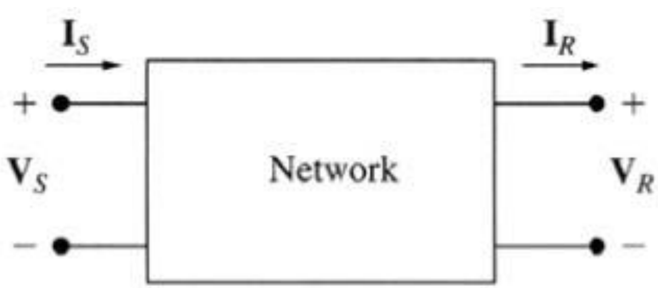
where *r*, *x* are resistance and reactance per unit length and *d* is the length of the transmission line.

- Two-port network model:

$$\boxed{V_R = V_S - RI - jX_L I}$$

$$\boxed{\begin{aligned} V_S &= AV_R + BI_R \\ I_S &= CV_R + DI_R \end{aligned}}$$

$$I_S = I_R$$



$$A = 1$$

$$B = Z$$

$$C = 0$$

$$D = 1$$

The equation is similar to that of a synchronous generator and transformer (w/o shunt impedance)

Short line

Voltage Regulation: $$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \cdot 100\%$$

1. If lagging (inductive) loads are added at the end of a line, the voltage at the end of the transmission line **decreases significantly** – large positive VR.
2. If unity-PF (resistive) loads are added at the end of a line, the voltage at the end of the transmission line **decreases slightly** – small positive VR.
3. If leading (capacitive) loads are added at the end of a line, the voltage at the end of the transmission line **increases** – negative VR.

Short line – simplified

- If the resistance of the line is ignored, then

$$I\cos\theta = \frac{V_S \sin\delta}{X_L}$$

$$\boxed{P = \frac{3V_S V_R \sin\delta}{X_L}}$$

- Therefore, the power flow through a transmission line depends on the angle between the input and output voltages.
- Maximum power flow occurs when $\delta = 90^o$.

$$\boxed{P_{\max} = \frac{3V_S V_R}{X_L}}$$

- Notes:
  - The maximum power handling capability of a transmission line is a function of the **square of its voltage.**
  - The maximum power handling capability of a transmission line is inversely proportional to its series reactance (some very long lines include series capacitors to reduce the total series reactance).
  - The angle δ controls the power flow through the line. Hence, it is possible to control power flow by placing a phase-shifting transformer.

Line Characteristics

- To prevents excessive voltage variations in a power system, the ratio of the magnitude of the receiving end voltage to the magnitude of the ending end voltage is generally within

$$0.95 \leq V_S/V_R \leq 1.05$$

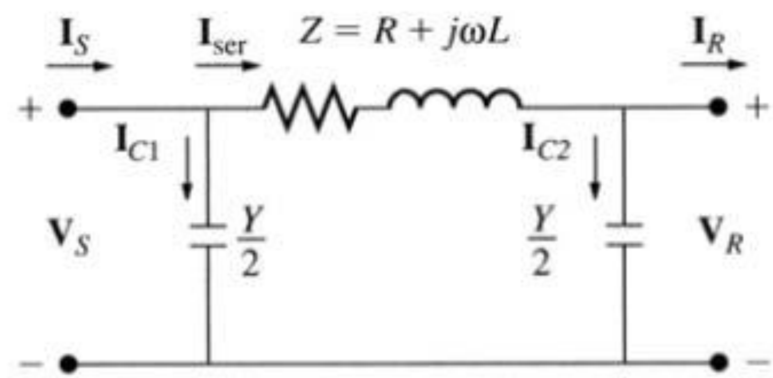
- The angle δ in a transmission line should typically be $\leq 30^o$ to ensure that the power flow in the transmission line is well below the static stability limit.
- Any of these limits can be more or less important in different circumstances.
  - In short lines, where series reactance *X* is relatively small, the **resistive heating** usually limits the power that the line can supply.
  - In longer lines operating at lagging power factors, the **voltage drop** across the line is usually the limiting factor.
  - In longer lines operating at leading power factors, the **maximum angle δ** can be the limiting f actor.

Example) A line with reactance X and negligible resistance supplies a pure resistive load from a fixed source V_S. Determine the maximum power transfer, and the load voltage V_R at which this occurs. *(Hint: recall the maximum power transfer theorem from your basic circuits course)*

Medium Line (50-150 mi)

•the shunt admittance must be included in calculations. However, the total admittance is usually modeled (π model) as two capacitors of equal values (each corresponding to a half of total admittance) placed at the sending and receiving ends.

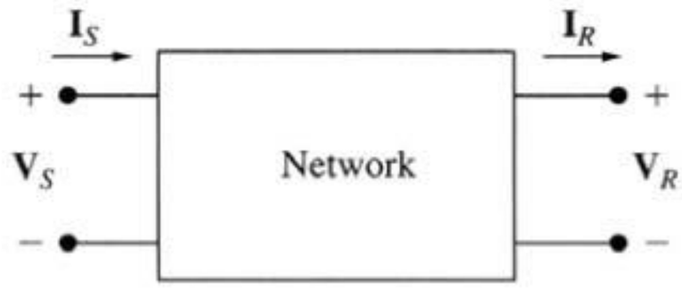
•The total series resistance and series reactance are calculated as before. Similarly, the total shunt admittance is given by



•where y is the shunt admittance per unit length and d is the length of the transmission line.

$$Y = yd$$

•Two-port network:



$$\boxed{\begin{aligned} V_S &= AV_R + BI_R \\ I_S &= CV_R + DI_R \end{aligned}}$$

$$A = \frac{ZY}{2} + 1$$

$$B = Z$$

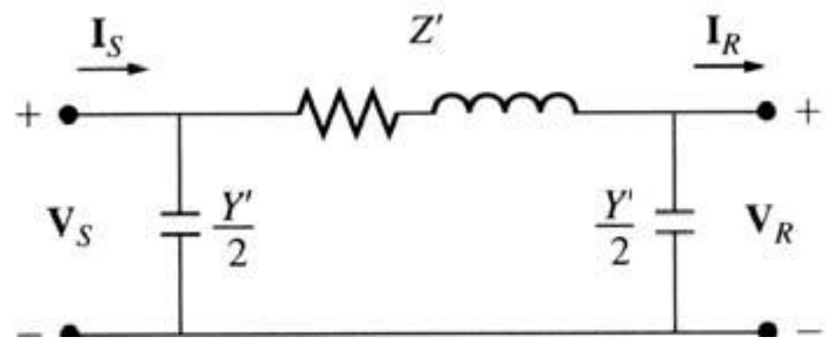
$$C = Y\left(\frac{ZY}{4} + 1\right)$$

$$D = \frac{ZY}{2} + 1$$

Long Lines (> 150 mi)

•For long lines, both the shunt capacitance and the series impedance must be treated as distributed quantities. The voltages and currents on the line are found by solving differential equations of the line.

•However, it is possible to model a long transmission line as a π model with a *modified* series impedance *Z'* and a *modified* shunt admittance *Y'* and to perform calculations on that model using ABCD constants.



where the propagation constant is defined by

$$\gamma = \sqrt{yz}$$

HVDC Transmission

Because of the large fixed cost necessary to convert ac to dc and then back to ac, dc transmission is only practical in specialized applications

–long distance overhead power transfer (> 400 miles)
–long underwater cable power transfer
–providing an asynchronous means of joining different power systems.

Physical Characteristics – underground cables

•Cable lines are designed to be placed underground or under water. The conductors are insulated from one another and surrounded by protective sheath.

•Cable lines are more expensive and harder to maintain. They also have capacitance problem – not suitable for long distance.

TRANSMISSION LINES

The electric parameters of transmission lines (i.e. resistance, inductance, and capacitance) can be determined from the specifications for the conductors, and from the geometric arrangements of the conductors.

Transmission Line Resistance

Resistance to d.c. current is given by

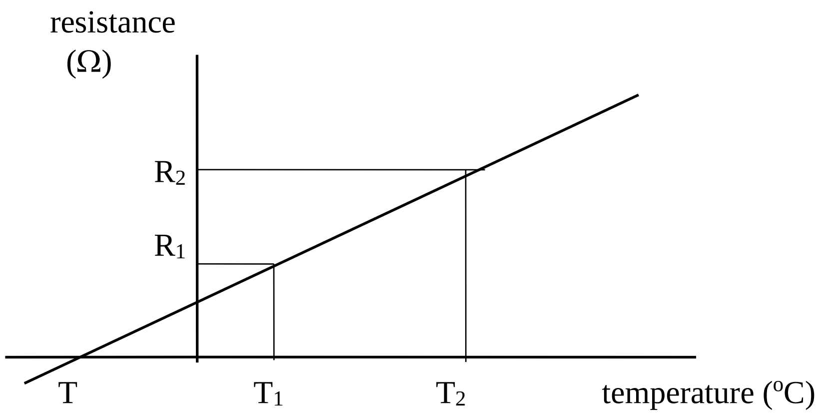
$$R_{dc} = \rho \frac{\ell}{A}$$

where ρ is the resistivity at 20^o C
ℓ is the length of the conductor
A is the cross sectional area of the conductor

Because of skin effect, the d.c. resistance is different from ac resistance. The ac resistance is referred to as effective resistance, and is found from power loss in the conductor

$$R = \frac{\text{power loss}}{I^2}$$

The variation of resistance with temperature is linear over the normal temperature range



Graph of Resistance vs Temperature

$$\frac{(R_1 - 0)}{(T_1 - T)} = \frac{(R_2 - 0)}{(T_2 - T)}$$

$$R_2 = \frac{T_2 - T}{T_1 - T} R_1$$

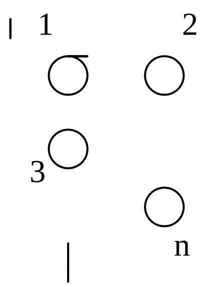
Transmission Line Inductive Reactance

Inductance of transmission lines is calculated per phase. It consists of self-inductance of the phase conductor and mutual inductance between the conductors. It is given by:

$$L = 2 \times 10^{-7} \ln \frac{GMD}{GMR} \qquad \text{[H/m]}$$

Where:
GMR is the geometric mean radius (available from manufacturer's tables)
GMD is the geometric mean distance (must be calculated for each line configuration)

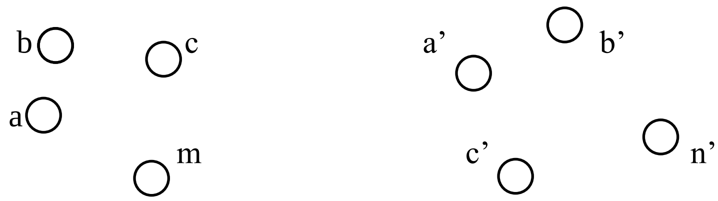
Geometric Mean Radius: There are magnetic flux lines not only outside of the conductor, but also inside. GMR is a hypothetical radius that replaces the actual conductor with a hollow conductor of radius equal to GMR such that the self-inductance of the inductor remains the same. If each phase consists of several conductors, the GMR is given by;



$$GMR = \sqrt[n^2]{(d_{11} d_{12} d_{13} \ldots . d_{1n}).(d_{21}.d_{22} \ldots . d_{2n}) \ldots \ldots (d_{n1}.d_{n2} \ldots . d_{nn})}$$

where $d_{11}=GMR_1$
$d_{22}=GMR_2$
.
.
.
$d_{nn}=GMR_n$

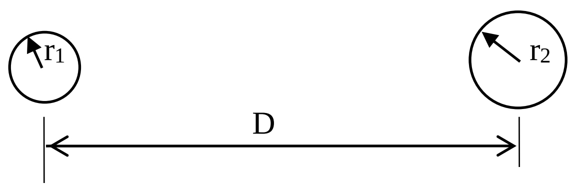
Note: for a solid conductor, GMR = $r.e^{-1/4}$, where r is the radius of the conductor.
Geometric Mean Distance:
GMD replaces the actual arrangement of conductors by a hypothetical mean distance such that the mutual inductance of the arrangement remains the same.



$$GMD = \sqrt[mn']{(D_{aa'}D_{ab'}\ldots D_{an'}).(D_{ba'}D_{bb'}\ldots D_{bn'}).....(D_{ma'}D_{mb'}.....D_{mn'})}$$

Where $D_{aa'}$ is the distance between conductors “a” and “a’” etc.

Inductance between Two Single Phase Conductors:



$$L_1 = 2\times10^{-7}\times\ln\frac{D}{r_1'} \qquad L_2 = 2\times10^{-7}\times\ln\frac{D}{r_2'}$$

Where r1’ is GMR of conductor 1
r_2’ is GMR of conductor 2
D is the GMD between the conductors

The total inductance of the line is then

$$L_T = L_1 + L_2 = 2\times10^{-7}\times\left[\ln\frac{D}{r_1'}+\ln\frac{D}{r_2'}\right] = 2\times10^{-7}\times\ln\frac{D^2}{r_1'r_2'} = 2\times10^{-7}\times2\times\frac{1}{2}\times\ln\frac{D^2}{r_1'r_2'}$$

$$L_T = 4\times10^{-7}\times\ln\left[\frac{D^2}{r_1'r_2'}\right]^{1/2} = 4\times10^{-7}\times\ln\frac{D}{\sqrt{r_1'r_2'}}$$

If, r1 = r_2, then

$$L_T = 4\times10^{-7}\times\ln\frac{D}{r_1'}$$

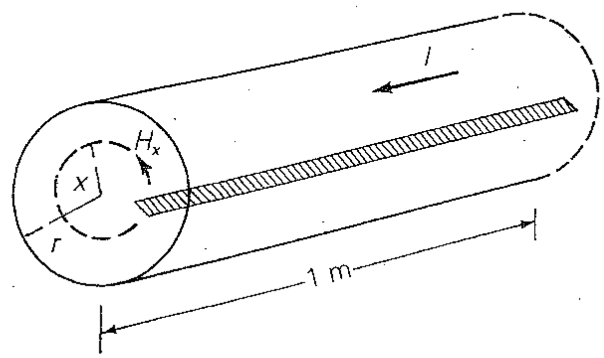
Inductance of Transmission Lines

The Inductance of a Single, Solid Cylindrical Conductor:

This is made up of two inductances:

A- Internal Inductance which is given by:

$$L_{int} = \frac{\lambda_{int}}{I} = \frac{\mu_0}{8\pi} = \frac{1}{2} \times 10^{-7} \quad \mathrm{H/m}$$



B- External Inductance whichis given by:

$$L_{12} = \frac{\lambda_{12}}{I} = 2 \times 10^{-7} \ln\left(\frac{D_2}{D_1}\right) \quad \mathrm{H/m}$$

The total Inductance of the conductor is given by:

$$L_P = \frac{\lambda_P}{I} = 2 \times 10^{-7} \ln\left(\frac{D}{r'}\right) \quad \mathrm{H/m}$$

Where:

$$r' = e^{-1/4} r = 0.7788r$$

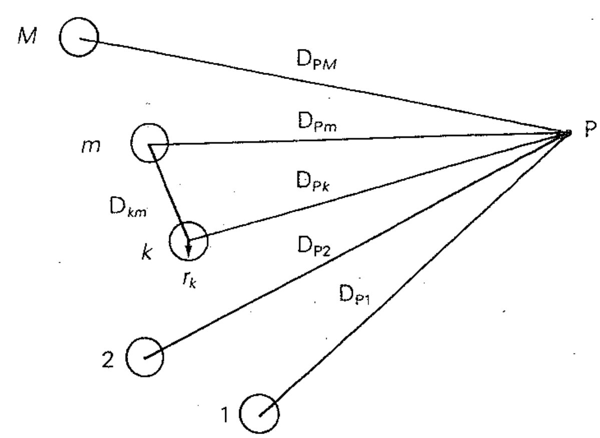
The Inductance of an Array of Solid Conductors:

The total flux linking conductor m in an array of M conductors carrying currents $I_1, I_2, \ldots I_M$ where:

$I_1 + I_2 + \ldots\ldots + I_M = 0$

Is:

$$\lambda_k = 2 \times 10^{-7} \sum_{m=1}^{M} I_m \ln \frac{1}{D_{km}} \quad \mathrm{Wb\text{-}t/m}$$



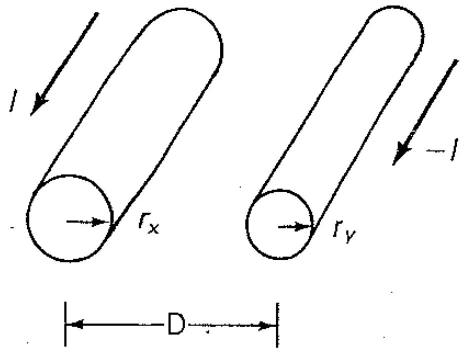
This formula can be used to develop the expressions for the inductance of :

a- Single-Phase Two-Wire line

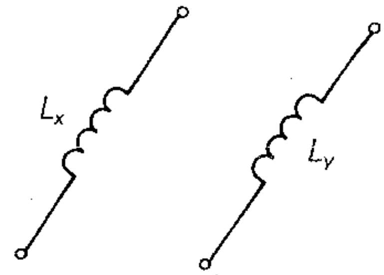
b- Three-Phase, Three-wire Line with Equal Phase Spacing

1- *The Inductance of Single-Phase Two-Wire line:*

Consider now the two-wire single-phase line shown in the following figure.



(a) Geometry



b- Inductances

The total inductance of the single-phase circuit, also called loop inductance, is:

$$L = L_x + L_y = 2 \times 10^{-7} \left(\ln \frac{D}{r'_x} + \ln \frac{D}{r'_y} \right)$$

$$= 2 \times 10^{-7} \ln \frac{D^2}{r'_x r'_y}$$

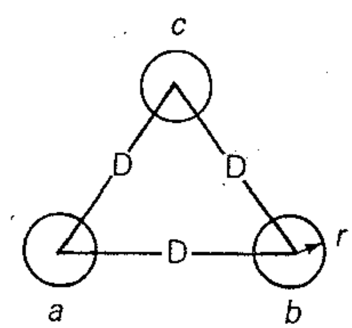
Or,

$$L_{Total} = 4 \times 10^{-7} \ln \frac{D}{\sqrt{r'_x r'_y}} \quad \text{H/m per circuit}$$

2- *The Inductance of Three-Phase, Three-wire Line with Equal Phase Spacing*

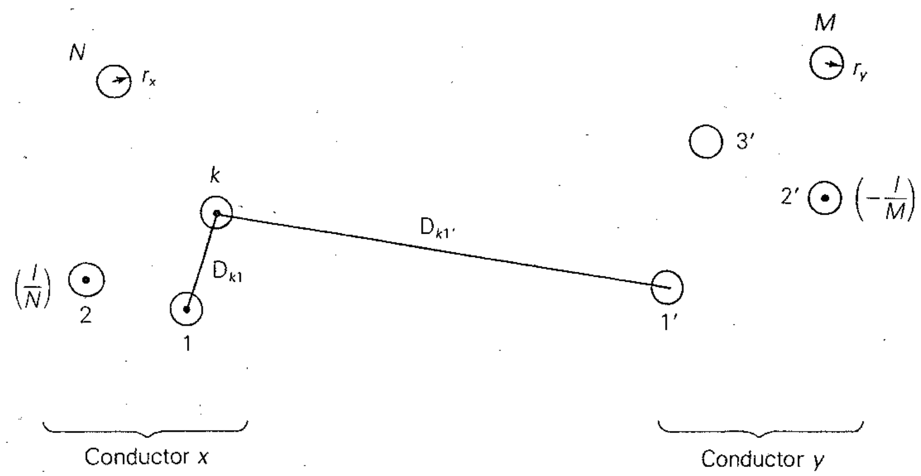
The inductance of phase a can be shown to be given by:

$$L_a = \frac{\lambda_a}{I_a} = 2 \times 10^{-7} \ln \frac{D}{r'} \quad \text{H/m per phase}$$



The Inductance of Multi-Filament Single-Phase Line:

The inductance of a single-phase, transmission line consisting of two multi-filament (composite) conductors x and y is given by:



$$L_x = 2 \times 10^{-7} \ln \frac{D_{xy}}{D_{xx}} \quad \text{H/m per conductor}$$

$$D_{xy} = \sqrt[MN]{\prod_{k=1}^{N} \prod_{m=1'}^{M} D_{km}}$$

$$D_{xx} = \sqrt[N^2]{\prod_{k=1}^{N} \prod_{m=1}^{N} D_{km}}$$

Where:

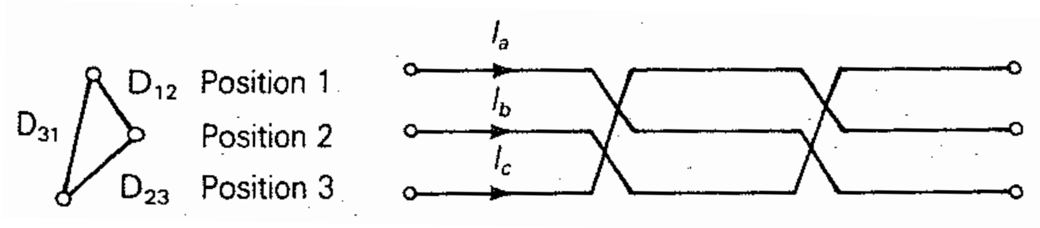
D_{xy} is called the Geometric Mena Distance (GMD) between conductors x and y.
D_{xx} is called the Geometric Mean Radius (GMR) of conductor x.

Similar equations can be used to find Ly, D_{yx} and D_{yy}.

The total inductance, L, for the single-phase circuit is:

$$L = L_x + L_y \quad \text{H/m per circuit}$$

The Inductance of a Three-Phase Three-Wire Line with Unequal Spacing:



The average inductance of phase a is given by

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_S} \quad \text{H/m}$$

$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$

Where:
D_{eq} is the GMD between the phases, and
D_s is the GMR for stranded conductor or **r'** for solid conductors.

The Inductance of Bundled Conductors:

It is common for EHV lines to have bundled phase conductors, each phase constitute 2, 3 or 4 sub-conductors. The inductance equation that was developed earlier for transposed conductors is.

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_S} \quad \text{H/m}$$

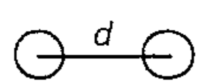
Can still be used, but now Ds is replaced by the GMR of the bundle, which can be considered as a composite conductor for which GMR is given by:

$$D_{xx} = \sqrt[N^2]{\prod_{k=1}^{N} \prod_{m=1}^{N} D_{km}}$$

DSL (DLS is the Geometric Mean Distance or D_{eq} in the above equation) can be found for 2,3 & 4 sub-conductors from:

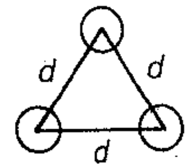
Two-Conductor Bundle:

$$D_{SL} = \sqrt[4]{(D_S \times d)^2} = \sqrt{D_S d}$$



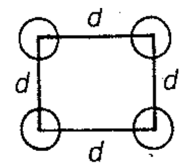
Three-Conductor Bundle:

$$D_{SL} = \sqrt[9]{(D_S \times d \times d)^3} = \sqrt[3]{D_S d^2}$$



Four-Conductor Bundle:

$$D_{SL} = \sqrt[16]{(D_S \times d \times d \times d\sqrt{2})^4} = 1.091\sqrt[4]{D_S d^3}$$



And the inductance of phase a is:

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_S} \quad \text{H/m}$$

Note that in this equation:

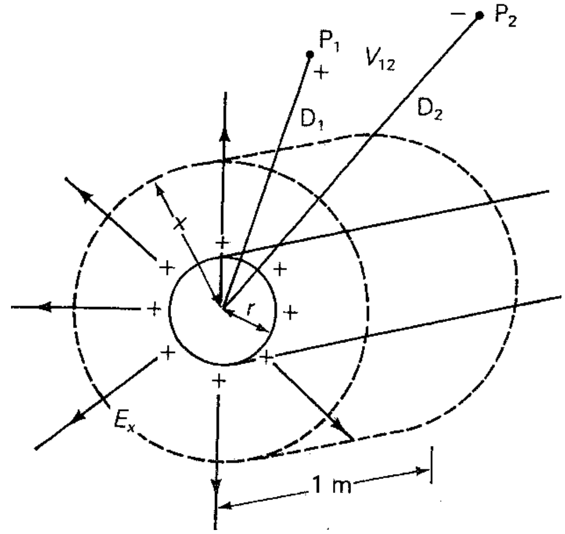
1- D_S is r' of each of the conductors in the bundle

2- D_{SL} is the GMR of the bundle, which means that the bundle is replaced by an equivalent conductor having GMR = D_{SL}.

Capacitance of Transmission Lines

Electric Field & Voltage for Solid Cylindrical Conductor:

Assume a solid cylindrical conductor having radius r and having a charge q coulombs/meter, as shown in the following figure.



The electric filed inside the conductor can be shown to be zero and that outside the conductor E_x at distance x is given by:

$$E_x = \frac{q}{2\pi\varepsilon x} \quad \text{V/m}$$

where, for a conductor in free space, $\varepsilon = \varepsilon_0 = 8.854 \times 10^{-12}$ F/m.

The potential difference between two concentric cylinders at distances D_1 and D_2 from the conductor center is:

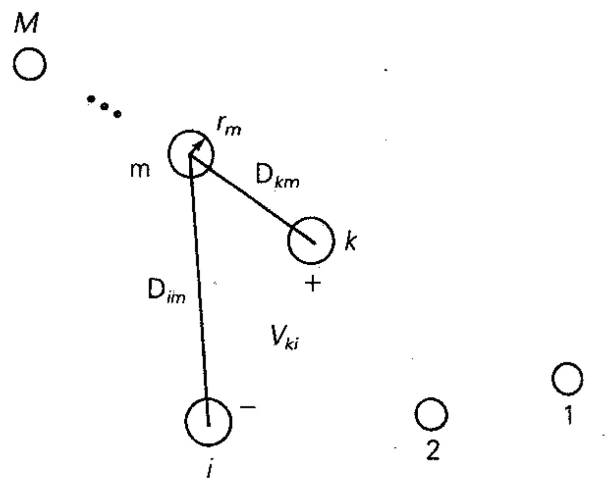
$$V_{12} = \int_{D_1}^{D_2} E_x \, dx$$

And this will become:

$$V_{12} = \int_{D_1}^{D_2} \frac{q}{2\pi\varepsilon x} \, dx = \frac{q}{2\pi\varepsilon} \ln \frac{D_2}{D_1} \quad \text{volts}$$

The Voltage Between Two Conductors in an Array of Solid Conductors:

Consider the array of conductors shown in the following figure.



The voltage V_{ki} between conductors k and I due to all other conductors is:

$$V_{ki} = \frac{1}{2\pi\varepsilon}\sum_{m=1}^{M} q_m \ln\frac{D_{im}}{D_{km}} \quad \text{volts}$$

Were $D_{mm} = r_m$ when k = m or I = m.

Note:
Because of the fact that E inside the conductor is zero, $D_{mm} = r_m$ and NOT r'_m as in inductance.

1- Capacitance of a Single-Phase Two-Wire Line:

Consider the single-phase two-wire line shown in the figure in Part 1 of the inductance equations. Assume conductor x has a uniform charge q C/m and conductor y has a negative charge –q C/m.
Applying the voltage equation of the previous section, we find the capacitance C_{xy} between the two conductors to be:

$$C_{xy} = \frac{q}{V_{xy}} = \frac{\pi\varepsilon}{\ln\left(\frac{D}{\sqrt{r_x r_y}}\right)} \quad \text{F/m line-to-line}$$

and if $r_x = r_y = r$,

$$C_{xy} = \frac{\pi\varepsilon}{\ln(D/r)} \quad \text{F/m line-to-line}$$

And the capacitance from either line to the grounded neutral is:

$$C_n = C_{xn} = C_{yn} = \frac{q}{V_{xn}} = 2C_{xy}$$
$$= \frac{2\pi\varepsilon}{\ln(D/r)} \quad \text{F/m line-to-neutral}$$

2- Capacitance of a Three-Phase Three-Wire Line with Equal Spacing:

For the three-phase line with equal phase spacing, as shown in the figure in Part 2 of the inductance equations, the capacitance-to-neutral per line length C_{an} is:

$$C_{an} = \frac{q_a}{V_{an}} = \frac{2\pi\varepsilon}{\ln(D/r)} \quad \text{F/m line-to-neutral}$$

3- Capacitance of Three-Phase Three-Wire Line with Unequal Spacing (Transposed Line):

As in the case of inductance of three-phase line with unequal phase spacing (Part 3), the inductance is:

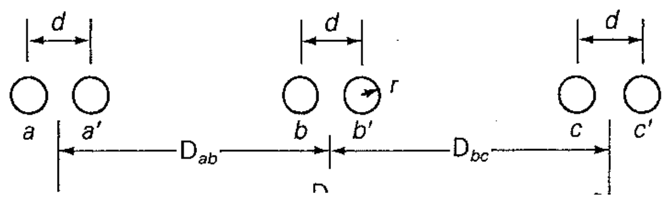
$$C_{an} = \frac{2\pi\varepsilon}{\ln(D_{eq}/r)} \quad \text{F/m}$$

where

$$D_{eq} = \sqrt[3]{D_{ab}D_{bc}D_{ac}}$$

4- Capacitance of Bundled Three-Phase Line:

For a bundled conductor having two coil sides, as shown in the following figure,



the capacitance with respect to ground of conductor a is:

$$C_{an} = \frac{2\pi\varepsilon}{\ln(D_{eq}/D_{SC})} \quad \text{F/m}$$

where

$$D_{SC} = \sqrt{rd} \quad \text{for a two-conductor bundle}$$

Similarly,

$$D_{SC} = \sqrt[3]{rd^2} \quad \text{for a three-conductor bundle}$$

$$D_{SC} = 1.091\sqrt[4]{rd^3} \quad \text{for a four-conductor bundle}$$

Note the similarities between these equations and those developed for inductance.

In all the above, the Capacitive Reactance of the line X_c is given by:

$$Xc = \frac{1}{2\pi fC}$$

Solved Examples for LO4

Example: A solid cylindrical aluminum conductor 25 km long has an area of 336,400 circular miles. The resistivity of aluminum at 20 C is 2.8×10^{-8} Ohms-meter. 1 square centimeter × 197= 1 circular mils. Obtain the conductor resistance at:

a) 20 C
b) 50 C

Example 2: A 3-phase transmission line is designed to deliver 190.5 MVA at 220 kV over a distance of 63 km. The total transmission loss is not to exceed 2.5 percent of the rated line MVA. If the resistivity of the conductor material at 20 C is 2.8×10^{-8} Ohms-meter, determine the required conductor diameter and the conductor size in circular miles.

The total transmission line loss is

$$P_L = \frac{2.5}{100}(190.5) = 4.7625 \ \text{MW}$$

$$|I| = \frac{S}{\sqrt{3}V_L} = \frac{(190.5)10^3}{\sqrt{3}(220)} = 500 \ \text{A}$$

From $P_L = 3R|I|^2$, the line resistance per phase is

$$R = \frac{4.7625 \times 10^6}{3(500)^2} = 6.35 \ \Omega$$

The conductor cross sectional area is

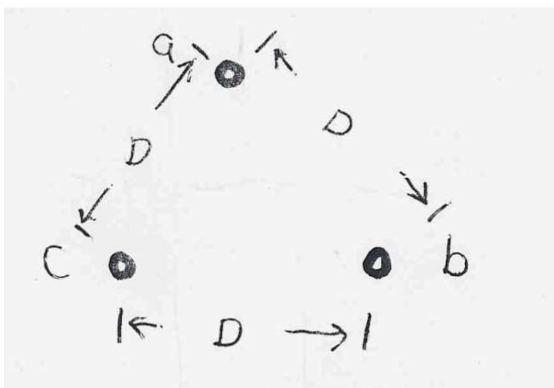
$$A = \frac{(2.84 \times 10^{-8})(63 \times 10^3)}{6.35} = 2.81764 \times 10^{-4} \ \text{m}^2$$

Therefore

d=1.894 cm= 0.7456 inches= 556000 cmil

Line Inductance Example

Calculate the reactance for a balanced 3ϕ, 60Hz transmission line with a conductor geometry of an equilateral triangle with D = 5m, r = 1.24cm (Rook conductor) and a length of 5 miles.



Since system is assumed balanced

$$i_a = -i_b - i_c$$

$$\lambda_a = \frac{\mu_0}{2\pi}\left[i_a \ln(\frac{1}{r'}) + i_b \ln(\frac{1}{D}) + i_c \ln(\frac{1}{D}) \right]$$

Substituting $i_a = -i_b - i_c$, obtain:

$$\lambda_{\mathrm{a}} = \frac{\mu_0}{2\pi}\left[i_a \ln\left(\frac{1}{r'}\right) - i_a \ln\left(\frac{1}{D}\right)\right]$$

$$= \frac{\mu_0}{2\pi} i_a \ln\left(\frac{D}{r'}\right).$$

$$L_a = \frac{\mu_0}{2\pi} \ln\left(\frac{D}{r'}\right) = \frac{4\pi \times 10^{-7}}{2\pi} \ln\left(\frac{5}{9.67 \times 10^{-3}}\right)$$

$$= 1.25 \times 10^{-6} \text{ H/m.}$$

Again note logarithm of ratio of distance between phases to the size of the conductor.

$$L_a = 1.25 \times 10^{-6} \text{ H/m}$$

Converting to reactance

$$X_{\text{a}} = 2\pi \times 60 \times 1.25 \times 10^{-6}$$

$$= 4.71 \times 10^{-4} \ \Omega/\text{m}$$

$$= 0.768 \ \Omega/\text{mile}$$

$$X_{\text{Total for 5 mile line}} = 3.79 \ \Omega$$

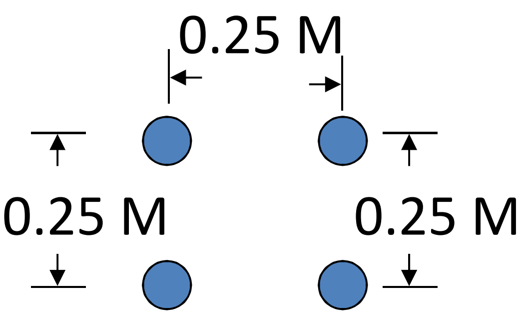
(this is the total per phase)

The reason we did NOT have mutual inductance was because of the symmetric conductor spacing

Bundle Inductance Example

Consider the previous example of the three phases symmetrically spaced 5 meters apart using wire with a radius of *r* = 1.24 cm. Except now assume each phase has 4 conductors in a square bundle, spaced 0.25 meters apart. What is the new inductance per meter?

$$r = 1.24 \times 10^{-2}\ \text{m} \quad r' = 9.67 \times 10^{-3}\ \text{m}$$



$$R_b = \left(9.67 \times 10^{-3} \times 0.25 \times 0.25 \times (\sqrt{2} \times 0.25)\right)^{1/4}$$

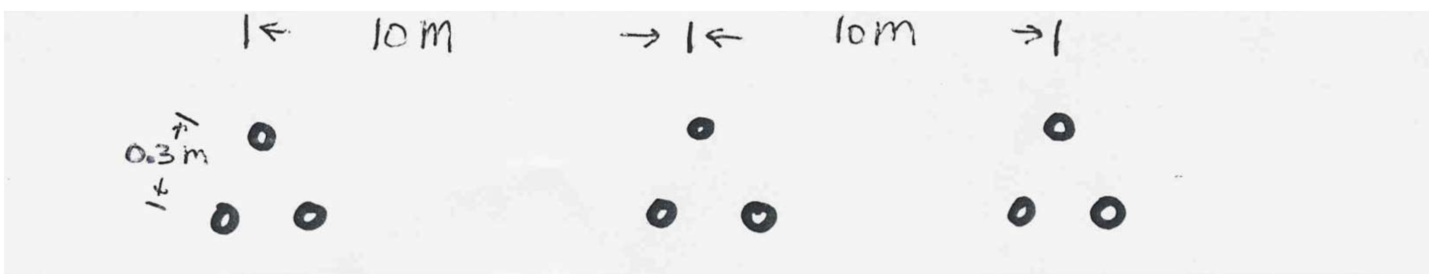
$$= 0.12\ \text{m}\ \ (\text{ten times bigger than } r!)$$

$$L_a = \frac{\mu_0}{2\pi} \ln \frac{5}{0.12} = 7.46 \times 10^{-7}\ \text{H/m}$$

Bundling reduces inductance.

Inductance Example

- Calculate the per phase inductance and reactance of a balanced 3ϕ, 60 Hz, line with:
  - horizontal phase spacing of 10m
  - using three conductor bundling with a spacing between conductors in the bundle of 0.3m.
- Assume the line is uniformly transposed and the conductors have a 1cm radius.



$$D_m = (d_{12} d_{13} d_{23})^{1/3},$$

$$= (10 \times (2 \times 10) \times 10)^{1/3} = 12.6\,\mathrm{m},$$

$$r' = r\, e^{-\mu_r/4} = 0.0078\,\mathrm{m},$$

$$R_b = (r' d_{12} d_{13})^{1/3},$$

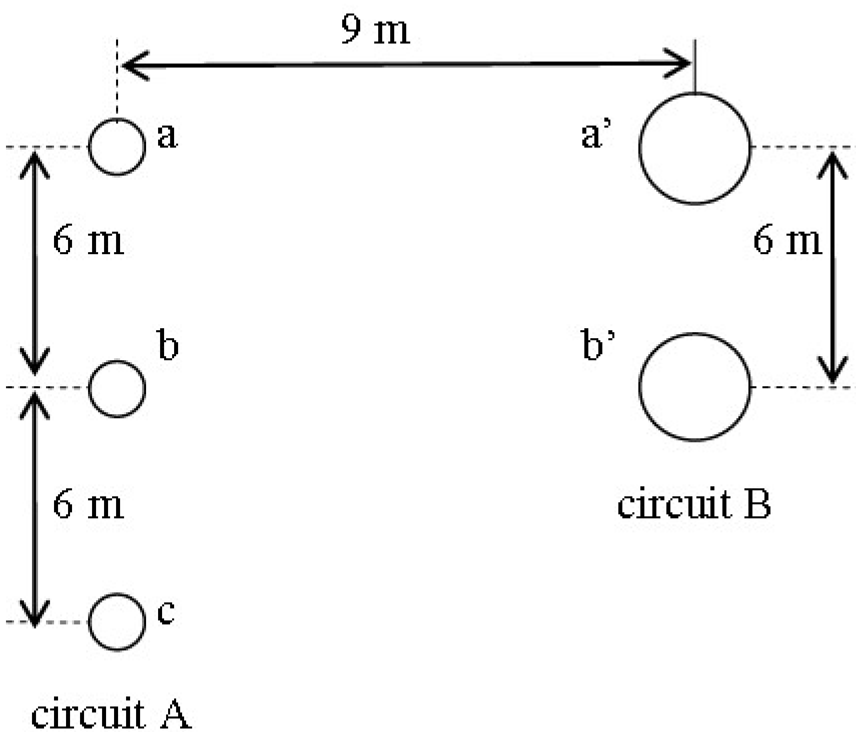
$$= (r' \times 0.3 \times 0.3)^{1/3} = 0.0888\,\mathrm{m},$$

$$L_a = \frac{\mu_0}{2\pi} \ln \frac{D_m}{R_b}$$

$$= 9.9 \times 10^{-7}\,\mathrm{H/m},$$

$$X_a = 2\pi f L_a (1600\,\mathrm{m/mile}) = 0.6\,\Omega/\mathrm{mile}.$$

Example: Find GMD, GMR for each circuit, inductance for each circuit, and total inductance per meter for two circuits that run parallel to each other. One circuit consists of three 0.25 cm radius conductors. The second circuit consists of two 0.5 cm radius conductor



Solution:

$m = 3,\ n' = 2,\ \therefore m \cdot n' = 6$

$$GMD = \sqrt[6]{(D_{aa'} D_{ab'})(D_{ba'} B_{bb'})(D_{ca'} D_{cb'})}$$

where

$$D_{aa'} = D_{bb'} = 9 \text{ m}$$

$$D_{ab'} = D_{ba'} = D_{cb'} = \sqrt{6^2 + 9^2} = \sqrt{117} \text{ m}$$

$$D_{ca'} = \sqrt{12^2 + 9^2} = 15 \text{ m}$$

$$\therefore\ GMD = 10.743 \text{ m}$$

Geometric Mean Radius for Circuit A:

$$GMR_A = \sqrt[3^2]{D_{aa}D_{ab}D_{ac}D_{ba}D_{bb}D_{bc}D_{ca}D_{cb}D_{cc}} = \sqrt[9]{\left(0.25\times10^{-2}\times e^{-1/4}\right)^3\times6^4\times12^2} = 0.481\text{m}$$

Geometric Mean Radius for Circuit B:

$$GMR_B = \sqrt[2^2]{D_{a'a'}D_{a'b'}D_{b'b'}D_{b'a'}} = \sqrt[4]{\left(0.5\times10^{-2}\times e^{-1/4}\right)^2\times6^2} = 0.153\text{m}$$

Inductance of circuit A

$$L_A = 2\times10^{-7}\ln\frac{GMD}{GMR_A} = 2\times10^{-7}\ln\frac{10.743}{0.481} = 6.212\times10^{-7} \qquad H/m$$

Inductance of circuit B

$$L_B = 2\times10^{-7}\ln\frac{GMD}{GMR_B} = 2\times10^{-7}\ln\frac{10.743}{0.153} = 8.503\times10^{-7} \qquad H/m$$

The total inductance is then

$$L_T = L_A + L_B = 14.715\times10^{-7} \qquad H/m$$

Example

An 8000 V, 60 Hz, single-phase, transmission line consists of two hard-drawn aluminum conductors with a radius of 2 cm spaced 1.2 m apart. If the transmission line is 30 km long and the temperature of the conductors is 20^0C,

a. What is the series resistance per kilometer of this line?
b. What is the series inductance per kilometer of this line?
c. What is the total series impedance of this line?

a. The series resistance of the transmission line is

$$R = \frac{\rho l}{A}$$

Ignoring the skin effect, the resistivity of the line at 20^0 will be $2.83 \cdot 10^{-8}$ Ω-m and the resistance per kilometer of the line is

$$r = \frac{\rho l}{A} = \frac{2.83 \cdot 10^{-8} \cdot 1000}{\pi \cdot 0.02^2} = 0.0225 \quad \Omega / km$$

b. The series inductance per kilometer of the transmission line is

$$l = \frac{\mu}{\pi}\left(\frac{1}{4} + \ln\frac{D}{r}\right)\cdot 1000 = \frac{\mu}{\pi}\left(\frac{1}{4} + \ln\frac{1.2}{0.02}\right)\cdot 1000 = 1.738\cdot 10^{-3} \quad H/km$$

d. The series impedance per kilometer of the transmission line is

$$z_{se} = r + jx = r + j2\pi fl = 0.0225 + j2\pi\cdot 60\cdot 1.738\cdot 10^{-3} = 0.0225 + j0.655 \;\; \Omega/km$$

Then the total series impedance of the line is

$$Z_{se} = \left(0.0225 + j0.655\right)\cdot 30 = 0.675 + j19.7 \;\; \Omega$$

Example

An 8000 V, 60 Hz, single-phase, transmission line consists of two hard-drawn aluminum conductors with a radius of 2 cm spaced 1.2 m apart. If the transmission line is 30 km long and the temperature of the conductors is 20^0C,

a. What is the series resistance per kilometer of this line?
b. What is the series inductance per kilometer of this line?
c. What is the shunt capacitance per kilometer of this line?
d. What is the total series impedance of this line?
e. What is the total shunt admittance of this line?

a. The series resistance of the transmission line is

$$R = \frac{\rho l}{A}$$

Ignoring the skin effect, the resistivity of the line at 20^0 will be $2.83 \cdot 10^{-8}$ Ω-m and the resistance per kilometer of the line is

$$r = \frac{\rho l}{A} = \frac{2.83 \cdot 10^{-8} \cdot 1000}{\pi \cdot 0.02^2} = 0.0225 \quad \Omega/km$$

b. The series inductance per kilometer of the transmission line is

$$l = \frac{\mu}{\pi}\left(\frac{1}{4} + \ln\frac{D}{r}\right)\cdot 1000 = \frac{\mu}{\pi}\left(\frac{1}{4} + \ln\frac{1.2}{0.02}\right)\cdot 1000 = 1.738\cdot 10^{-3} \quad H/km$$

c. The shunt capacitance per kilometer of the transmission line is

$$c_{ab} = \frac{\pi\varepsilon}{\ln\frac{D}{r}}\cdot 1000 = \frac{\pi\cdot 8.854\cdot 10^{-12}}{\ln\frac{1.2}{0.02}}\cdot 1000 = 6.794\cdot 10^{-9} \quad F/km$$

d. The series impedance per kilometer of the transmission line is

$$z_{se} = r + jx = r + j2\pi fl = 0.0225 + j2\pi\cdot 60\cdot 1.738\cdot 10^{-3} = 0.0225 + j0.655 \;\; \Omega/km$$

Then the total series impedance of the line is

$$Z_{se} = \left(0.0225 + j0.655\right)\cdot 30 = 0.675 + j19.7 \;\; \Omega$$

e. The shunt admittance per kilometer of the transmission line is

$$y_C = j2\pi fc = j2\pi \cdot 60 \cdot 6.794 \cdot 10^{-9} = j2.561 \cdot 10^{-6} \quad S/m$$

The total shunt admittance will be

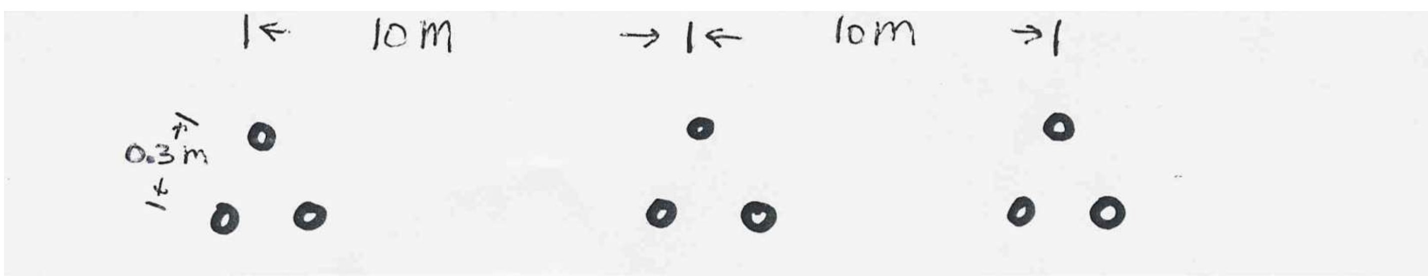
$$Y_{se} = \left(j2.561 \cdot 10^{-6} \right) \cdot 30 = j7.684 \cdot 10^{-5} \;\; S$$

The corresponding shunt capacitive reactance is

$$Z_{sh} = \frac{1}{Y_{sh}} = \frac{1}{j7.684 \cdot 10^{-5}} = -j13.0 \;\; k\Omega$$

Example

Calculate the per phase capacitance and susceptance of a balanced 3ϕ, 60 Hz, transmission line with horizontal phase spacing of 10m using three conductor bundling with a spacing between conductors in the bundle of 0.3m. Assume the line is uniformly transposed and the conductors have a 1cm radius.



$$R_b^c = (0.01 \times 0.3 \times 0.3)^{1/3} = 0.0963 \text{ m}$$

$$D_m = (10 \times 10 \times 20)^{1/3} = 12.6 \text{ m}$$

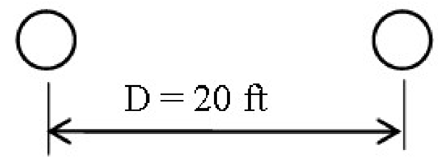
$$C = \frac{2\pi \times 8.854 \times 10^{-12}}{\ln \dfrac{12.6}{0.0963}} = 1.141 \times 10^{-11} \text{F/m}$$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi 60 \times 1.141 \times 10^{-11} \text{F/m}}$$

$$= 2.33 \times 10^{8} \ \Omega\text{-m (not } \Omega / \text{m)}$$

Example 1

Find the inductive reactance per mile and the capacitive reactance in MΩ.miles of a single phase line operating at 60 Hz. The conductor used is Partridge, with 20 ft spacing between the conductor centers.



From the Tables, for Partridge conductor,

GMR = 0.0217 ft

and inductive reactance at 1 ft spacing

X_a = 0.465 Ω/mile, which matches the table.

The spacing factor for 20 ft spacing is

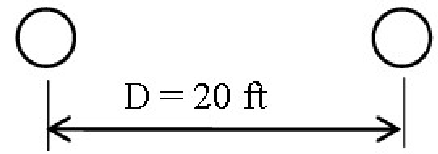
X_d = 0.3635 Ω /mile.

The inductance of the line is then

$X_L = X_a + X_d = 0.465 + 0.3635 = 0.8285$ Ω/ mile

Example 1

Find the inductive reactance per mile and the capacitive reactance in MΩ.miles of a single phase line operating at 60 Hz. The conductor used is Partridge, with 20 ft spacing between the conductor centers.



The outside radius of the Partridge conductor is $r = \frac{0.642}{2}$ in $= 0.0268$ ft

The capacitive reactance is

$$X_C = \frac{1.779 \times 10^6}{f} \ln\frac{D}{r} = \frac{1.779 \times 10^6}{f} \ln\frac{20}{0.0268} = 0.1961 \quad \mathrm{M\Omega.mile}$$

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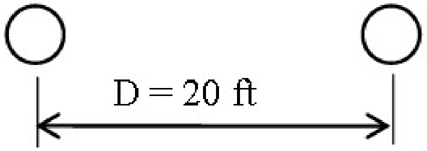

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This is the capacitive reactance between the conductor and the neutral.

OR

From tables $X_a^{'} = 0.1074 \quad M\Omega.\text{mile}$

$X_d^{'} = 0.0889 \quad M\Omega.\text{mile}$ for 20' spacing

$$\therefore X_C = X_a^{'} + X_d^{'} = 0.1963 \quad M\Omega.\text{mile}$$

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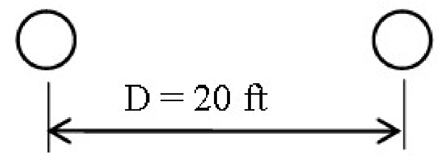

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OR From tables $X'_a = 0.1074 \quad M\Omega.\text{mile}$

$X'_d = 0.0889 \quad M\Omega.\text{mile}$ for 20' spacing

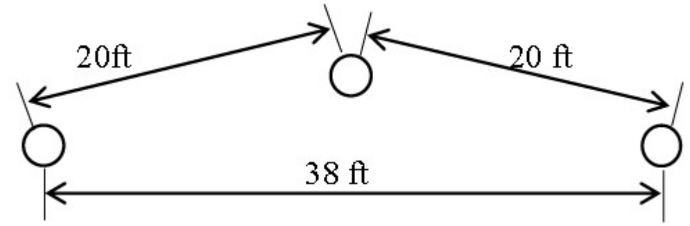
$$\therefore X_C = X'_a + X'_d = 0.1963 \quad M\Omega.\text{mile}$$

This is the capacitive reactance between the conductor and the neutral. Line-to-line capacitive reactance is:

$$X_C^{L-L} = \frac{X_C}{2} = 0.0981 \quad M\Omega.\text{mile}$$

Example 2

A three phase line operated at 60 Hz is arranged as shown. The conductors are ACSR Drake. If the length of the line is 175 miles and the normal operating voltage is 220 kV, Find:

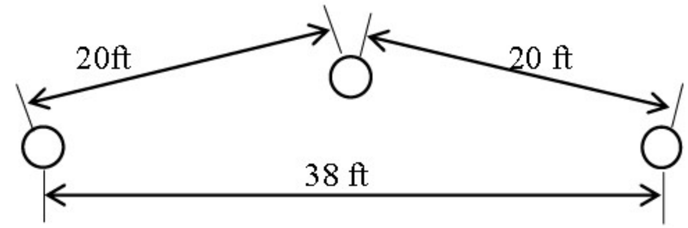


1. the inductive reactance per mile
2. the inductive reactance for the entire length of the line
3. the capacitive reactance for one mile
4. the capacitive reactance to neutral for the entire length of the line
5. the charging current for the line
6. the charging reactive power

Example 2

1. The inductive reactance per mile:

For ACSR Drake conductor, GMR = 0.0373 ft



$$D_{eq} = \sqrt[3]{20 \times 20 \times 38} = 24.8 \quad \text{ft}$$

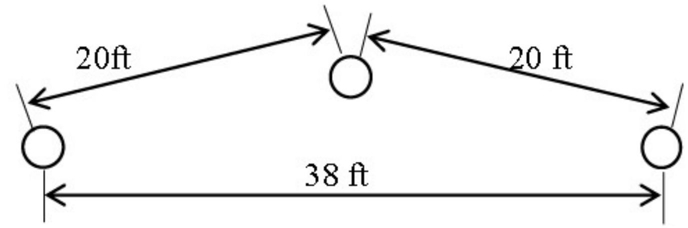
$$L = 2 \times 10^{-7} \ln \frac{24.8}{0.0373} = 13 \times 10^{-7} \qquad \text{H / m}$$

$$X_L = 2\pi \times 60 \times 1609 \times 13 \times 10^{-7} = 0.788 \qquad \Omega \text{ / mile}$$

Example 2

1. The inductive reactance per mile:

For ACSR Drake conductor, GMR = 0.0373 ft



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$$X_L = 2\pi \times 60 \times 1609 \times 13 \times 10^{-7} = 0.788 \qquad \Omega \text{ / mile}$$

or,

from the tables $X_a = 0.399 \quad \Omega$ / mile

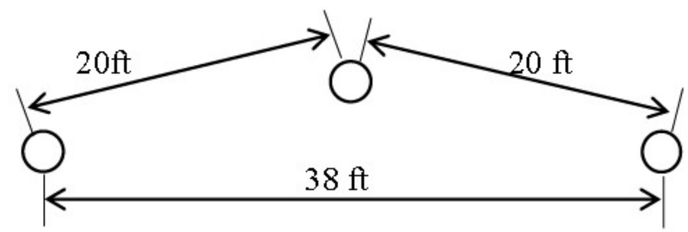
The spacing factor is calculated for spacing equal the geometric mean distance between the conductors, that is, $X_d = 2.022 \times 10^{-3} \times 60 \ln 24.8 = 0.389 \qquad \Omega$ / mile

Then the line inductance is $X_{line} = X_a + X_d = 0.788 \quad \Omega$ / mile per phase

Example 2

2. The inductive reactance the entire length of the line:

$X_L = 0.788 \times 175 = 137.9\ \Omega$



Example 2

3. The capacitive reactance for one mile:

1 foot = 12 inch

The outside radius for Drake conductors is $r = \dfrac{1.108}{2}$ in $= 0.0462$ ft

The geometric mean distance for this line is

$$D_{eq} = \sqrt[3]{20 \times 20 \times 38} = 24.8 \quad \text{ft}$$

From tables, $X'_a = 0.0912$ MΩ.mile

$$X'_d = \frac{1.779 \times 10^6}{f} \ln D_{eq} = \frac{1.779 \times 10^6}{60} \ln 24.8 = 0.0952 \quad \text{M}\Omega.\text{mile}$$

$$\therefore X_{cn} = X'_a + X'_d = 0.1864 \quad \text{M}\Omega.\text{mile}$$

This is the capacitive reactance to neutral.

Example 2

4. The capacitive reactance to neutral for the entire length of the line:

For the length of 175 miles,

$$X_{Ctotal} = \frac{X_{cn}}{175} = 1065 \quad \Omega$$

Example 2

5. the charging current for the line:

$$I_C = \frac{V_{LN}}{X_{Ctotal}} = \frac{\frac{220k}{\sqrt{3}}}{1065} = 119 \quad A$$

6. the charging reactive power:

$$Q_C = \sqrt{3} V_{LL} I_C = \sqrt{3} \times 220k \times 119 = 45.45 \quad MVAr$$