5.7.4 VARIOUS EQUIVALENT CIRCUIT CONFIGURATIONS

The equivalent circuit shown in Fig. 5.13*e* is not convenient to use for predicting the performance of the induction machine. As a result, several simplified versions have been proposed in various textbooks on electric machines. There is no general agreement on how to treat the shunt branch (i.e., R_c and X_m), particularly the resistance R_c representing the core loss in the machine. Some of the commonly used versions of the equivalent circuit are discussed here.

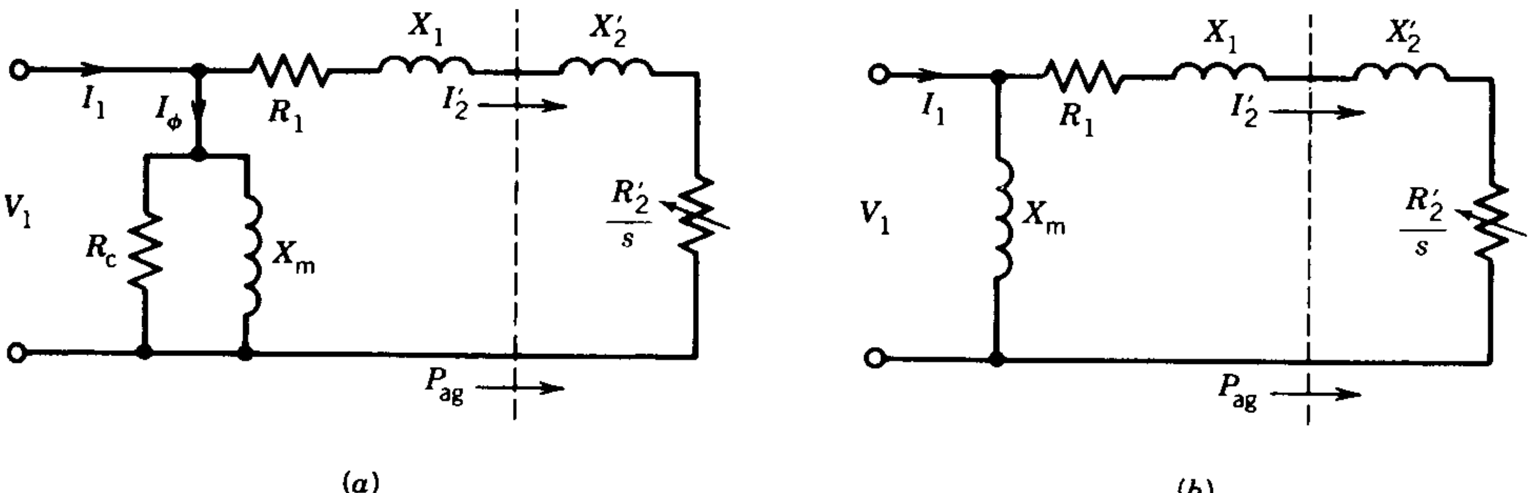


FIGURE 5.14 Approximate equivalent circuit.

Approximate Equivalent Circuit

If the voltage drop across R_1 and X_1 is small and the terminal voltage V_1 does not appreciably differ from the induced voltage E_1, the magnetizing branch (i.e., R_c and X_m) can be moved to the machine terminals as shown in Fig. 5.14*a*. This approximation of the equivalent circuit will considerably simplify computation, because the excitation current (I_ϕ) and the load component (I_2') of the machine current can be directly computed from the terminal voltage V_1 by dividing it by the corresponding impedances.

Note that if the induction machine is connected to a supply of fixed voltage and frequency, the stator core loss is fixed. At no load, the machine will operate close to synchronous speed. Therefore, the rotor frequency f_2 is very small and hence rotor core loss is very small. At a lower speed f_2 increases and so does the rotor core loss. The total core losses thus increase as the speed falls. On the other hand, at no load, friction and windage losses are maximum and as speed falls these losses decrease. Therefore, if a machine operates from a constant-voltage and constant-frequency source, the sum of core losses and friction and windage losses remains essentially constant at all operating speeds. These losses can thus be lumped together and termed the constant *rotational losses* of the induction machine. If the core loss is lumped with the windage and friction loss, R_c can be removed from the equivalent circuit, as shown in Fig. 5.14*b*.

IEEE-Recommended Equivalent Circuit

In the induction machine, because of its air gap, the exciting current I_ϕ is high—of the order of 30 to 50 percent of the full-load current. The leakage

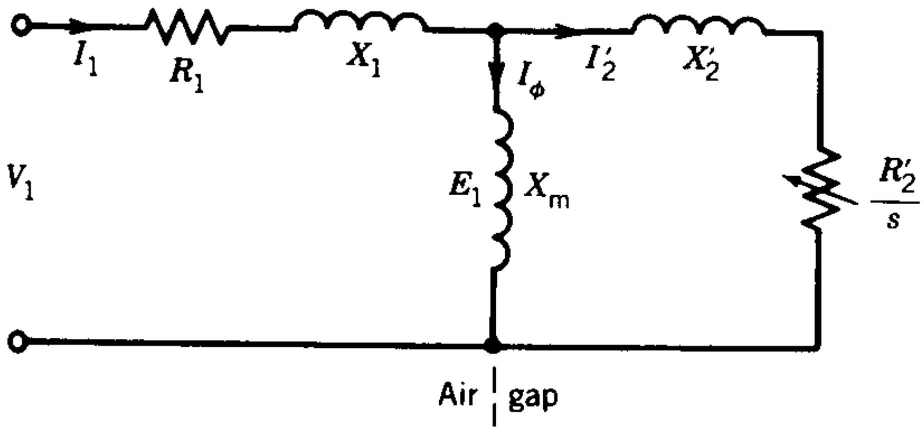


FIGURE 5.15 IEEE-recommended equivalent circuit.

reactance X_1 is also high. The IEEE recommends that, in such a situation, the magnetizing reactance X_m not be moved to the machine terminals (as is done in Fig. 5.14*b*) but be retained at its appropriate place, as shown in Fig. 5.15. The resistance R_c is, however, omitted, and the core loss is lumped with the windage and friction losses. This equivalent circuit (Fig. 5.15) is to be preferred for situations in which the induced voltage E_1 differs appreciably from the terminal voltage V_1.

5.7.5 THEVENIN EQUIVALENT CIRCUIT

In order to simplify computations, V_1, R_1, X_1, and X_m can be replaced by the Thevenin equivalent circuit values V_{th}, R_{th}, and X_{th}, as shown in Fig. 5.16, where

$$V_{th} = \frac{X_m}{[R_1^2 + (X_1 + X_m)^2]^{1/2}} V_1 \tag{5.45}$$

If $R_1^2 \ll (X_1 + X_m)^2$, as is usually the case,

$$V_{th} \approx \frac{X_m}{X_1 + X_m} V_1 \tag{5.45a}$$

$$= K_{th} V_1 \tag{5.45b}$$

The Thevenin impedance is

$$Z_{th} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$$

$$= R_{th} + jX_{th}$$

If $R_1^2 \ll (X_1 + X_m)^2$,

$$R_{th} \simeq \left(\frac{X_m}{X_1 + X_m}\right)^2 R_1 \tag{5.46}$$

$$= K_{th}^2 R_1 \tag{5.46a}$$

and since $X_1 \ll X_m$,

$$X_{th} \simeq X_1 \tag{5.47}$$

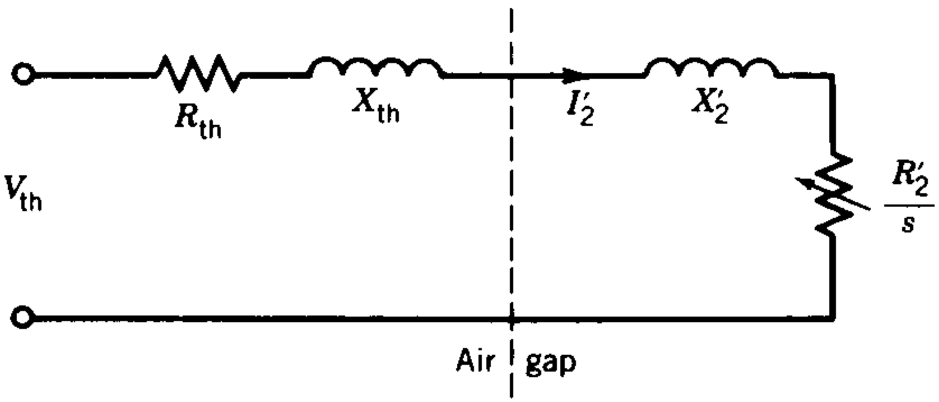


FIGURE 5.16 Thevenin equivalent circuit.

5.8 NO-LOAD TEST, BLOCKED-ROTOR TEST, AND EQUIVALENT CIRCUIT PARAMETERS

The parameters of the equivalent circuit, R_c, X_m, R_1, X_1, X_2, and R_2, can be determined from the results of a no-load test, a blocked-rotor test and from measurement of the dc resistance of the stator winding. The no-load test on an induction machine, like the open-circuit test on a transformer, gives information about exciting current and rotational losses. This test is performed by applying balanced polyphase voltages to the stator windings at the rated frequency. The rotor is kept uncoupled from any mechanical load. The small power loss in the machine at no load is due to the core loss and the friction and windage loss. The total rotational loss at the rated voltage and frequency under load is usually considered to be constant and equal to its value at no load.

The blocked-rotor test on an induction machine, like the short-circuit test on a transformer, gives information about leakage impedances. In this test the rotor is blocked so that the motor cannot rotate, and balanced polyphase voltages are applied to the stator terminals. The blocked-rotor test should be performed under the same conditions of rotor current and frequency

that will prevail in the normal operating conditions. For example, if the performance characteristics in the normal running condition (i.e., low-slip region) are required, the blocked-rotor test should be performed at a reduced voltage and rated current. The frequency also should be reduced because the rotor effective resistance and leakage inductance at the reduced frequency (corresponding to lower values of slip) may differ appreciably from their values at the rated frequency. This will be particularly true for double-cage or deep-bar rotors, as discussed in Section 5.11, and also for high-power motors.

The IEEE recommends a frequency of 25 percent of the rated frequency for the blocked-rotor test. The leakage reactances at the rated frequency can then be obtained by considering that the reactance is proportional to frequency. However, for normal motors of less than 20-hp rating, the effects of frequency are negligible and the blocked-rotor test can be performed directly at the rated frequency.

The determination of the equivalent circuit parameters from the results of the no-load and blocked-rotor tests is illustrated by the following example.

EXAMPLE 5.3

The following test results are obtained from a 3ϕ, 60 hp, 2200 V, six-pole, 60 Hz squirrel-cage induction motor.

(1) No-load test:

$$\text{Supply frequency} = 60\ \text{Hz}$$

$$\text{Line voltage} = 2200\ \text{V}$$

$$\text{Line current} = 4.5\ \text{A}$$

$$\text{Input power} = 1600\ \text{W}$$

(2) Blocked-rotor test:

$$\text{Frequency} = 15\ \text{Hz}$$

$$\text{Line voltage} = 270\ \text{V}$$

$$\text{Line current} = 25\ \text{A}$$

$$\text{Input power} = 9000\ \text{W}$$

(3) Average dc resistance per stator phase:

$$R_1 = 2.8\ \Omega$$

(a) Determine the no-load rotational loss.

(b) Determine the parameters of the IEEE-recommended equivalent circuit of Fig. 5.15.

(c) Determine the parameters (V_{th}, R_{th}, X_{th}) for the Thevenin equivalent circuit of Fig. 5.16.

Solution

(a) From the no-load test, the no-load power is

$$P_{NL} = 1600\ \mathrm{W}$$

The no-load rotational loss is

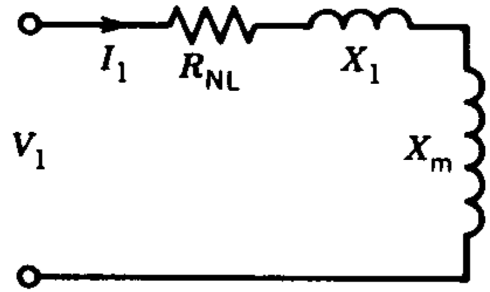
$$\begin{aligned} P_{Rot} &= P_{NL} - 3I_1^2R_1 \\ &= 1600 - 3 \times 4.5^2 \times 2.8 \\ &= 1429.9\ \mathrm{W} \end{aligned}$$

(b) *IEEE-recommended equivalent circuit.* For the no-load condition, R'_2/s is very high. Therefore, in the equivalent circuit of Fig. 5.15, the magnetizing reactance X_m is shunted by a very high resistive branch representing the rotor circuit. The reactance of this parallel combination is almost the same as X_m. Therefore the total reactance X_{NL}, measured at no load at the stator terminals, is essentially $X_1 + X_m$. The equivalent circuit at no load is shown in Fig. E5.3*a*.

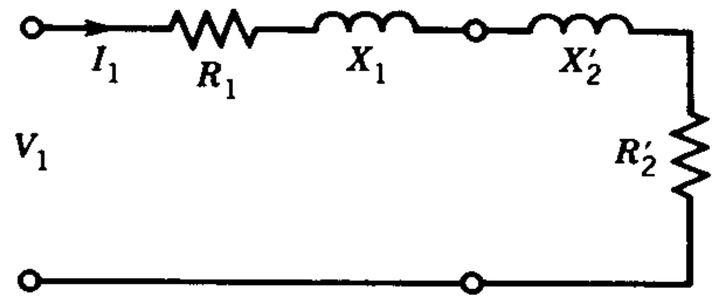
$$V_1 = \frac{2200}{\sqrt{3}} = 1270.2 \text{ V/phase}$$

The no-load impedance is

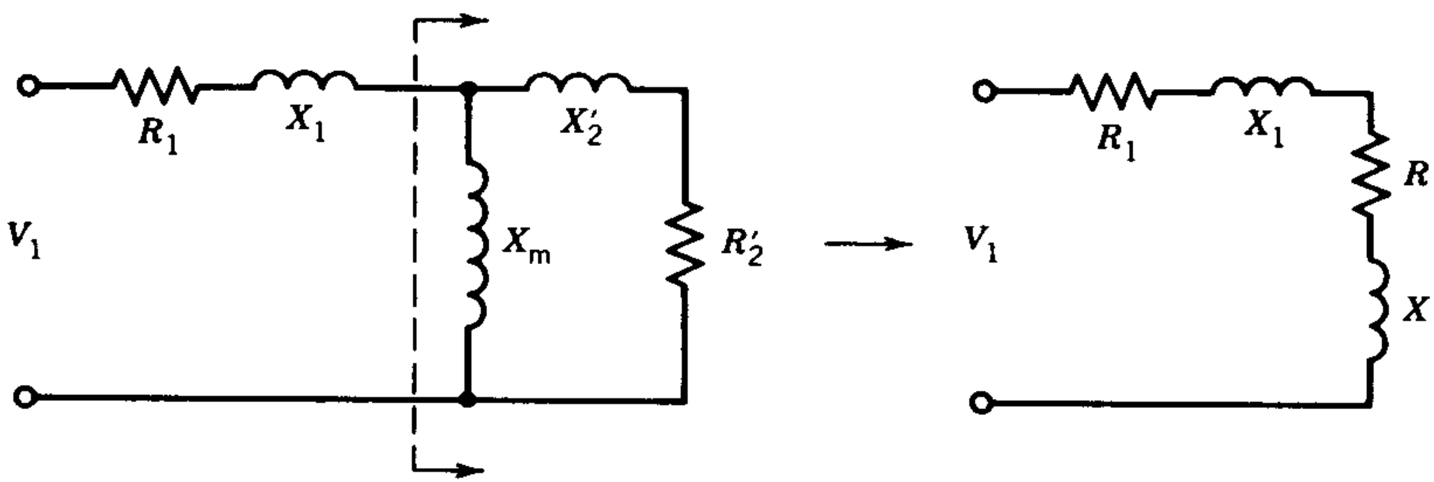
$$Z_{NL} = \frac{V_1}{I_1} = \frac{1270.2}{4.5} = 282.27\ \Omega$$



(*a*) No-load equivalent circuit based on Fig. 5.15



(*b*) Blocked-rotor equivalent circuit based on Fig. 5.15



(*c*) Blocked-rotor equivalent circuit for improved value for R_2'

FIGURE E5.3

The no-load resistance is

$$R_{NL} = \frac{P_{NL}}{3I_1^2} = \frac{1600}{3 \times 4.5^2} = 26.34\ \Omega$$

The no-load reactance is

$$X_{NL} = (Z_{NL}^2 - R_{NL}^2)^{1/2}$$

$$= (282.27^2 - 26.34^2)^{1/2} = 281.0\ \Omega$$

Thus, $X_1 + X_m = X_{NL} = 281.0\ \Omega$.

For the blocked-rotor test the slip is 1. In the equivalent circuit of Fig. 5.15, the magnetizing reactance X_m is shunted by the low-impedance branch $jX_2' + R_2'$. Because $|X_m| \gg |R_2' + jX_2'|$, the impedance X_m can be neglected and the equivalent circuit for the blocked-rotor test reduces to the form shown in Fig. E5.3*b*. From the blocked-rotor test, the blocked-rotor resistance is

$$R_{BL} = \frac{P_{BL}}{3I_1^2} = \frac{9000}{3 \times 25^2} = 4.8\ \Omega$$

Therefore, $R_2' = R_{BL} - R_1 = 4.8 - 2.8 = 2\ \Omega$. The blocked-rotor impedance at 15 Hz is

$$Z_{BL} = \frac{V_1}{I_1} = \frac{270}{\sqrt{3} \times 25} = 6.24\ \Omega$$

The blocked-rotor reactance at 15 Hz is

$$X_{BL} = (6.24^2 - 4.8^2)^{1/2}$$
$$= 3.98\ \Omega$$

Its value at 60 Hz is

$$X_{BL} = 3.98 \times \frac{60}{15} = 15.92\ \Omega$$

$$X_{BL} \simeq X_1 + X_2'$$

Hence,

$$X_1 = X_2' = \frac{15.92}{2} = 7.96\ \Omega \quad \text{(at 60 Hz)}$$

The magnetizing reactance is therefore

$$X_m = 281.0 - 7.96 = 273.04\ \Omega$$

Comments: The rotor equivalent resistance R_2' plays an important role in the performance of the induction machine. A more accurate determination of R_2' is recommended by the IEEE as follows: The blocked resistance R_{BL} is the sum of R_1 and an equivalent resistance, say R, which is the resistance of $R_2' + jX_2'$ in parallel with X_m as shown in Fig. E5.3*c;* therefore,

$$R = \frac{X_m^2}{R_2'^2 + (X_2' + X_m)^2} R_2'$$

If $X_2' + X_m \gg R_2'$, as is usually the case,

$$R \simeq \left(\frac{X_m}{X_2' + X_m}\right)^2 R_2'$$

or

$$R_2' = \left(\frac{X_2' + X_m}{X_m}\right)^2 R$$

Now $R = R_{BL} - R_1 = 4.8 - 2.8 = 2\ \Omega$. So,

$$R_2' = \left(\frac{7.96 + 273.04}{273.04}\right)^2 \times 2 = 2.12\ \Omega$$

(c) From Eq. 5.45

$$V_{th} \simeq \frac{273.04}{7.96 + 273.04} V_1$$

$$= 0.97 V_1$$

From Eq. 5.46a

$$R_{th} \simeq 0.97^2 R_1 = 0.97^2 \times 2.8 = 2.63\ \Omega$$

From Eq. 5.47

$$X_{th} \simeq X_1 = 7.96\ \Omega \quad \blacksquare$$

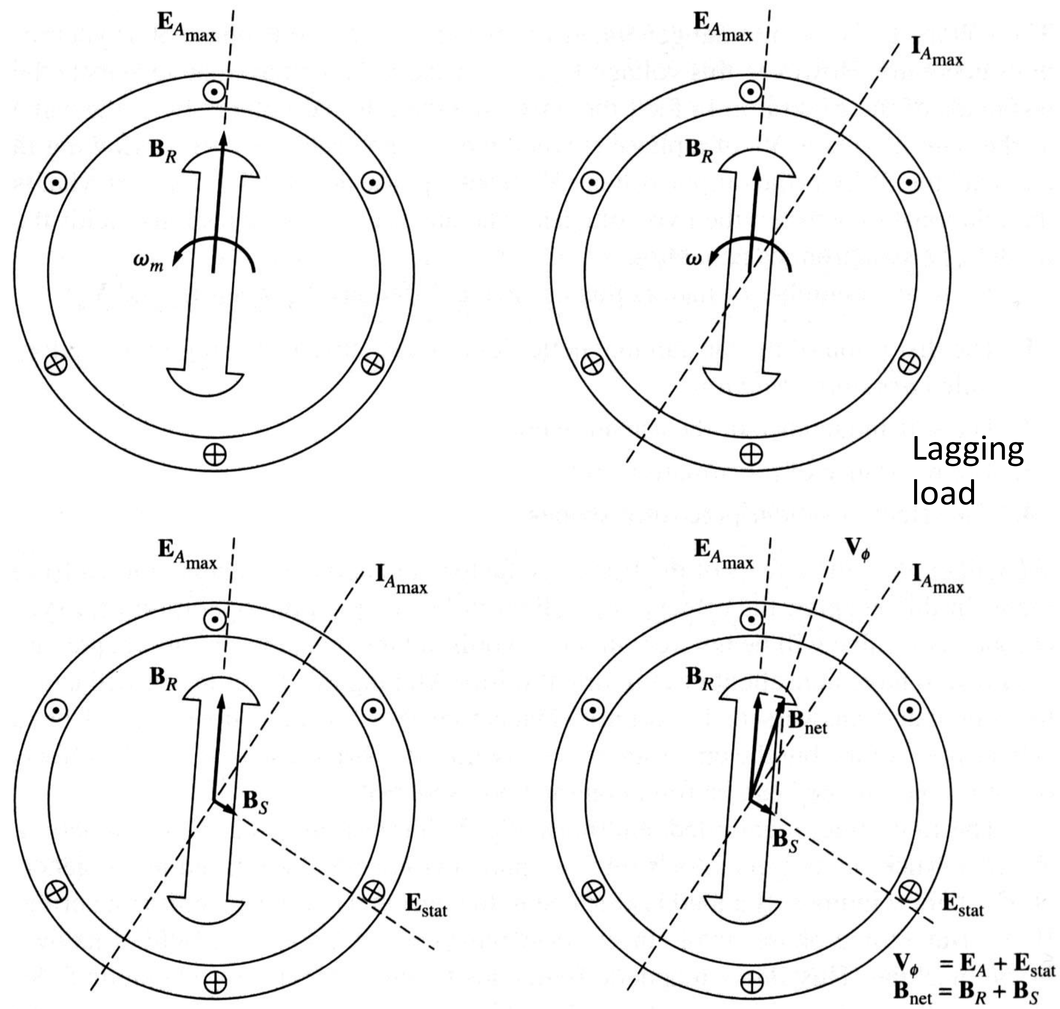
Equivalent circuit of a synchronous generator

The internally generated voltage in a single phase of a synchronous machine E_A is not usually the voltage appearing at its terminals. It equals to the output voltage V_ϕ only when there is no armature current in the machine. The reasons that the armature voltage E_A is not equal to the output voltage V_ϕ are:

1. Distortion of the air-gap magnetic field caused by the current flowing in the stator (armature reaction);
2. Self-inductance of the armature coils;
3. Resistance of the armature coils;
4. Effect of salient-pole rotor shapes.

- Armature reaction (the largest effect):

When the rotor of a synchronous generator is spinning, a voltage E_A is induced in its stator. When a load is connected, a current starts flowing creating a magnetic field in machine's stator. This stator magnetic field B_S adds to the rotor (main) magnetic field B_R affecting the total magnetic field and, therefore, the phase voltage.



- Assuming that the generator is connected to a lagging load, the load current I_A will create a stator magnetic field B_S, which will produce the armature reaction voltage E_{stat}. Therefore, the phase voltage will be

$$V_\phi = E_A + E_{stat}$$

- The net magnetic flux will be

$$B_{net} = \underset{\text{Rotor field}}{B_R} + \underset{\text{Stator field}}{B_S}$$

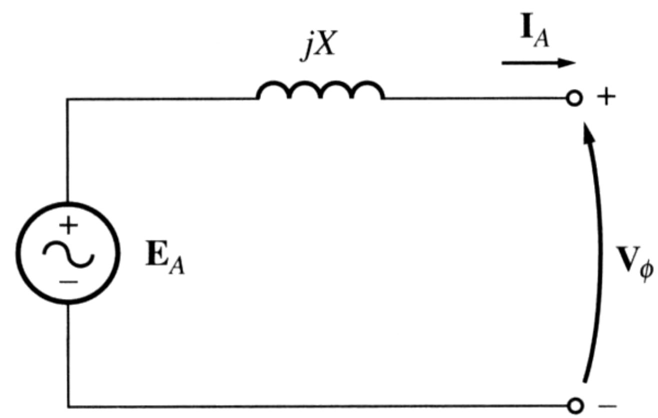
- Note that the directions of the net magnetic flux and the phase voltage are the same.

- Assuming that the load reactance is *X*, the armature reaction voltage is

$$E_{stat} = -jXI_A$$

- The phase voltage is then $$V_\phi = E_A - jXI_A$$

Armature reactance can be modeled by the following circuit…



- However, in addition to armature reactance effect, the stator coil has a self-inductance L_A (X_A is the corresponding reactance) and the stator has resistance R_A. The phase voltage is thus

$$V_\phi = E_A - jXI_A - jX_AI_A - RI_A$$

➢ Often, armature reactance and self-inductance are combined into the synchronous reactance of the machine:

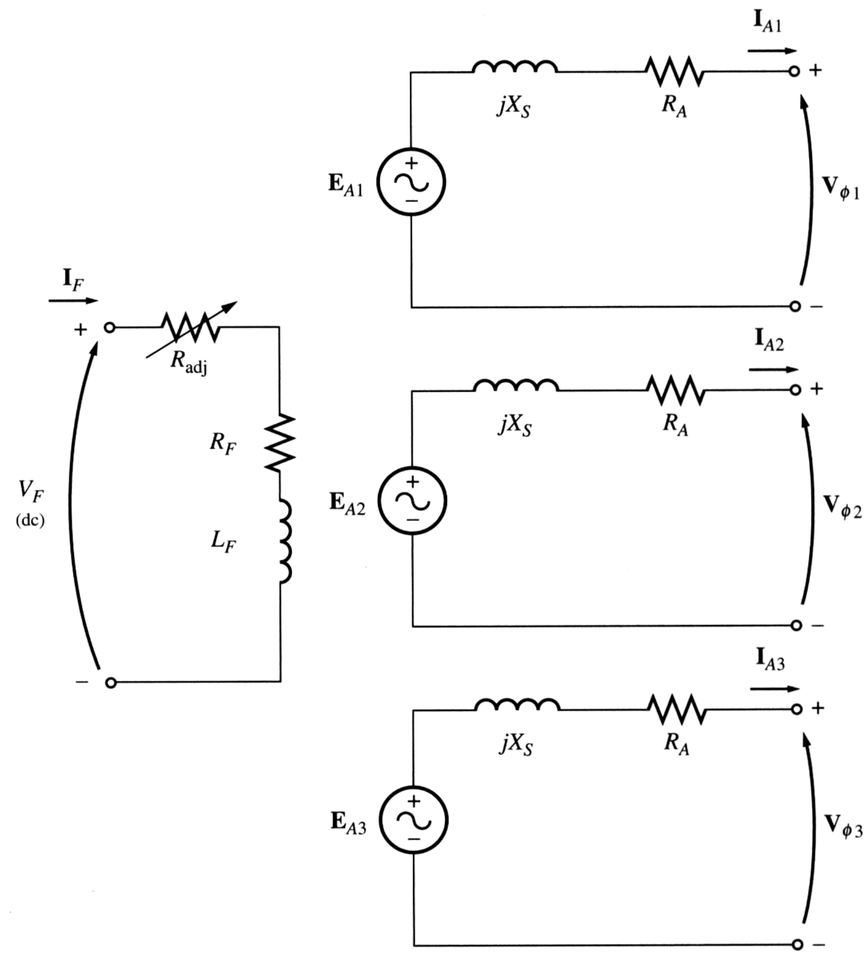
$$X_S = X + X_A$$

Therefore, the phase voltage is

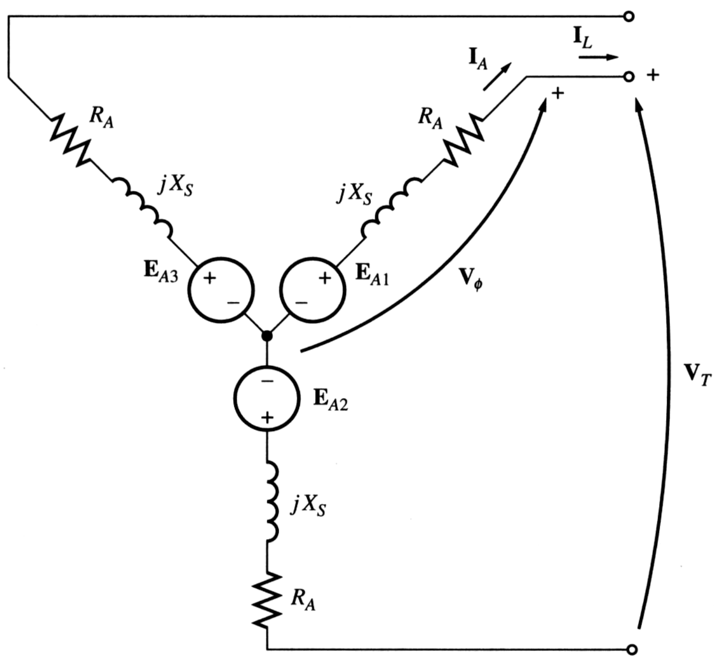
$$V_\phi = E_A - jX_S I_A - RI_A$$

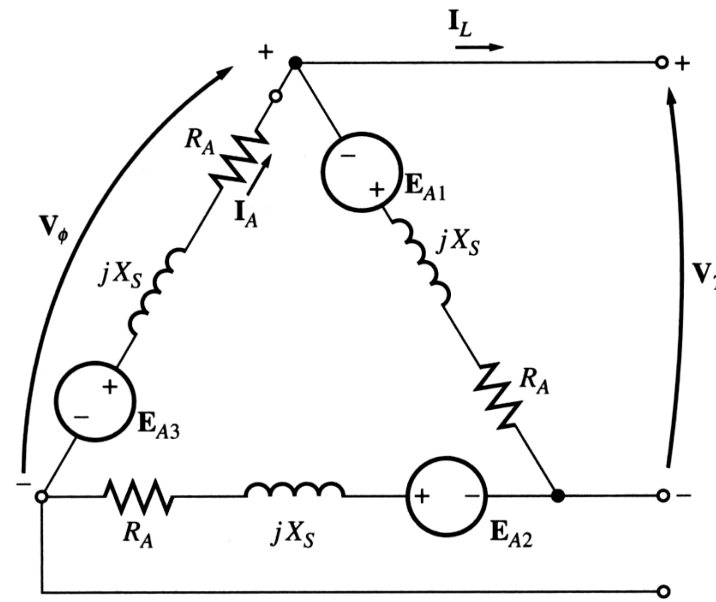
➢ The equivalent circuit of a 3-phase synchronous generator is shown.

➢ The adjustable resistor R_{adj} controls the field current and, therefore, the rotor magnetic field.



- A synchronous generator can be Y- or Δ-connected:



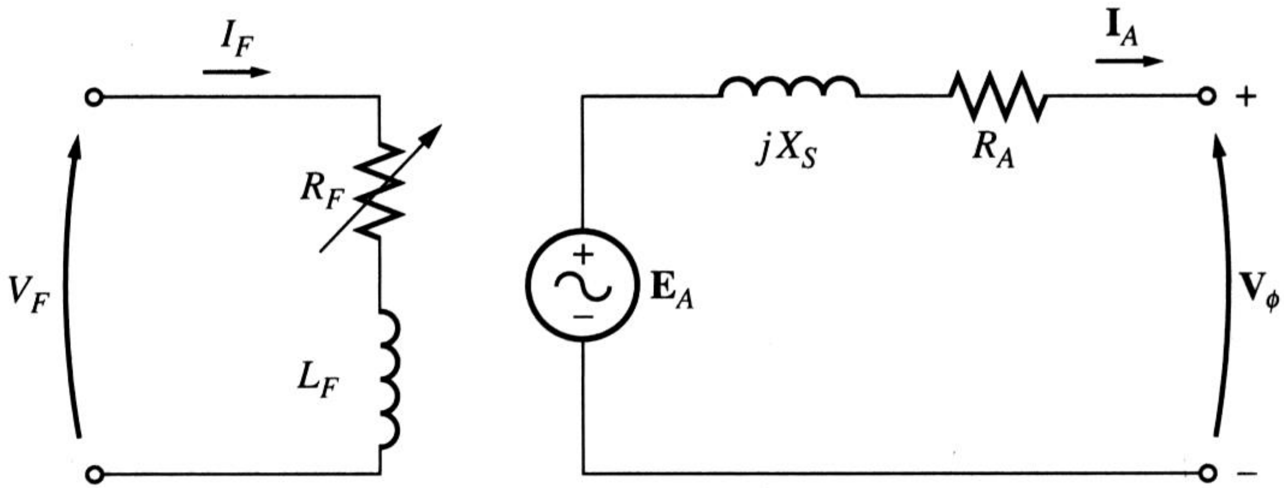


- The terminal voltage will be

$$V_T = \sqrt{3}V_\phi \quad - for\ Y \qquad\qquad V_T = V_\phi \quad - for\ \Delta$$

- Note: the discussion above assumed a balanced load on the generator!

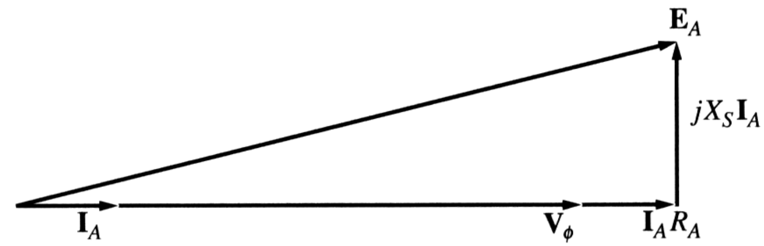
- Since – for balanced loads – the three phases of a synchronous generator are identical except for phase angles, per-phase equivalent circuits are often used.



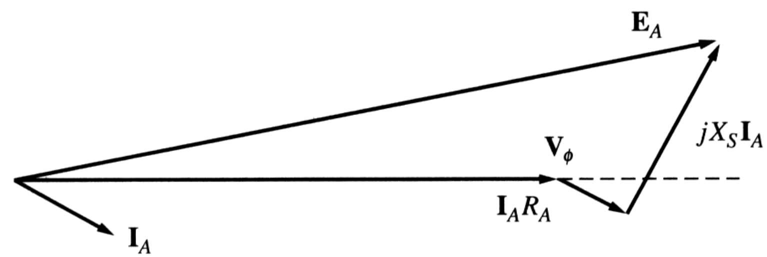
Phasor diagram of a synchronous generator

- Since the voltages in a synchronous generator are AC voltages, they are usually expressed as phasors. A vector plot of voltages and currents within one phase is called a phasor diagram.

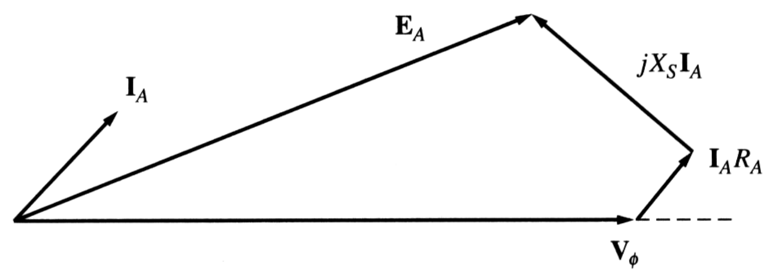
- A phasor diagram of a synchronous generator with a unity power factor (resistive load)



- Lagging power factor (inductive load): a larger than for leading PF internal generated voltage E_A is needed to form the same phase voltage.



- Leading power factor (capacitive load).



Power and torque in synchronous generators

- A synchronous generator needs to be connected to a prime mover whose speed is reasonably constant (to ensure constant frequency of the generated voltage) for various loads.

The applied mechanical power

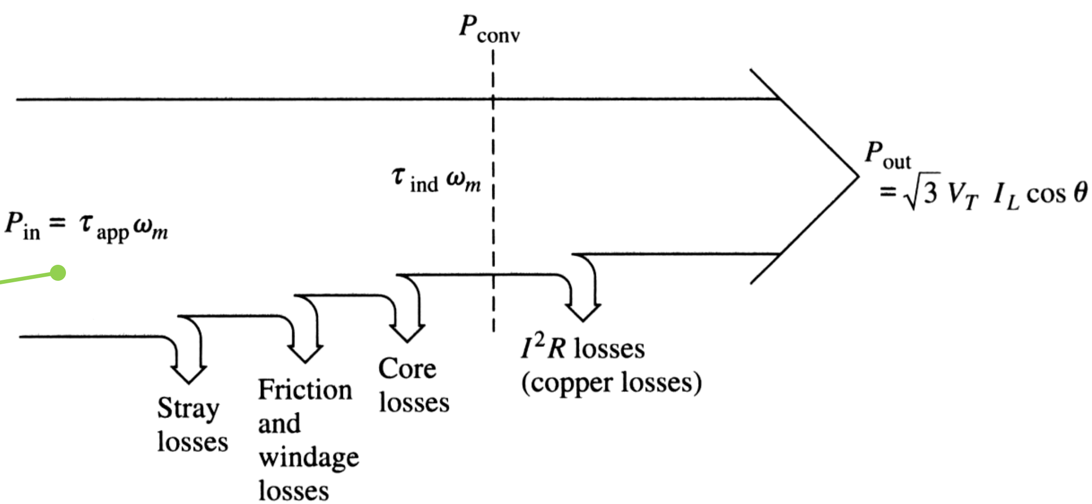
$$P_{in} = \tau_{app}\omega_m$$

is partially converted to electricity

$$P_{conv} = \tau_{ind}\omega_m = 3E_A I_A \cos\gamma$$

Where γ is the angle between E_A and I_A.

The power-flow diagram of a synchronous generator.



- The real output power of the synchronous generator is

$$P_{out} = \sqrt{3} V_T I_L \cos\theta = 3 V_\phi I_A \cos\theta$$

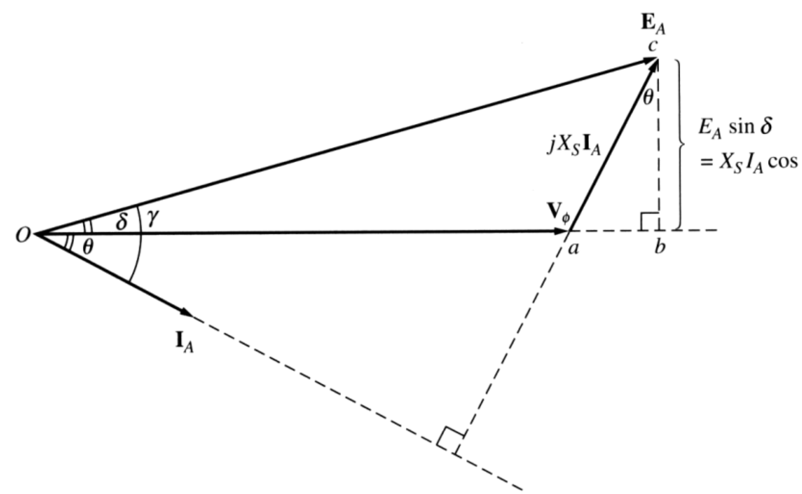
- The reactive output power of the synchronous generator is

$$Q_{out} = \sqrt{3} V_T I_L \sin\theta = 3 V_\phi I_A \sin\theta$$

- Recall that the power factor angle θ is the angle between V_ϕ and I_A and **not** the angle between V_T and I_L.

- In real synchronous machines of any size, the armature resistance $R_A << X_S$ and, therefore, the armature resistance can be ignored. Thus, a simplified phasor diagram indicates that

$$I_A \cos\theta = \frac{E_A \sin\delta}{X_S}$$



- Then the real output power of the synchronous generator can be approximated as

$$P_{out} \approx \frac{3V_\phi E_A \sin\delta}{X_S}$$

- We observe that electrical losses are assumed to be zero since the resistance is neglected. Therefore:

$$P_{conv} \approx P_{out}$$

- Here δ is the torque angle of the machine – the angle between V_ϕ and E_A.

- The maximum power can be supplied by the generator when $\delta = 90^0$:

$$P_{\max} = \frac{3V_\phi E_A}{X_S}$$

- The maximum power specified is called the static stability limit of the generator. Normally, real generators do not approach this limit: full-load torque angles are usually between 15^0 and 20^0.

- The induced torque is

$$\tau_{ind} = kB_R \times B_S = kB_R \times B_{net} = kB_R B_{net} \sin\delta$$

- Notice that the torque angle δ is also the angle between the rotor magnetic field B_R and the net magnetic field B_{net}.

- Alternatively, the induced torque is

$$\tau_{ind} = \frac{3V_\phi E_A \sin\delta}{\omega_m X_S} \qquad (7.25.2)$$

6.5 POWER AND TORQUE CHARACTERISTICS

A synchronous machine is normally connected to a fixed-voltage bus and operates at a constant speed. There is a limit on the power a synchronous generator can deliver to the infinite bus and on the torque that can be applied to the synchronous motor without losing synchronism. Analytical expressions for the steady-state power transfer between the machine and the constant-voltage bus or the torque developed by the machine are derived in this section in terms of bus voltage, machine voltage, and machine parameters.

The per-phase equivalent circuit is shown again in Fig. 6.18 for convenience, where V_t is the constant bus voltage per phase and is considered as the reference phasor. Let

$$V_t = |V_t|\angle 0° \tag{6.16}$$

$$E_f = |E_f|\angle \delta \tag{6.17}$$

$$Z_s = R_a + jX_s = |Z_s|\angle \theta_s \tag{6.18}$$

where the quantities inside the vertical bars represent the magnitudes of the phasors.

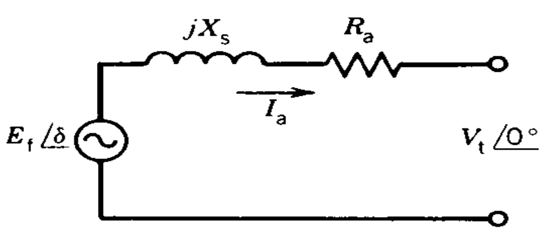


FIGURE 6.18 Per-phase equivalent circuit.

The per-phase complex power S at the terminals is

$$S = V_t I_a^* \tag{6.19}$$

The conjugate of the current phasor I_a is used to conform with the convention that lagging reactive power is considered as positive and leading reactive power as negative, as shown in Fig. 6.19.

From Fig. 6.18

$$\begin{aligned} I_a^* &= \left| \frac{E_f - V_t}{Z_s} \right|^* = \frac{E_f^*}{Z_s^*} - \frac{V_t^*}{Z_s^*} \\ &= \frac{|E_f|\underline{/-\delta}}{|Z_s|\underline{/-\theta_s}} - \frac{|V_t|\underline{/0}}{|Z_s|\underline{/-\theta_s}} \\ &= \frac{|E_f|}{|Z_s|}\underline{/\theta_s - \delta} - \frac{|V_t|}{|Z_s|}\underline{/\theta_s} \end{aligned} \tag{6.20}$$

From Eqs. 6.19 and 6.20

$$S = \frac{|V_t||E_f|}{|Z_s|}\underline{/\theta_s - \delta} - \frac{|V_t|^2}{|Z_s|}\underline{/\theta_s}\ \text{VA/phase} \tag{6.21}$$

The real power P and the reactive power Q per phase are

$$P = \frac{|V_t||E_f|}{|Z_s|}\cos(\theta_s - \delta) - \frac{|V_t|^2}{|Z_s|}\cos\theta_s \text{ watt/phase} \tag{6.22}$$

$$Q = \frac{|V_t||E_f|}{|Z_s|}\sin(\theta_s - \delta) - \frac{|V_t|^2}{|Z_s|}\sin\theta_s \text{ VAR/phase} \tag{6.23}$$

If R_a is neglected, then $Z_s = X_s$ and $\theta_s = 90°$. From Eqs. 6.22 and 6.23 for a 3ϕ machine,

$$P_{3\phi} = \frac{3|V_t||E_f|}{|X_s|}\sin\delta \tag{6.24}$$

$$= P_{max}\sin\delta \text{ watts} \tag{6.25}$$

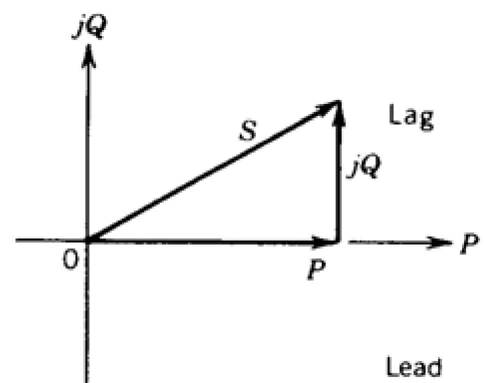


FIGURE 6.19 Complex power phasor.

where $$P_{\max} = \frac{3|V_t||E_f|}{|X_s|} \tag{6.25a}$$

$$Q_{3\phi} = \frac{3|V_t||E_f|}{|X_s|}\cos\delta - \frac{3|V_t|^2}{|X_s|}\,\text{VAR} \tag{6.26}$$

Because the stator losses are neglected in this analysis, the power developed at the terminals is also the air gap power. The developed torque of the machine is

$$T = \frac{P_{3\phi}}{\omega_{\text{syn}}} \tag{6.27}$$

$$= \frac{3}{\omega_{\text{syn}}}\frac{|V_t||E_f|}{X_s}\sin\delta \tag{6.28}$$

$$= T_{\max}\sin\delta\ \text{N}\cdot\text{m} \tag{6.29}$$

where $$T_{\max} = \frac{3}{\omega_{\text{syn}}}\frac{|V_t||E_f|}{X_s} = \frac{P_{\max}}{\omega_{\text{syn}}} \tag{6.29a}$$

$$\omega_{\text{syn}} = \frac{n_{\text{syn}}}{60}2\pi$$

n_{syn} is the synchronous speed in rpm

Both power and torque vary sinusoidally with the angle δ (as shown in Fig. 6.20), which is called the *power angle* or *torque angle*. The machine can be loaded *gradually* up to the limit of P_{max} or T_{max}, which are known as *static stability limits*. The machine will lose synchronism if δ becomes greater than 90°. The maximum torque T_{max} is also known as the *pull-out* torque. Note that since V_t is constant, the pull-out torque can be increased by increasing the excitation voltage E_f. If a synchronous motor tends to pull out of synchronism because of excessive load torque, the field current can be increased to develop high torque to prevent loss of synchronism. Similarly, in a synchronous generator, if the prime mover tends to drive the machine to supersynchronous speed by excessive driving torque, the field current can be increased to produce more counter torque to oppose such a tendency.

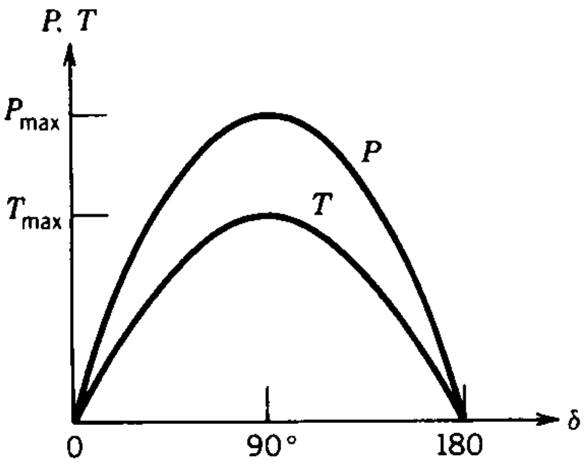


FIGURE 6.20 Power and torque–angle characteristics.

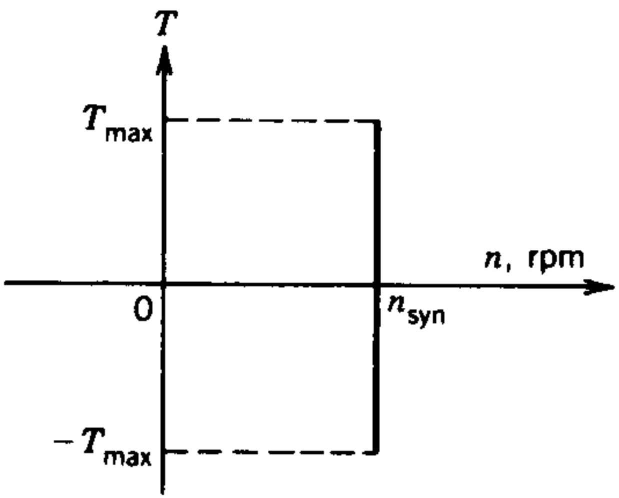


FIGURE 6.21 Torque–speed characteristics.

As the speed remains constant in a synchronous machine, the speed–torque characteristic is a straight line, parallel to the torque axis, as shown in Fig. 6.21.

Power and torque can also be expressed in terms of currents I_a, I_f' and the angle β between their phasors. Consider the equivalent circuit and the phasor diagram of Fig. 6.17*c*. The complex power S_a across the air gap is

$$S_a = I_a E_a^* \tag{6.29b}$$

Let

$$I_a = |I_a| \angle 0° \tag{6.29c}$$

Now

$$\begin{aligned} E_a &= jX_s I_m \\ &= jX_s(I_a + I_f') \\ &= |X_s||I_a| \angle 90° + |X_s||I_f'| \angle 90 + \beta \qquad (6.29d) \\ E_a^* &= |X_s|I_a| \angle -90° + |X_s|I_f'| \angle -90 - \beta \qquad (6.29e) \end{aligned}$$

From Eqs. 6.29b, 6.29c, and 6.29e,

$$S_a = |X_s|I_a^2|\underline{/-90^\circ} + |X_s|I_a|I_f'|\underline{/-90-\beta} \tag{6.29f}$$

From Eq. 6.29f, the real power transferred across the air gap is

$$\begin{aligned} P_a &= Re[S_a] \\ &= X_s I_a^2 \cos(-90^\circ) + |X_s|I_a|I_f'|\cos(-90-\beta) \\ &= -\omega L_s I_a I_f' \sin\beta \end{aligned} \tag{6.29g}$$

where $\omega = 2\pi f$.

The torque developed is

$$T = \frac{3P_a}{\omega_{syn}} \tag{6.29h}$$

Now,

$$\omega_{\text{syn}} = \frac{n_{\text{syn}} 2\pi}{60} = \frac{120f}{p}\frac{2\pi}{60} = \frac{4\pi f}{p} = \frac{2}{p}\,\omega \tag{6.29i}$$

From Eqs. 6.29g–i, the torque is

$$T = -\frac{3p}{2} L_{\text{s}} I_{\text{a}} I_{\text{f}}' \sin\beta \tag{6.29j}$$

Both power and torque vary sinusoidally with the angle β.

EXAMPLE 6.3

A 3ϕ, 5 kVA, 208 V, four-pole, 60 Hz, star-connected synchronous machine has negligible stator winding resistance and a synchronous reactance of 8 ohms per phase at rated terminal voltage.

The machine is first operated as a generator in parallel with a 3ϕ, 208 V, 60 Hz power supply.

(a) Determine the excitation voltage and the power angle when the machine is delivering rated kVA at 0.8 PF lagging. Draw the phasor diagram for this condition.

(b) If the field excitation current is now increased by 20 percent (without changing the prime mover power), find the stator current, power factor, and reactive kVA supplied by the machine.

(c) With the field current as in (a) the prime mover power is slowly increased. What is the steady-state (or static) stability limit? What are the corresponding values of the stator (or armature) current, power factor, and reactive power at this maximum power transfer condition?

Solution

The per-phase equivalent circuit for the synchronous generator is shown in Fig. E6.3*a*.

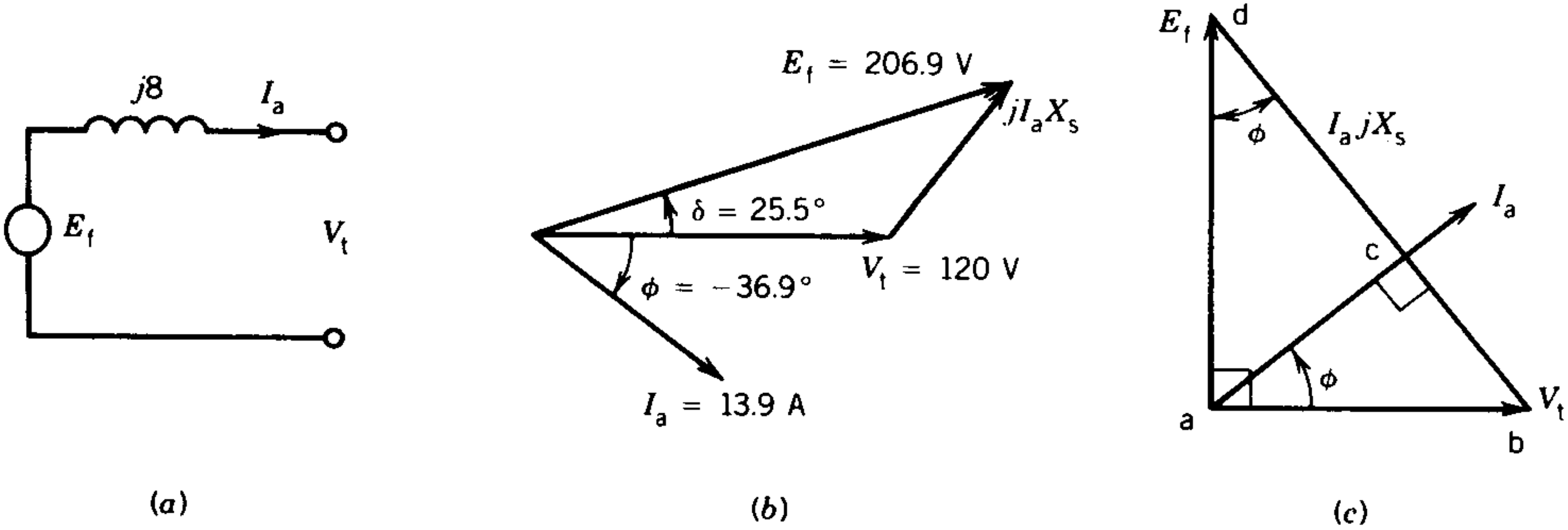


FIGURE E6.3

(a) $V_t = \dfrac{208}{\sqrt{3}} = 120$ V/phase

Stator current at rated kVA;

$$I_a = \frac{5000}{\sqrt{3} \times 208} = 13.9 \text{ A}$$

$$\phi = -36.9^\circ \quad \text{for lagging pf of } 0.8$$

From Fig. E6.3*a*

$$\begin{aligned} E_f &= V_t\underline{/0^\circ} + I_a jX_s \\ &= 120\underline{/0^\circ} + 13.9\underline{/-36.9^\circ} \cdot 8\underline{/90^\circ} \\ &= 206.9\underline{/25.5^\circ} \end{aligned}$$

$$\text{Excitation voltage} \qquad E_f = 206.9 \text{ V/phase}$$

$$\text{Power angle} \quad \delta = +25.5^\circ$$

Note that because of generator action the power angle is positive. The phasor diagram is shown in Fig. E6.3*b*.

(b) The new excitation voltage $E_f' = 1.2 \times 206.9 = 248.28$ V. Because power transfer remains same,

$$\frac{V_t E_f}{X_s} \sin\delta = \frac{V_t E_f'}{X_s} \sin\delta'$$

or

$$E_f \sin\delta = E_f' \sin\delta'$$

or

$$\sin\delta' = \frac{E_f}{E_f'} \sin\delta = \frac{\sin 25.5}{1.2}$$

$$\delta' = 21°$$

The stator current is

$$I_a = \frac{E_f - V_t}{jX_s}$$

$$= \frac{248.28\underline{/21°} - 120\underline{/0°}}{8\underline{/90°}}$$

$$= \frac{142.87\angle 38.52^\circ}{8\angle 90^\circ}$$

$$= 17.86\angle -51.5^\circ \text{ A}$$

$$\text{Power factor} = \cos 51.5^\circ = 0.62 \quad \text{lag}$$

$$\text{Reactive kVA} = 3|V_t||I_a| \sin 51.5^\circ$$

$$= 3 \times 120 \times 17.86 \times 0.78 \times 10^{-3}$$

$$= 5.03$$

or from Eq. 6.26

$$Q = 3\left(\frac{120 \times 248.28}{8} \cos 21^\circ - \frac{120^2}{8}\right) \times 10^{-3}$$

$$= 3(3476.86 - 1800)$$

$$= 5.03$$

(c) From Eq. 6.25 the maximum power transfer occurs at $\delta = 90°$.

$$P_{max} = \frac{3E_f V_t}{X_s} = \frac{3 \times 206.9 \times 120}{8} = 9.32 \text{ kW}$$

$$I_a = \frac{E_f - V_t}{jX_s} = \frac{206.9\underline{/+90°} - 120\underline{/0°}}{8\underline{/90°}}$$

$$= 29.9\underline{/30.1°} \text{ A}$$

$$\text{Stator current} \qquad I_a = 29.9 \text{ A}$$

$$\text{Power factor} = \cos 30.1° = 0.865 \quad \text{leading}$$

The stator current and power factor can also be obtained by drawing the phasor diagram for the maximum power transfer condition. The phasor diagram is shown in Fig. E6.3*c*.

Because $\delta = +90°$, E_f leads V_t by 90°. The distance *bd* between phasors V_t and E_f is the voltage drop $I_a X_s$ and the current phasor I_a is in quadrature with $I_a X_s$.

From the phasor diagram,

$$|I_a X_s|^2 = |E_f|^2 + |V_t|^2$$

$$I_a = \left(\frac{206.9^2 + 120^2}{8^2}\right)^{1/2} = 29.9 \text{ A}$$

From the two triangles *abc* and *abd*,

$$\underline{/bac} = \underline{/adb} = \phi$$

$$\tan \phi = \frac{ab}{ad} = \frac{120}{206.9} = 0.58$$

$$\phi = 30.1^\circ$$

$$\text{PF} = \cos 30.1^\circ = 0.865 \quad \text{lead}$$

EXAMPLE 6.4

The synchronous machine in Example 6.3 is operated as a synchronous motor from the 3ϕ, 208 V, 60 Hz power supply. The field excitation is adjusted so that the power factor is unity when the machine draws 3 kW from the supply.

(a) Find the excitation voltage and the power angle. Draw the phasor diagram for this condition.

(b) If the field excitation is held constant and the shaft load is slowly increased, determine the maximum torque (i.e., pull-out torque) that the motor can deliver.

Solution

The per-phase equivalent circuit for motoring operation is shown in Fig. E6.4*a*.

(a) $3V_tI_a \cos\phi = 3\text{ kW} = 3V_tI_a$ for $\cos\phi = 1$.

$$I_a = \frac{3000}{3 \times 120} = 8.33\text{ A}$$

$$E_f = V_t - I_a jX_s$$

$$= 120\angle 0^\circ - 8.33\angle 0^\circ \cdot 8\angle 90^\circ$$

$$= 137.35\angle -29^\circ$$

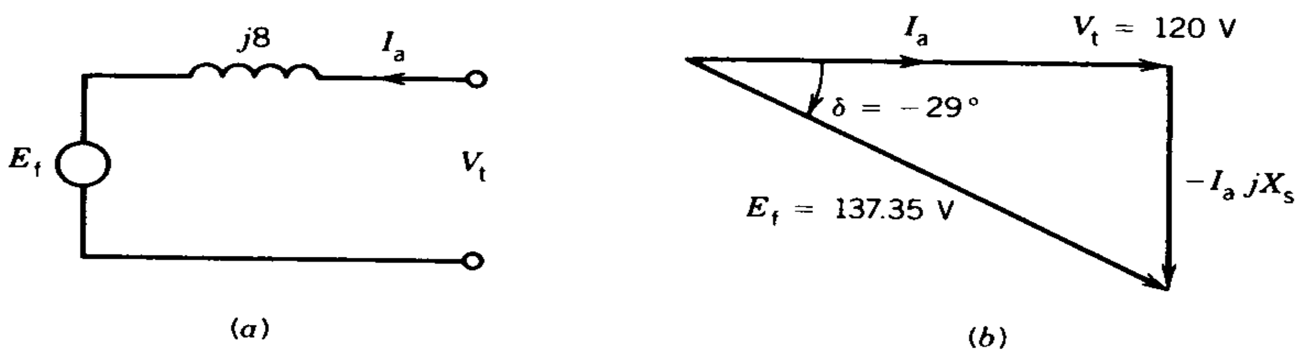


FIGURE E6.4

$$\text{Excitation voltage} \qquad E_f = 137.35 \text{ V/phase}$$

$$\text{Power angle} \qquad \delta = -29°$$

Note that because of motor action the power angle is negative.

The phasor diagram is shown in Fig. E.6.4*b*. E_f and δ can also be calculated from the phasor diagram.

$$E_f = \sqrt{|V_t|^2 + |I_a X_s|^2} = \sqrt{120^2 + (8.33 \times 8)^2}$$

$$= 137.35 \text{ V/phase}$$

$$\tan \delta = \frac{|I_a X_s|}{|V_t|} = \frac{8.33 \times 8}{120} = 0.555$$

$$|\delta| = 29°$$

$$\delta = -29°$$

(b) Maximum torque will be developed at $\delta = 90°$. From Eq. 6.25a,

$$P_{max} = \frac{3 \times 137.35 \times 120}{8} = 6180.75 \text{ W}$$

$$T_{max} = \frac{P_{max}}{\omega_{syn}} = \frac{6180.75}{(1800/60) \times 2\pi} = 32.8 \text{ N} \cdot \text{m}$$

Measuring parameters of synchronous generator model

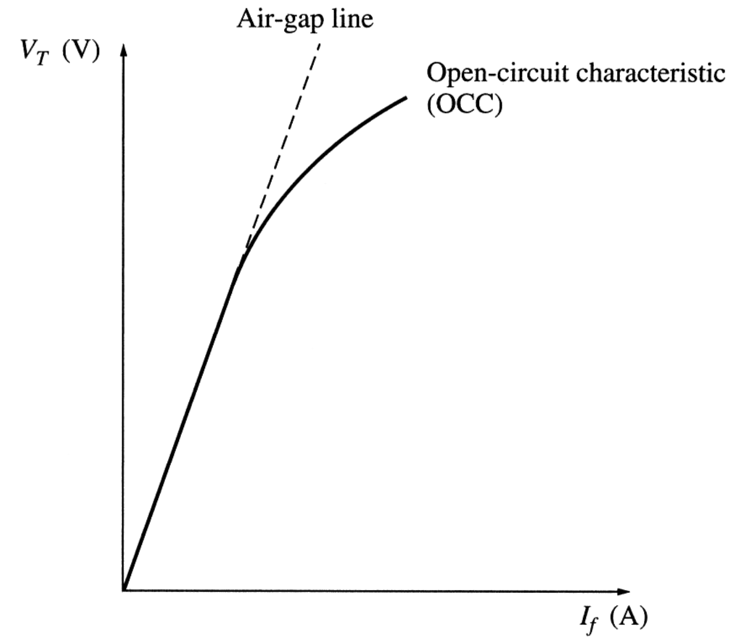
The three quantities must be determined in order to describe the generator model:

1. The relationship between field current and flux (and therefore between the field current I_F and the internal generated voltage E_A);
2. The synchronous reactance;
3. The armature resistance.

➢ We conduct first the open-circuit test on the synchronous generator: the generator is rotated at the rated speed, all the terminals are disconnected from loads, the field current is set to zero first. Next, the field current is increased in steps and the phase voltage (whish is equal to the internal generated voltage E_A since the armature current is zero) is measured.

➢ Therefore, it is possible to plot the dependence of the internal generated voltage on the field current – the open-circuit characteristic (OCC) of the generator.

➢ Since the unsaturated core of the machine has a reluctance thousands times lower than the reluctance of the air-gap, the resulting flux increases linearly first. When the saturation is reached, the core reluctance greatly increases causing the flux to increase much slower with the increase of the mmf.

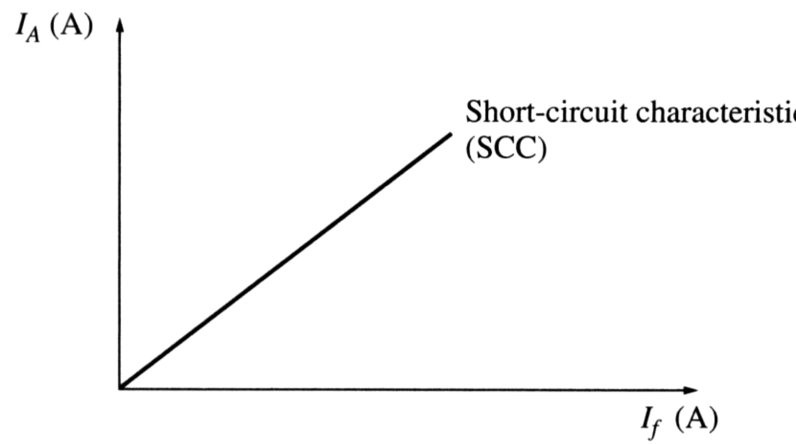


➢ We conduct next the short-circuit test on the synchronous generator: the generator is rotated at the rated speed, all the terminals are short-circuited through ammeters, the field current is set to zero first. Next, the field current is increased in steps and the armature current I_A is measured as the field current is increased.

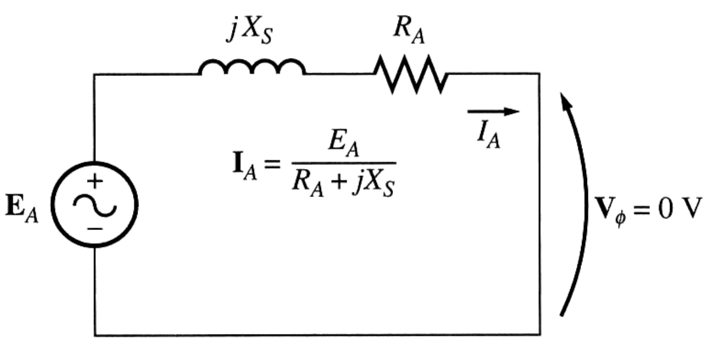
➢ The plot of armature current (or line current) vs. the field current is the short-circuit characteristic (SCC) of the generator.

- The SCC is a straight line since, for the short-circuited terminals, the magnitude of the armature current is

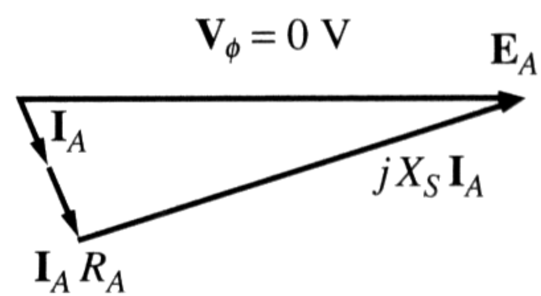
$$I_A = \frac{E_A}{\sqrt{R_A^2 + X_S^2}}$$



The equivalent generator's circuit during SC

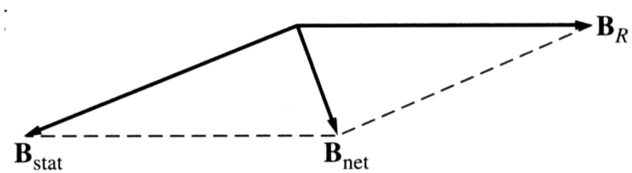


The resulting phasor diagram



Since B_S almost cancels B_R, the net field B_{net} is very small.

The magnetic fields during short-circuit test



An approximate method to determine the synchronous reactance X_S at a given field current:

1. Get the internal generated voltage E_A from the OCC at that field current.
2. Get the short-circuit current $I_{A,SC}$ at that field current from the SCC.
3. Find X_S from

$$X_S \approx \frac{E_A}{I_{A,SC}}$$

Since the internal machine impedance is

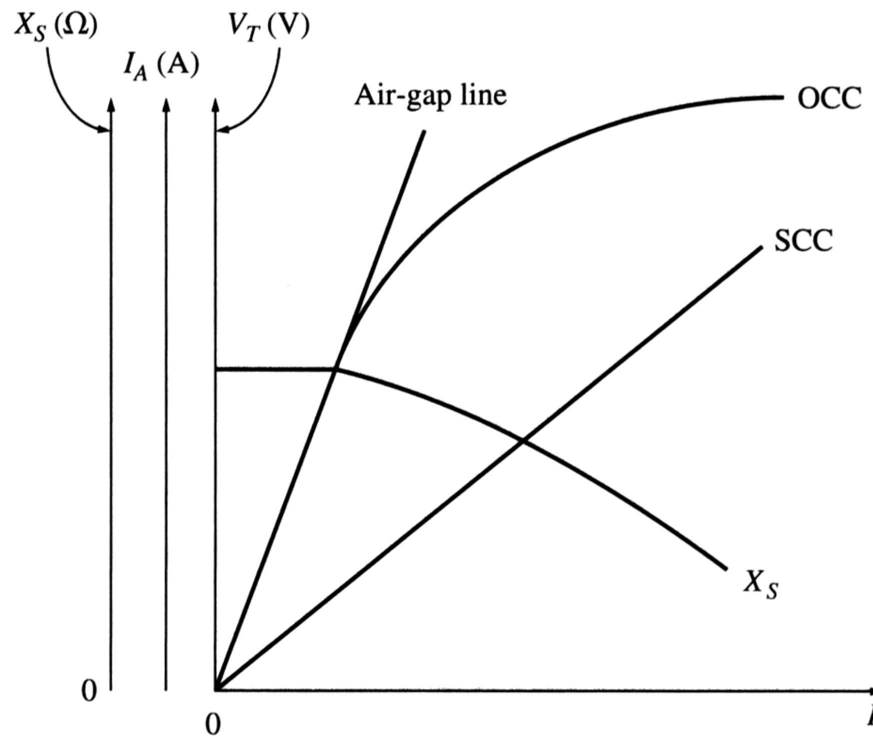
$$Z_S = \sqrt{R_A^2 + X_S^2} = \frac{E_A}{I_{A,SC}} \approx X_S \qquad \{\text{since } X_S > R_A\}$$

- A drawback of this method is that the internal generated voltage E_A is measured during the OCC, where the machine can be saturated for large field currents, while the armature current is measured in SCC, where the core is unsaturated. Therefore, this approach is accurate for unsaturated cores only.

- The approximate value of synchronous reactance varies with the degree of saturation of the OCC.

Therefore, the value of the synchronous reactance for a given problem should be estimated at the approximate load of the machine.

- The winding's resistance can be approximated by applying a DC voltage to a stationary machine's winding and measuring the current. However, AC resistance is slightly larger than DC resistance (skin effect).



Example 1: A 200 kVA, 480 V, 50 Hz, Y-connected synchronous generator with a rated field current of 5 A was tested and the following data were obtained:

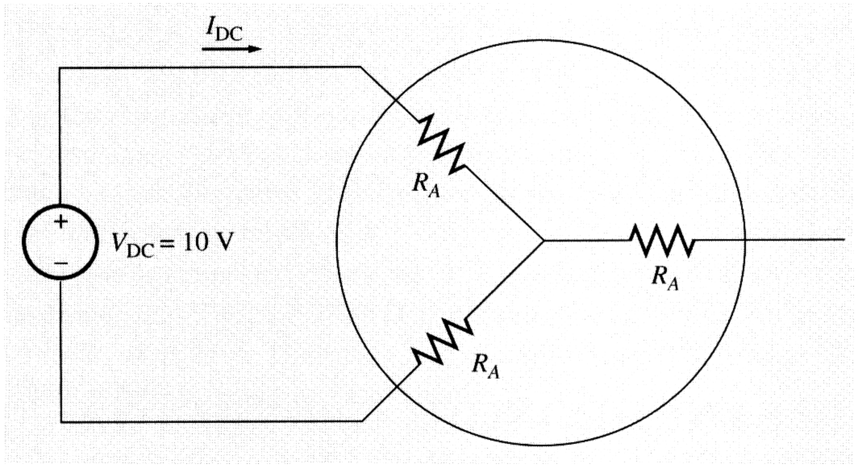
1. $V_{T,OC}$ = 540 V at the rated I_F.
2. $I_{L,SC}$ = 300 A at the rated I_F.
3. When a DC voltage of 10 V was applied to two of the terminals, a current of 25 A was measured.

Find the generator's model at the rated conditions (i.e., the armature resistance and the approximate synchronous reactance).

Since the generator is Y-connected, a DC voltage was applied between its ***two*** phases. Therefore:

$$2R_A = \frac{V_{DC}}{I_{DC}}$$

$$R_A = \frac{V_{DC}}{2I_{DC}} = \frac{10}{2 \cdot 25} = 0.2\,\Omega$$



- The internal generated voltage at the rated field current is

$$E_A = V_{\phi,OC} = \frac{V_T}{\sqrt{3}} = \frac{540}{\sqrt{3}} = 311.8\,V$$

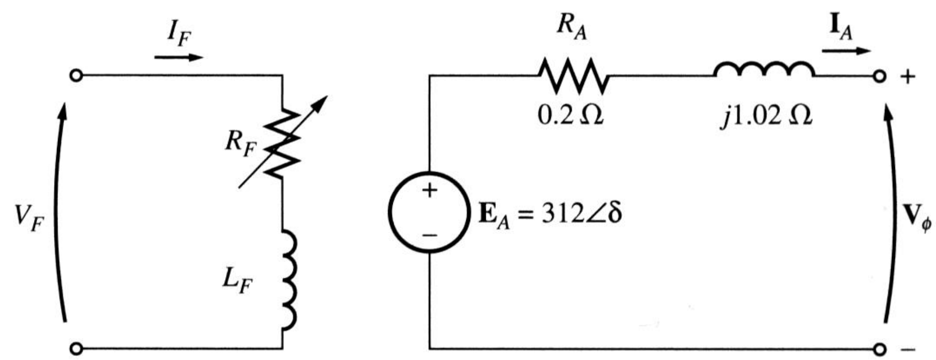
- The synchronous reactance at the rated field current is precisely

$$X_S = \sqrt{Z_S^2 - R_A^2} = \sqrt{\frac{E_A^2}{I_{A,SC}^2} - R_A^2} = \sqrt{\frac{311.8^2}{300^2} - 0.2^2} = 1.02\,\Omega$$

- We observe that if X_S was estimated via the approximate formula, the result would be:

$$X_S \approx \frac{E_A}{I_{A,SC}} = \frac{311.8}{300} = 1.04\,\Omega$$

- Which is close to the previous result. The error ignoring R_A is much smaller than the error due to core saturation.



The equivalent circuit

The Synchronous Generator Operating Alone

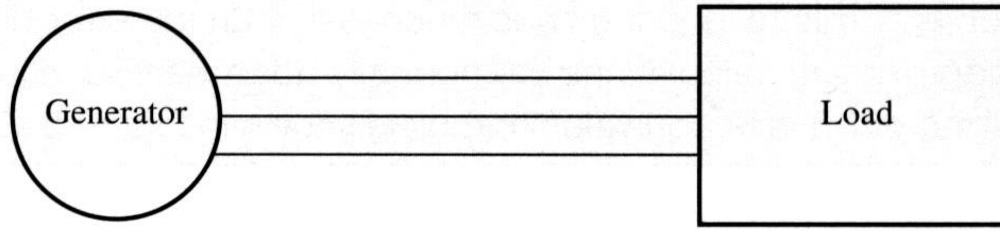
- The behavior of a synchronous generator varies greatly under load depending on the power factor of the load and on whether the generator is working alone or in parallel with other synchronous generators.

- Although most of the synchronous generators in the world operate as parts of large power systems, we start our discussion assuming that the synchronous generator works alone.

- Unless otherwise stated, the speed of the generator is assumed constant.

Effects of load changes

➢ A increase in the load is an increase in the real and/or reactive power drawn from the generator.



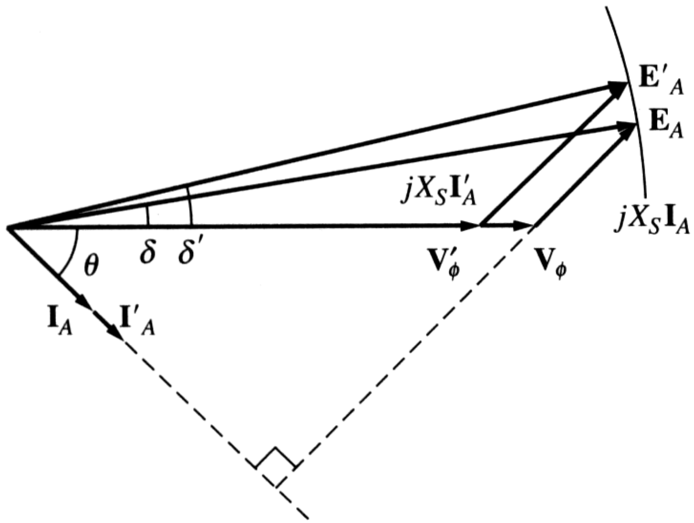
➢ Since the field resistor is unaffected, the field current is constant and, therefore, the flux ϕ is constant too. Since the speed is assumed as constant, the magnitude of the internal generated voltage is constant also.

➢ Assuming the same power factor of the load, change in load will change the magnitude of the armature current I_A. However, the angle will be the same (for a constant PF). Thus, the armature reaction voltage jX_SI_A will be larger for the increased load. Since the magnitude of the internal generated voltage is constant

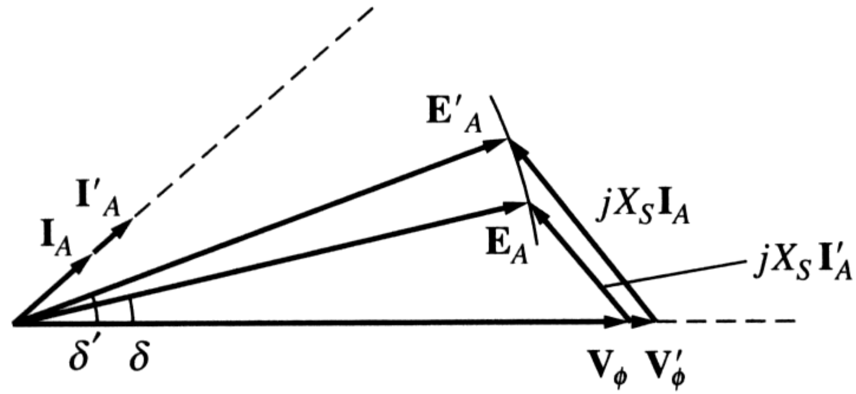
$$E_A = V_\phi + jX_S I_A$$

➢ Armature reaction voltage vector will "move parallel" to its initial position.

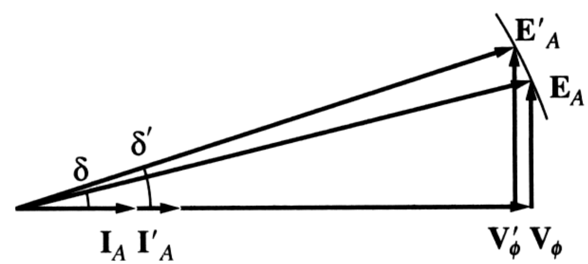
- Increase load effect on generators with



Lagging PF



Leading PF



Unity PF

- Generally, when a load on a synchronous generator is added, the following changes can be observed:

1. For lagging (inductive) loads, the phase (and terminal) voltage decreases significantly.
2. For unity power factor (purely resistive) loads, the phase (and terminal) voltage decreases slightly.
3. For leading (capacitive) loads, the phase (and terminal) voltage rises.

- Effects of adding loads can be described by the voltage regulation:

$$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} 100\%$$

- Where V_{nl} is the no-load voltage of the generator and V_{fl} is its full-load voltage.

➢ A synchronous generator operating at a lagging power factor has a fairly large positive voltage regulation. A synchronous generator operating at a unity power factor has a small positive voltage regulation. A synchronous generator operating at a leading power factor often has a negative voltage regulation.

➢ Normally, a constant terminal voltage supplied by a generator is desired. Since the armature reactance cannot be controlled, an obvious approach to adjust the terminal voltage is by controlling the internal generated voltage $E_A = K\phi\omega$. This may be done by changing flux in the machine while varying the value of the field resistance R_F, which is summarized:

1. Decreasing the field resistance increases the field current in the generator.
2. An increase in the field current increases the flux in the machine.
3. An increased flux leads to the increase in the internal generated voltage.
4. An increase in the internal generated voltage increases the terminal voltage of the generator.

➢ Therefore, the terminal voltage of the generator can be changed by adjusting the field resistance.

Example 2: A 480 V, 60 Hz, Y-connected six-pole synchronous generator has a per-phase synchronous reactance of 1.0 Ω. Its full-load armature current is 60 A at 0.8 PF lagging. Its friction and windage losses are 1.5 kW and core losses are 1.0 kW at 60 Hz at full load. Assume that the armature resistance (and, therefore, the I^2R losses) can be ignored. The field current has been adjusted such that the no-load terminal voltage is 480 V.

a. What is the speed of rotation of this generator?
b. What is the terminal voltage of the generator if
    1. It is loaded with the rated current at 0.8 PF lagging;
    2. It is loaded with the rated current at 1.0 PF;
    3. It is loaded with the rated current at 0.8 PF leading.

c. What is the efficiency of this generator (ignoring the unknown electrical losses) when it is operating at the rated current and 0.8 PF lagging?

d. How much shaft torque must be applied by the prime mover at the full load? how large is the induced counter torque?

e. What is the voltage regulation of this generator at 0.8 PF lagging? at 1.0 PF? at 0.8 PF leading?

- Since the generator is Y-connected, its phase voltage is

$$V_\phi = V_T / \sqrt{3} = 277\,V$$

- At no load, the armature current I_A = 0 and the internal generated voltage is E_A = 277 V and it is constant since the field current was initially adjusted that way.

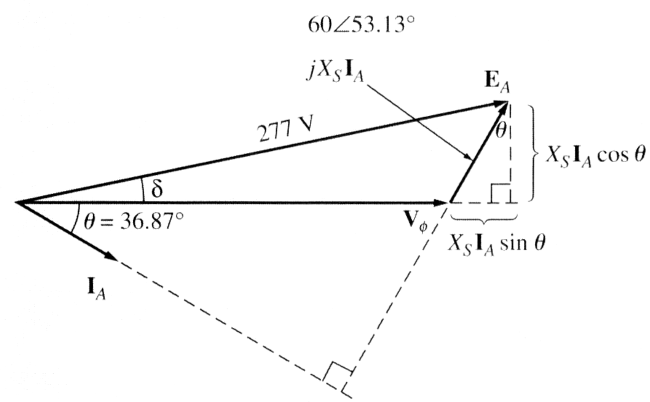
a. The speed of rotation of a synchronous generator is

$$n_m = \frac{120}{P} f_e = \frac{120}{6} 60 = 1200\,rpm$$

which is

$$\omega_m = \frac{1200}{60} 2\pi = 125.7\,rad/s$$

b.1. For the generator at the rated current and the 0.8 PF lagging, the phasor diagram is shown. The phase voltage is at 0^0, the magnitude of E_A is 277 V,



and that $$jX_S I_A = j \cdot 1 \cdot 60\angle -36.87° = 60\angle 53.13°$$

➢ Two unknown quantities are the magnitude of V_ϕ and the angle δ of E_A. From the phasor diagram:

$$E_A^2 = \left(V_\phi + X_S I_A \sin\theta\right)^2 + \left(X_S I_A \cos\theta\right)^2$$

Then:

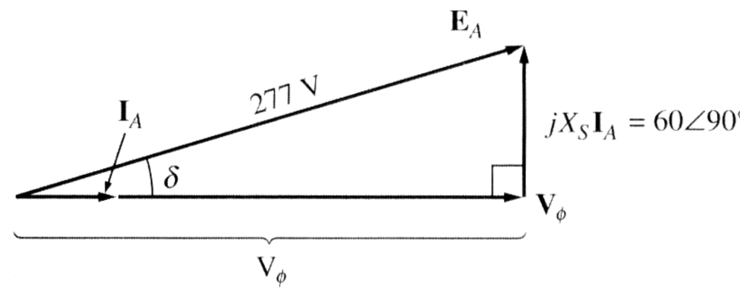
$$V_\phi = \sqrt{E_A^2 - \left(X_S I_A \cos\theta\right)^2} - X_S I_A \sin\theta = 236.8\,V$$

➢ Since the generator is Y-connected,

$$V_T = \sqrt{3}V_\phi = 410\,V$$

b.2. For the generator at the rated current and the 1.0 PF, the phasor diagram is shown.
Then:

$$V_\phi = \sqrt{E_A^2 - (X_S I_A \cos\theta)^2} - X_S I_A \sin\theta = 270.4\,V$$

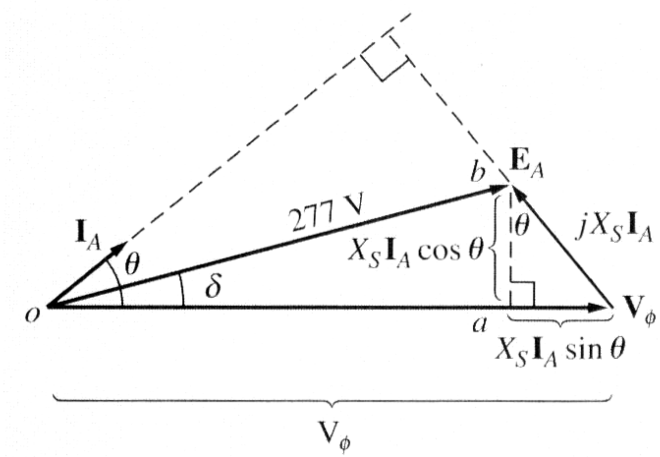


and

$$V_T = \sqrt{3}V_\phi = 468.4\,V$$

b.3. For the generator at the rated current and the 0.8 PF leading, the phasor diagram is shown.
Then:

$$V_\phi = \sqrt{E_A^2 - (X_S I_A \cos\theta)^2} - X_S I_A \sin\theta = 308.8\,V$$



and

$$V_T = \sqrt{3}V_\phi = 535\,V$$

c. The output power of the generator at 60 A and 0.8 PF lagging is

$$P_{out} = 3V_{\phi}I_A \cos\theta = 3 \cdot 236.8 \cdot 60 \cdot 0.8 = 34.1\,kW$$

- The mechanical input power is given by

$$P_{in} = P_{out} + P_{elec\ loss} + P_{core\ loss} + P_{mech\ loss} = 34.1 + 0 + 1.0 + 1.5 = 36.6\,kW$$

- The efficiency is

$$\eta = \frac{P_{out}}{P_{in}} \cdot 100\,\% = \frac{34.1}{36.6} \cdot 100\% = 93.2\%$$

d. The input torque of the generator is

$$\tau_{app} = \frac{P_{in}}{\omega_m} = \frac{36.6}{125.7} = 291.2\ \ N\text{-}m$$

- The induced counter torque of the generator is

$$\tau_{app} = \frac{P_{conv}}{\omega_m} = \frac{34.1}{125.7} = 271.3 \;\; N\text{-}m$$

e. The voltage regulation of the generator is

Lagging PF:

$$VR = \frac{480-410}{410} \cdot 100\% = 17.1\%$$

Unity PF:

$$VR = \frac{480-468}{468} \cdot 100\% = 2.6\%$$

Lagging PF:

$$VR = \frac{480-535}{535} \cdot 100\% = -10.3\%$$