

**Course:** EEL 2043 Principles of Electric Machines

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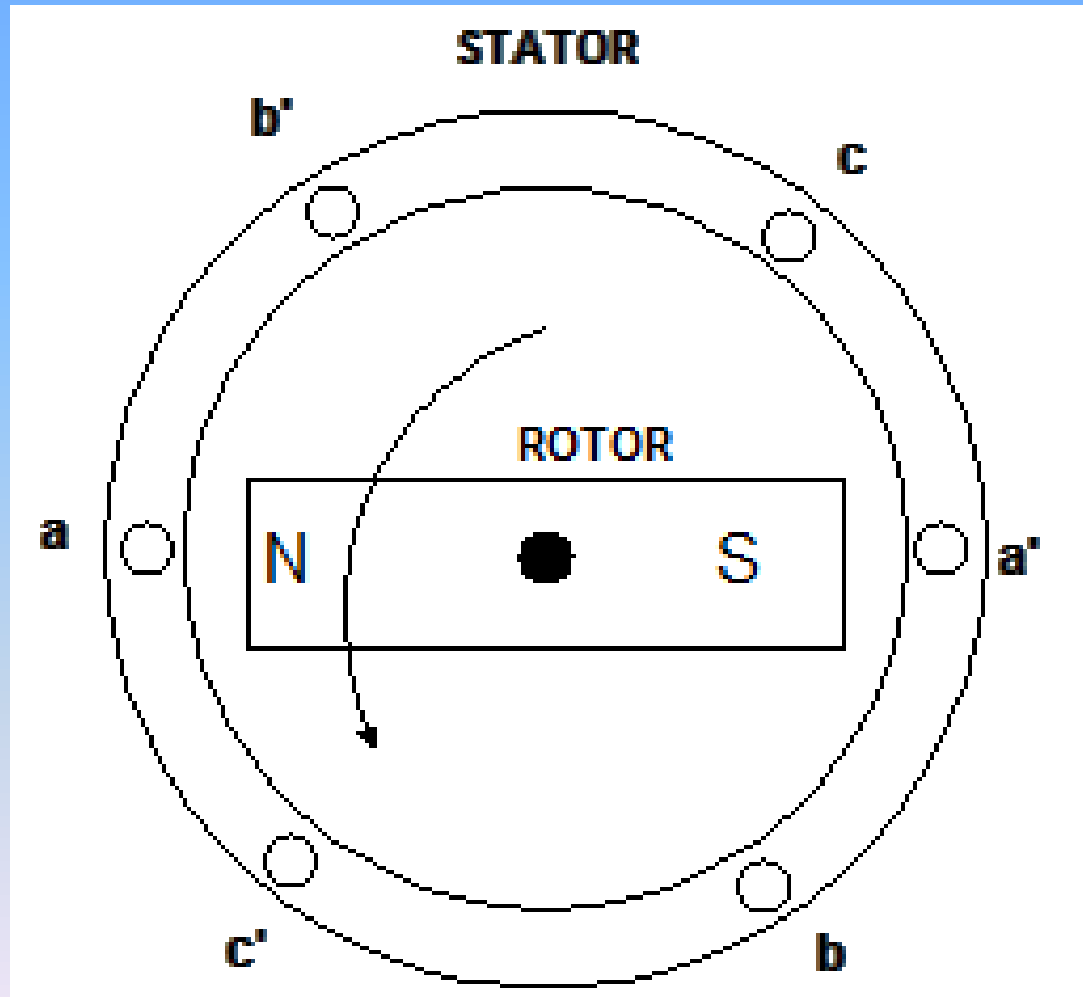
كليات التقنية العليا  
HIGHER COLLEGES OF TECHNOLOGY

## LO 1: Three Phase Circuits

# Three Phase AC System

- Three phase is generated by a generator with three sets of independent windings which are physically spaced 120 degrees around the stator.
- Voltages are labeled phase a, phase b, and phase c and have the same magnitude but differ in phase angle by 120 degrees.

# Three Phase AC System



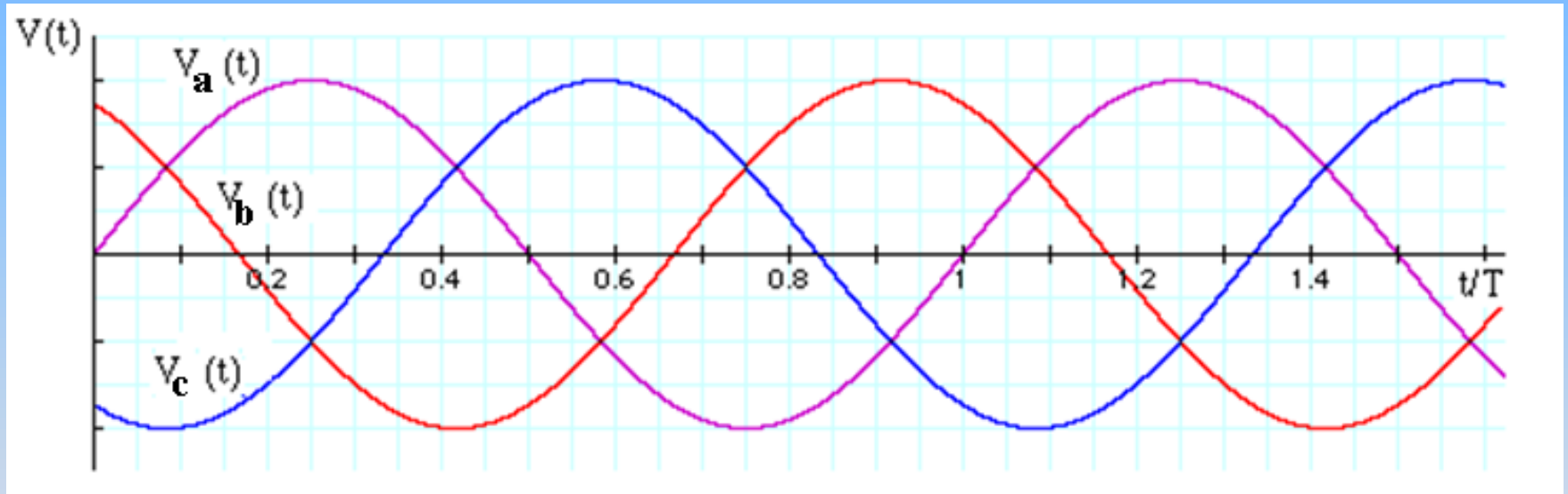
# Three Phase AC System

<http://www.youtube.com/watch?v=Cq3Yzyv88pU>

<http://www.youtube.com/watch?v=HLNugJwBRow>

# Three Phase AC System

The waveform of a three-phase ac generator

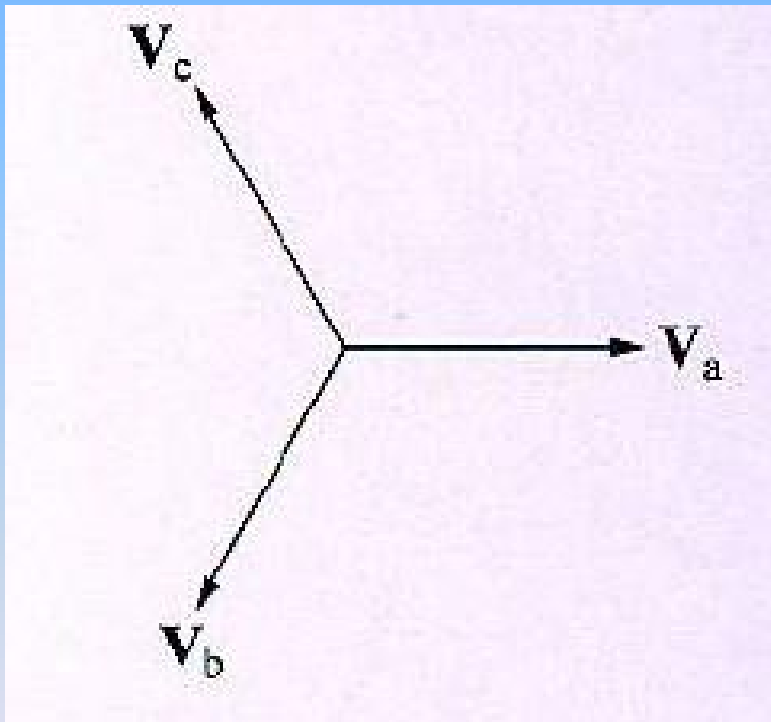


# Three Phase AC System

- Three-phase systems have either three or four conductors.
- There are three-phase conductors identified as A, B, and C.
- The three phases are 120 degrees out of phase with each other (360 divided by 3).
- There is sometimes a fourth conductor, which is the neutral.

# Three Phase AC System

The *abc* (or *positive*) phase sequence



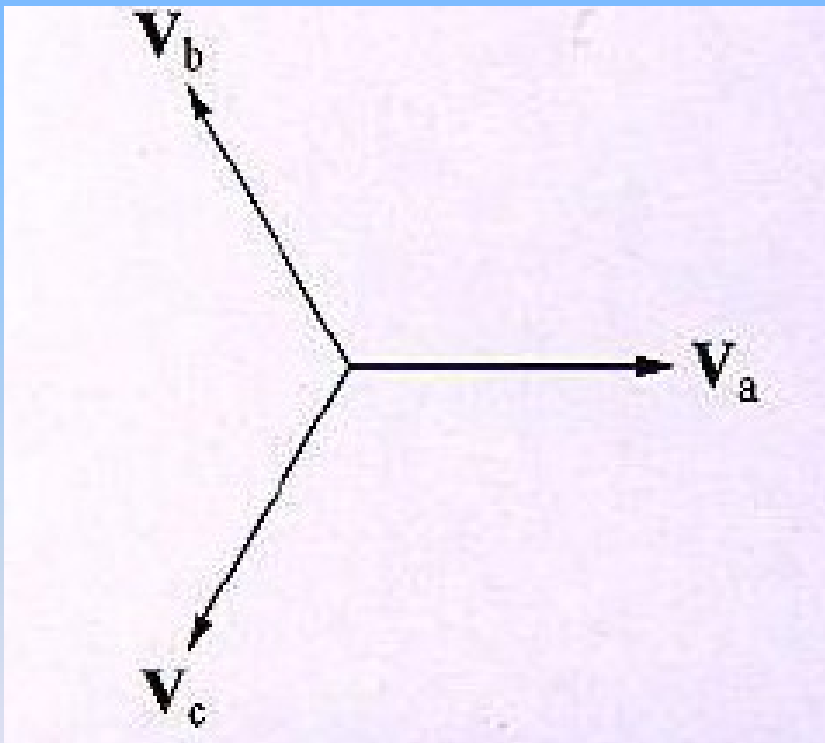
$$V_a = V_m \angle 0^\circ,$$

$$V_b = V_m \angle -120^\circ,$$

$$V_c = V_m \angle +120^\circ,$$

# Three Phase AC System

The *acb* (or *negative*) phase sequence



$$\mathbf{V}_a = V_m \angle 0^\circ,$$

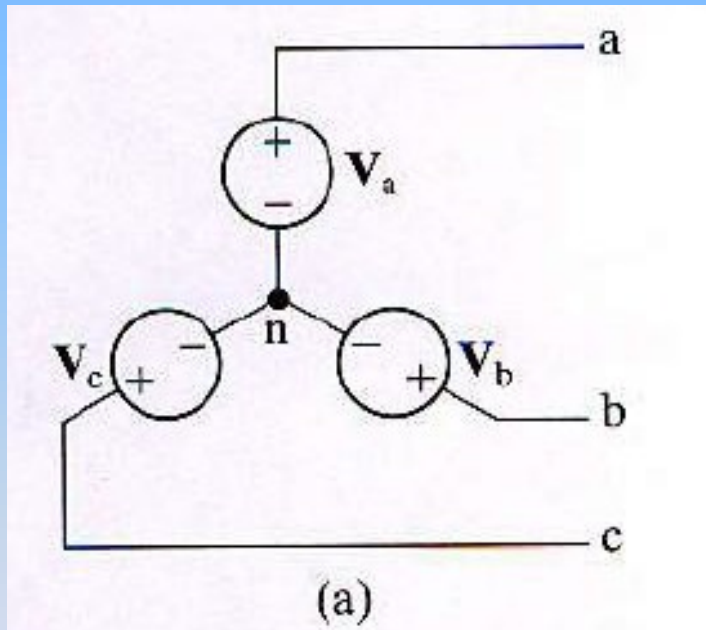
$$\mathbf{V}_b = V_m \angle +120^\circ,$$

$$\mathbf{V}_c = V_m \angle -120^\circ.$$

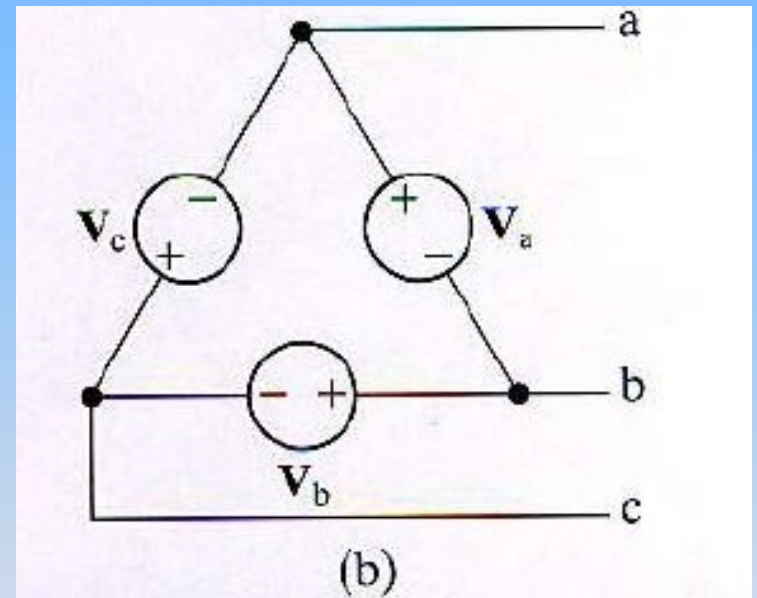


# Three Phase AC System

- The two basic connections of ac three-phase source

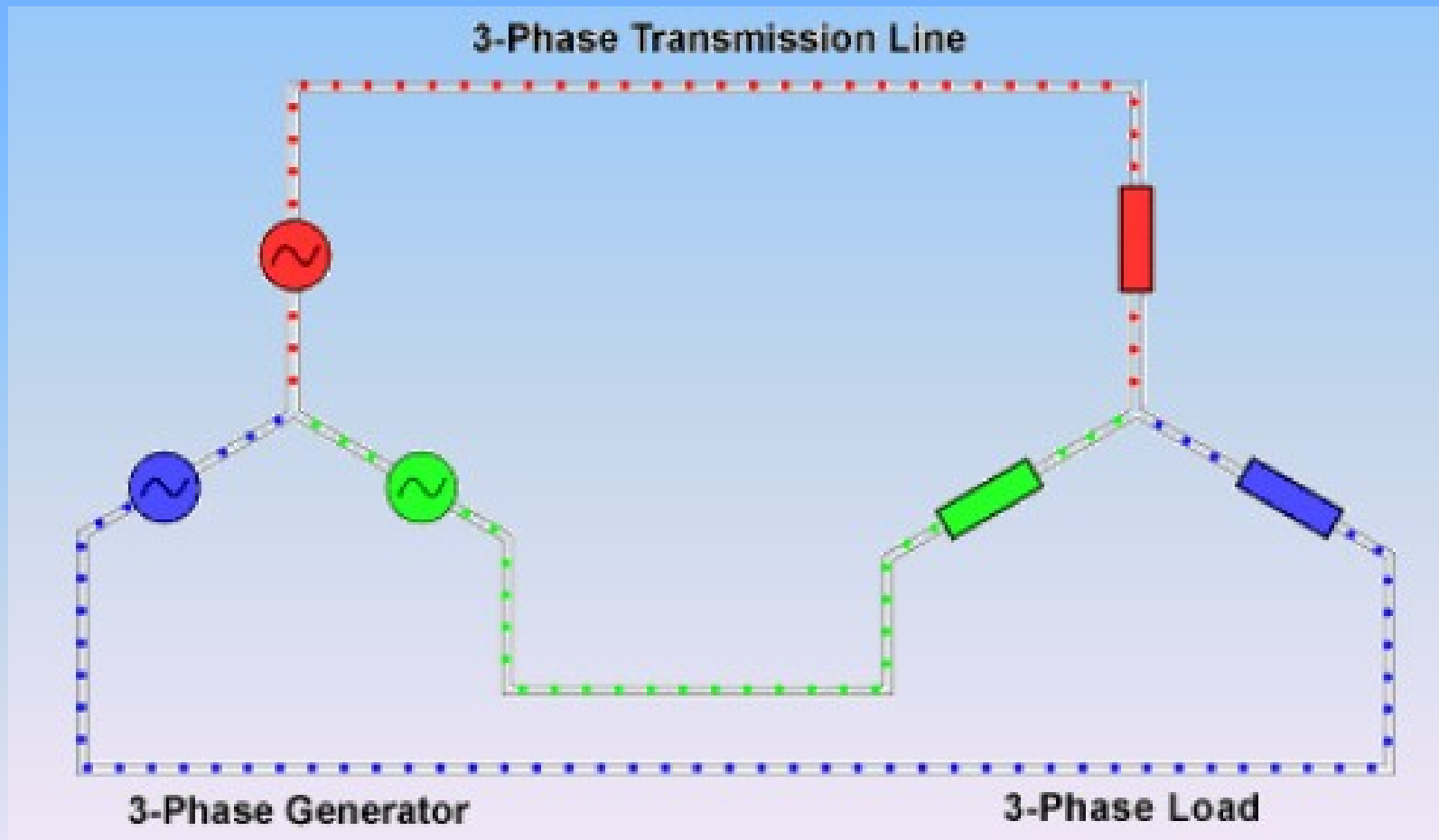


Y-connected source



$\Delta$ -connected source

# Three Phase AC System



# Three Phase AC System

- **Why use 3-phase?**
  - 3-phase motors, generators, and transformers are more efficient
  - Smooth torque on generator shaft
  - Delivery of constant power to a 3-phase load
  - 3 or 4 Wires and not 6
  - Can deliver more power for a given weight and cost
  - Voltage regulation is better

# Three Phase AC System

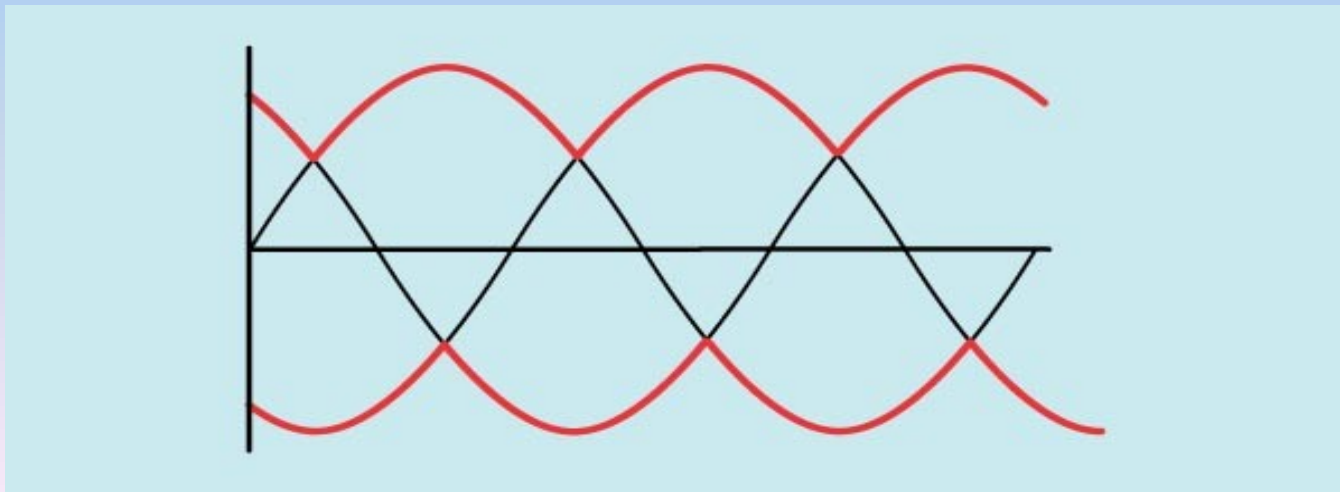
- **Why use 3-phase?**

The horsepower rating of three-phase motors and the kVA rating of three-phase transformers are 150% greater than single-phase motors or transformers of similar frame size.

# Three Phase AC System

- **Why use 3-phase?**

The power delivered by a single-phase system pulsates and falls to zero. The three-phase power never falls to zero. The power delivered to the load in a three-phase system is the same at any instant. This produces superior operating characteristics for three-phase motors.



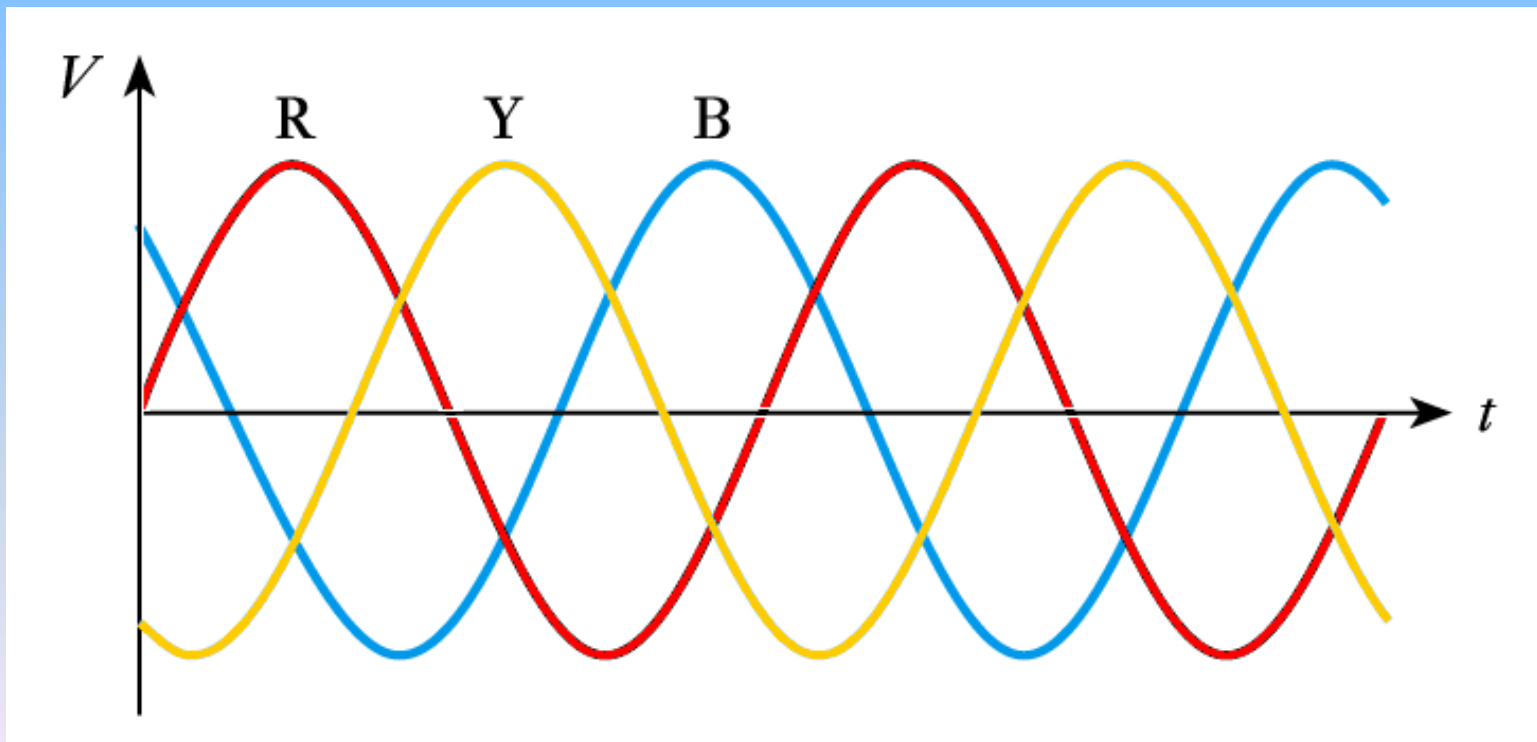
# Three Phase AC System

- **Why use 3-phase?**

A three-phase system needs three conductors; however, each conductor is only 75% the size of the equivalent kVA rated single-phase conductors.

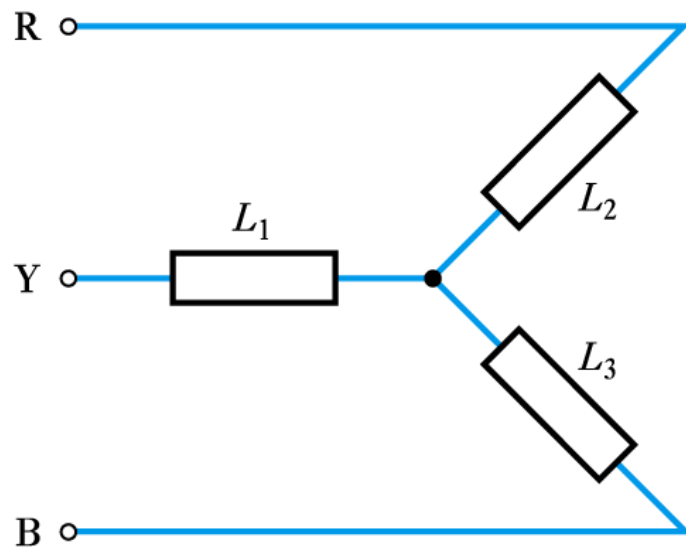
# Three-Phase Systems

- Relationship between the phases in a three-phase arrangement

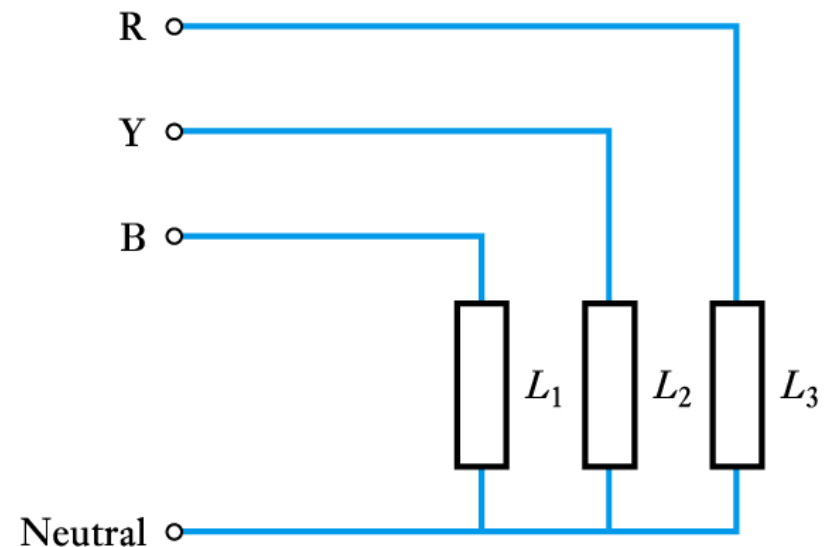


# Three-Phase Systems

- Three-phase arrangements may use either 3 or 4 conductors



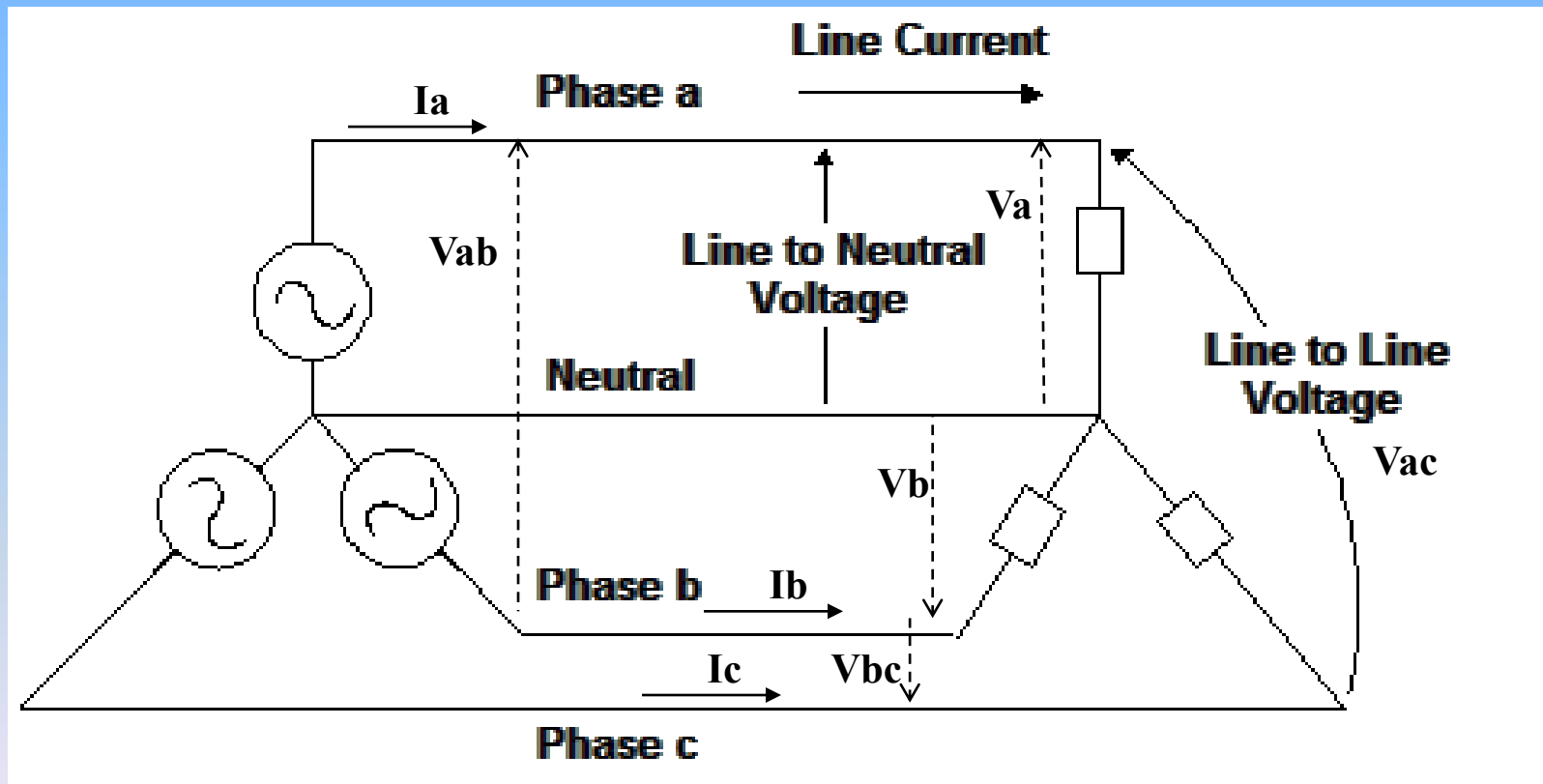
(a)



(b)

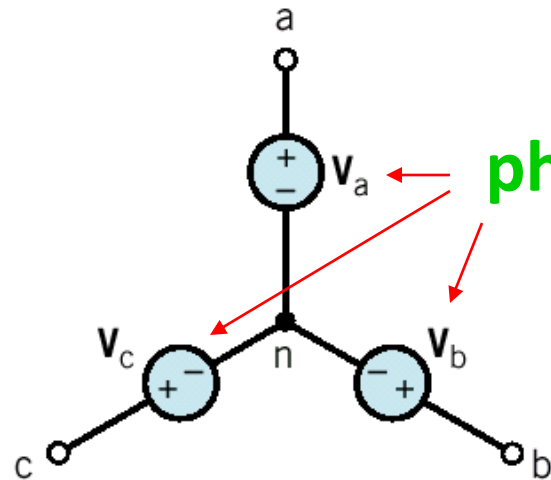


# Three Phase AC System



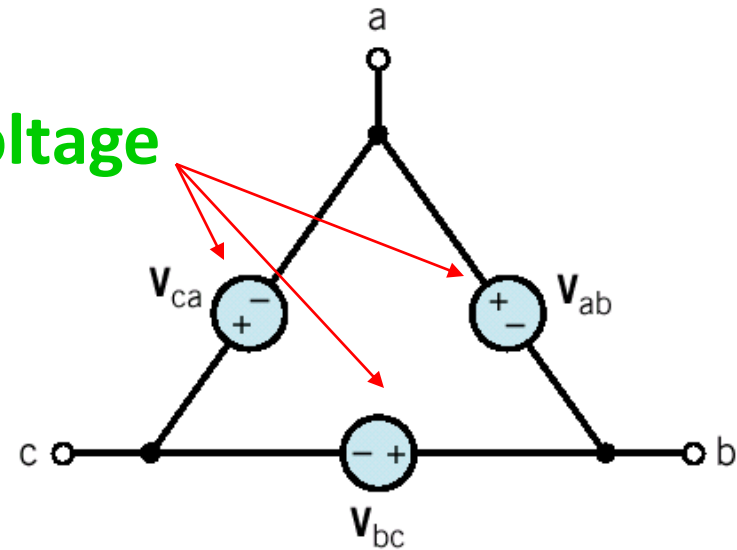
# Star/Delta Connections

# Two Common Methods Three-phase Connection



(a)

(a) Y-connected sources



(b)

(b)  $\Delta$ -connected sources

phase voltage

# Star Connection

- The wye, or star, connection is made by connecting one end of each of the phase windings together in a common node.
- Each phase winding has a voltage drop known as the phase voltage.
- The line voltage is measured from phase conductor to a different phase conductor.

# Star Connection

- The line voltage is higher than the phase voltage by a factor of the square root of 3 (1.732)

$$V_L = \sqrt{3} \times V_p$$

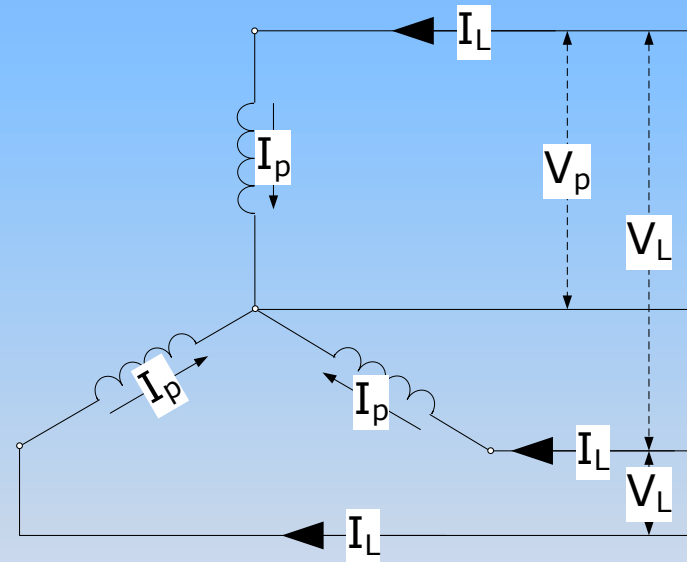
- The line current is equal to the phase current

$$I_L = I_p$$

# Star Connection

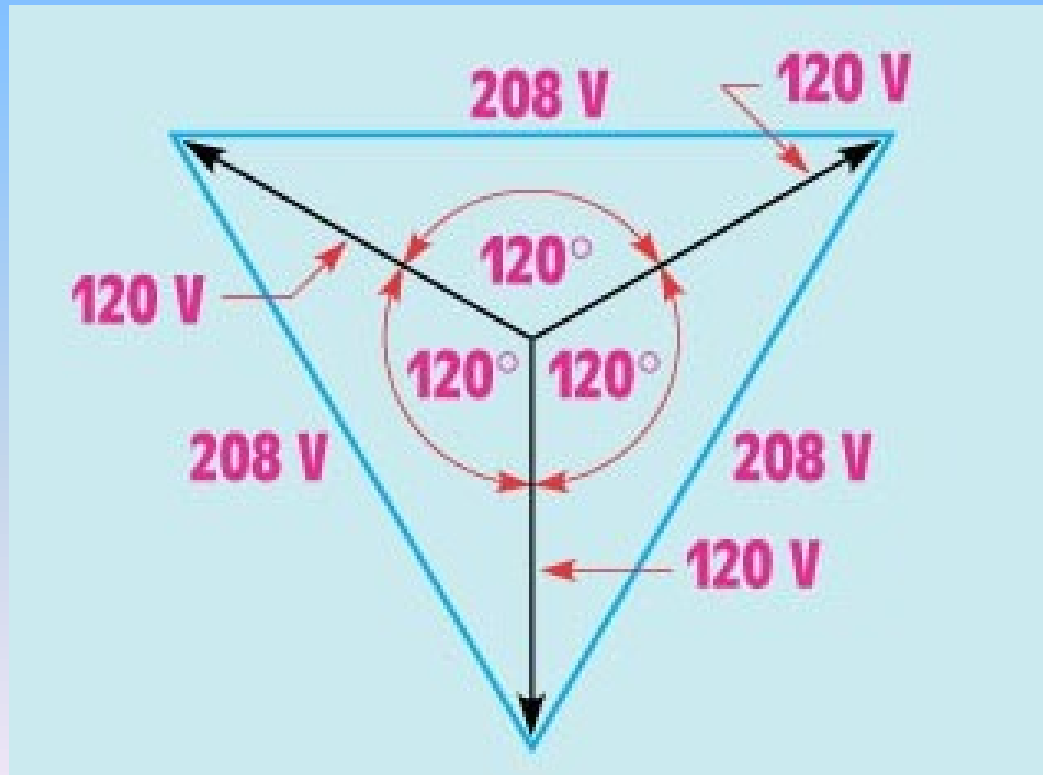
$$I_L = I_p$$

$$V_L = \sqrt{3} \times V_p$$

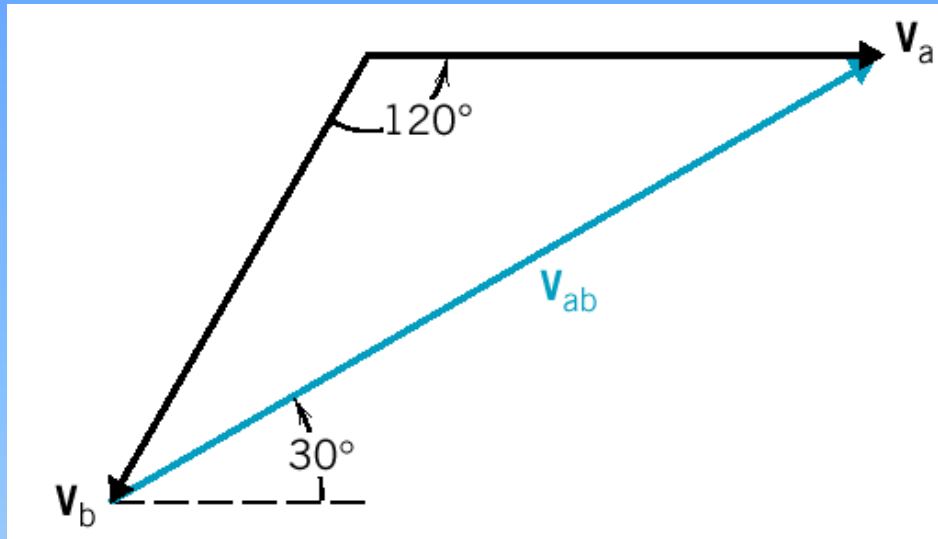


# Star Connection

- Vector sum of typical wye system voltages



# Phase and Line Voltages



The line-to-line voltage  $V_{ab}$  of the Y-connected source

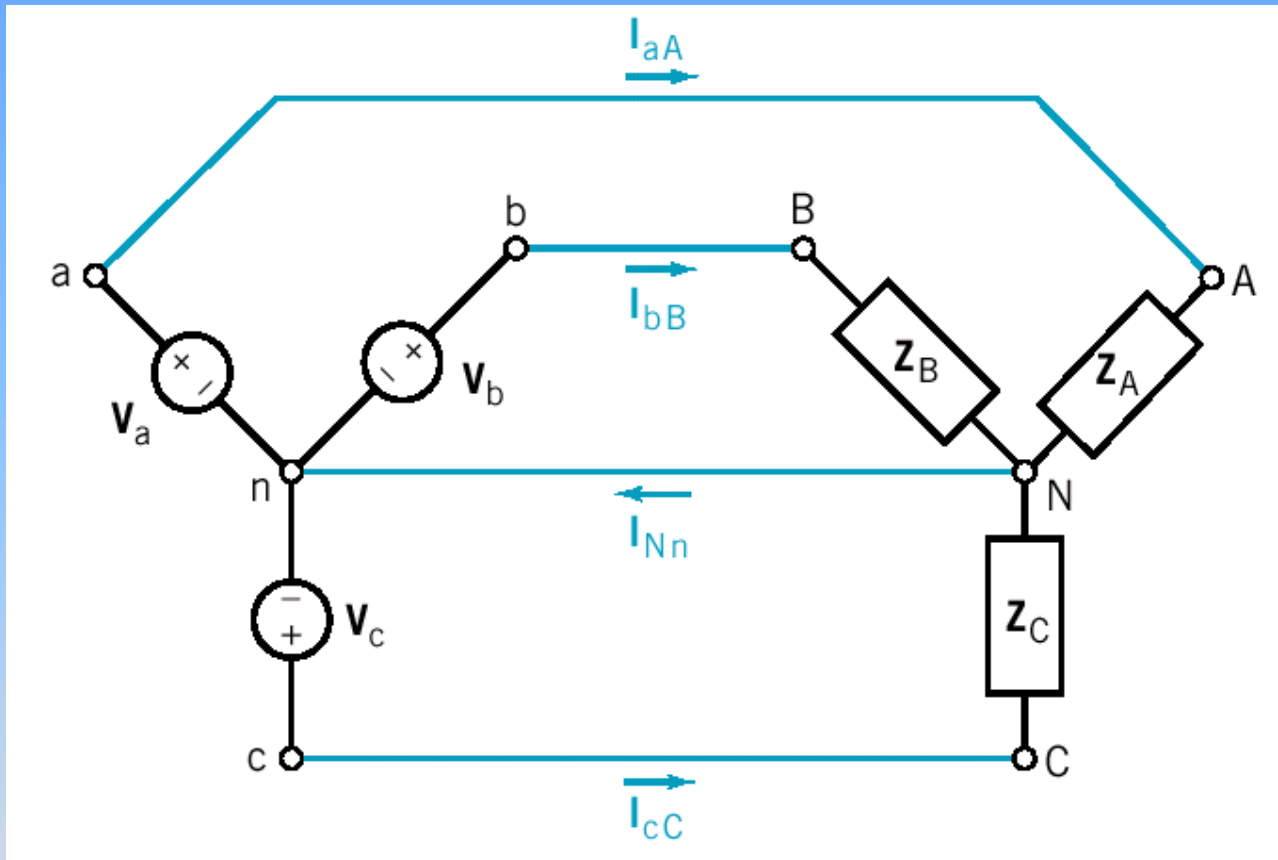
$$\begin{aligned} V_{ab} &= V_a - V_b \\ &= V_p \angle 0^\circ - V_p \angle -120^\circ \\ &= V_p - V_p(-0.5 - j0.866) \\ &= \sqrt{3}V_p \angle 30^\circ \end{aligned}$$

Similarly

$$\begin{aligned} V_{bc} &= \sqrt{3}V_p \angle -90^\circ \\ V_{ca} &= \sqrt{3}V_p \angle -210^\circ \end{aligned}$$



# The Y-to-Y Circuit



A four-wire Y-to-Y circuit

## *Four - wire*

$$\triangleright \mathbf{I}_{aA} = \frac{\mathbf{V}_a}{\mathbf{Z}_A},$$

$$\triangleright \mathbf{I}_{bB} = \frac{\mathbf{V}_b}{\mathbf{Z}_B}, \text{ and}$$

$$\triangleright \mathbf{I}_{cC} = \frac{\mathbf{V}_c}{\mathbf{Z}_C}$$

$$\mathbf{I}_{nN} = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC}$$

The average power delivered by the three-phase source to the three-phase load

$$P = P_A + P_B + P_C$$

➤ When  $\mathbf{Z}_A = \mathbf{Z}_B = \mathbf{Z}_C$  the load is said to be *balanced*

➤ 
$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a}{\mathbf{Z}_A} = \frac{V \angle 0^\circ}{Z \angle \theta} \longrightarrow \mathbf{I}_{aA} = \frac{V}{Z} \angle -\theta^\circ$$

➤ 
$$\mathbf{I}_{bB} = \frac{\mathbf{V}_b}{\mathbf{Z}_B} = \frac{V \angle -120^\circ}{Z \angle \theta} \longrightarrow \mathbf{I}_{bB} = \frac{V}{Z} \angle (-\theta - 120^\circ)$$

➤ 
$$\mathbf{I}_{cC} = \frac{\mathbf{V}_c}{\mathbf{Z}_C} = \frac{V \angle 120^\circ}{Z \angle \theta} \longrightarrow \mathbf{I}_{cC} = \frac{V}{Z} \angle (-\theta + 120^\circ)$$

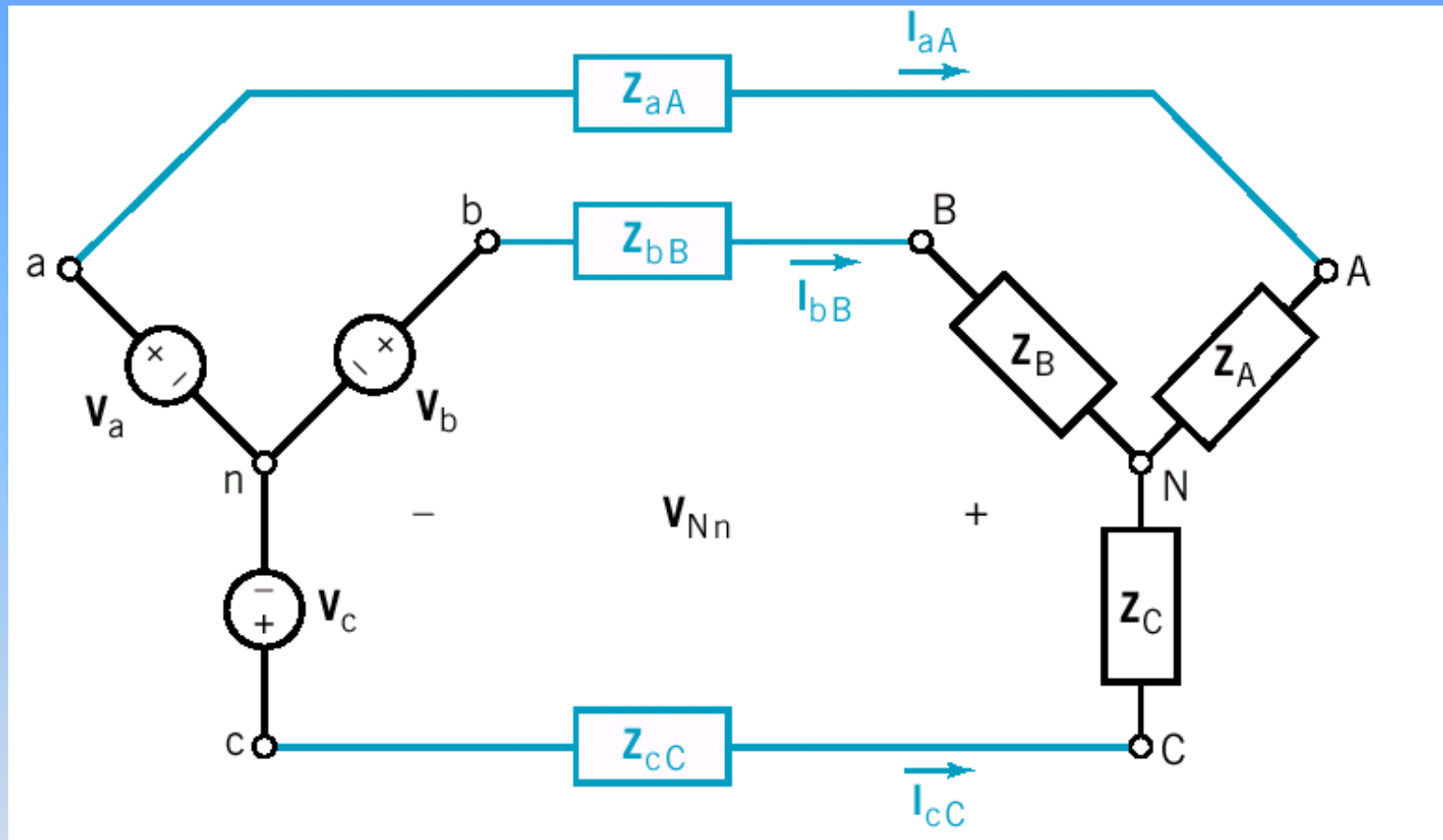
- There is no current in the wire connecting the neutral node of the source to the neutral node of the load.

$$\mathbf{I}_{nN} = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC} = 0$$

- The average power delivered to the load is:

$$\begin{aligned} P &= P_A + P_B + P_C \\ &= V \frac{V}{Z} \cos(-\theta) + V \frac{V}{Z} \cos(-\theta) + V \frac{V}{Z} \cos(-\theta) \\ &= 3 \frac{V^2}{Z} \cos(\theta) \end{aligned}$$

# The Y-to-Y Circuit (3-wire)



A three-wire Y-to-Y circuit

## *$\Upsilon$ - $\Upsilon$ Three - wire*

We need to solve for  $\mathbf{V}_{Nn}$

$$0 = I_{aA} + I_{bB} + I_{cC}$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a - \mathbf{V}_{Nn}}{\mathbf{Z}_A}, \mathbf{I}_{bB} = \frac{\mathbf{V}_b - \mathbf{V}_{Nn}}{\mathbf{Z}_B}, \text{ and } \mathbf{I}_{cC} = \frac{\mathbf{V}_c - \mathbf{V}_{Nn}}{\mathbf{Z}_C}$$

$$\begin{aligned} 0 &= \frac{\mathbf{V}_a - \mathbf{V}_{Nn}}{\mathbf{Z}_A} + \frac{\mathbf{V}_b - \mathbf{V}_{Nn}}{\mathbf{Z}_B} + \frac{\mathbf{V}_c - \mathbf{V}_{Nn}}{\mathbf{Z}_C} \\ &= \frac{V \angle 0^\circ - \mathbf{V}_{Nn}}{\mathbf{Z}_A} + \frac{V \angle -120^\circ - \mathbf{V}_{Nn}}{\mathbf{Z}_B} + \frac{V \angle 120^\circ - \mathbf{V}_{Nn}}{\mathbf{Z}_C} \end{aligned}$$

➤ Solve for  $\mathbf{V}_{Nn}$

$$\mathbf{V}_{Nn} = \frac{(V \angle -120^\circ) \mathbf{Z}_A \mathbf{Z}_C + V \angle 120^\circ \mathbf{Z}_A \mathbf{Z}_B + V \angle 0^\circ \mathbf{Z}_B \mathbf{Z}_C}{\mathbf{Z}_A \mathbf{Z}_C + \mathbf{Z}_A \mathbf{Z}_B + \mathbf{Z}_B \mathbf{Z}_C}$$

➤ When the circuit is **balanced** i.e.  $Z_A = Z_B = Z_C$

$$\begin{aligned} \mathbf{V}_{Nn} &= \frac{(V \angle -120^\circ)\mathbf{ZZ} + V \angle 120^\circ\mathbf{ZZ} + V \angle 0^\circ\mathbf{ZZ}}{\mathbf{ZZ} + \mathbf{ZZ} + \mathbf{ZZ}} \\ &= 0 \end{aligned}$$

➤ The average power delivered to the load is:

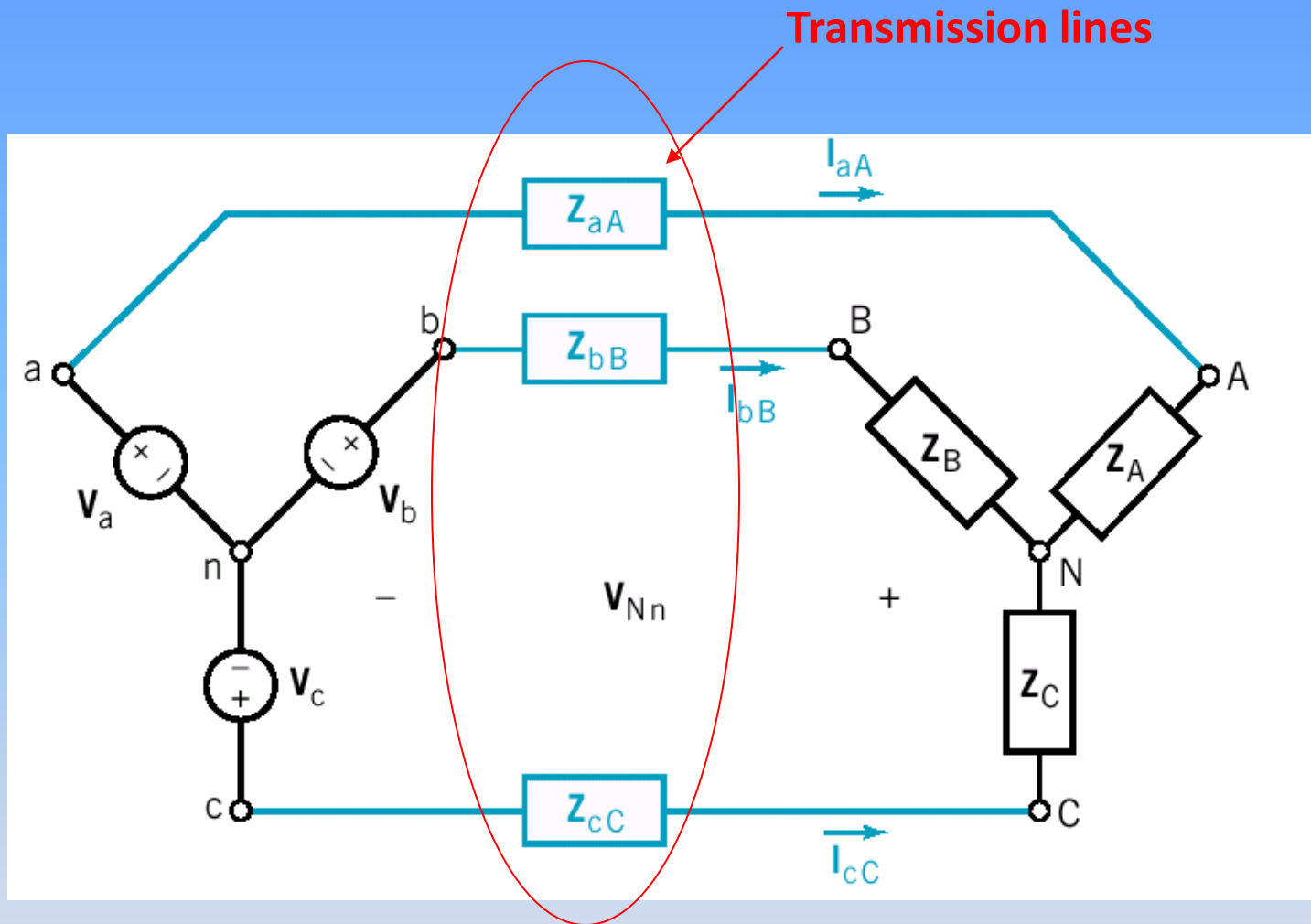
$$\begin{aligned} P &= P_A + P_B + P_C \\ &= 3 \frac{V^2}{Z} \cos(\theta) \end{aligned}$$

➤ or

$$P = 3 V I \cos \theta,$$

$$Q = 3 V I \sin \theta,$$

$$S = 3 V I^*$$

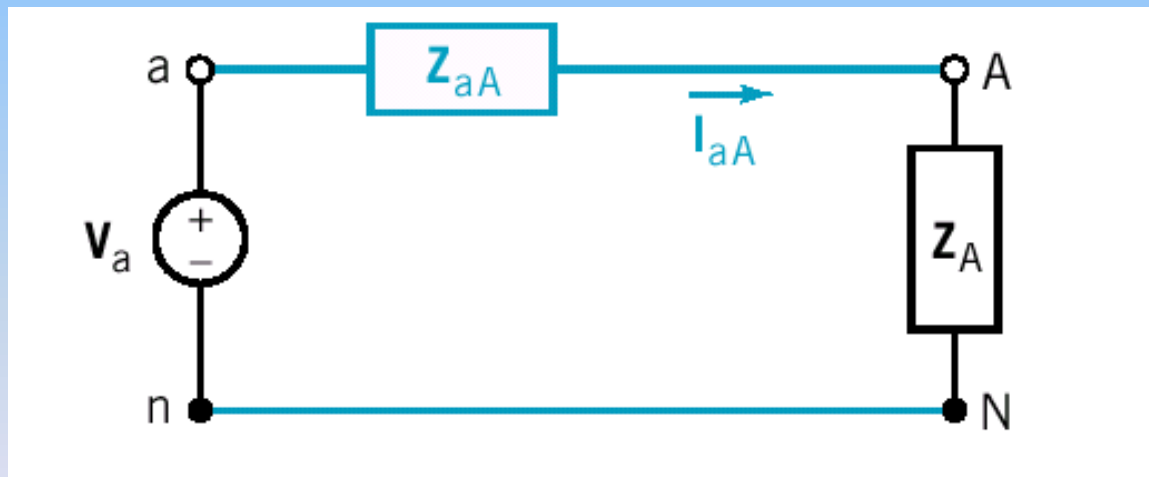


A three-wire Y-to-Y circuit with line impedances



➤ The analysis of **balanced Y-Y circuits** is **simpler** than the analysis of **unbalanced Y-Y circuits**.

- ✚  $V_{Nn} = 0$ . It is not necessary to solve for  $V_{Nn}$ .
- ✚ The line currents have equal magnitudes and differ in phase by 120 degree.
- ✚ Equal power is absorbed by each impedance.



Per-phase equivalent circuit

# Delta Connection

- The line current is higher than the phase current by a factor of the square root of 3 (1.732)

$$I_L = \sqrt{3} \times I_P$$

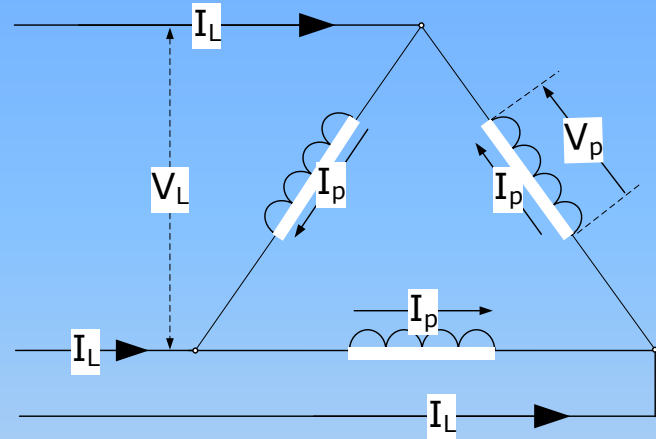
- The line current is equal to the phase current

$$V_L = V_P$$

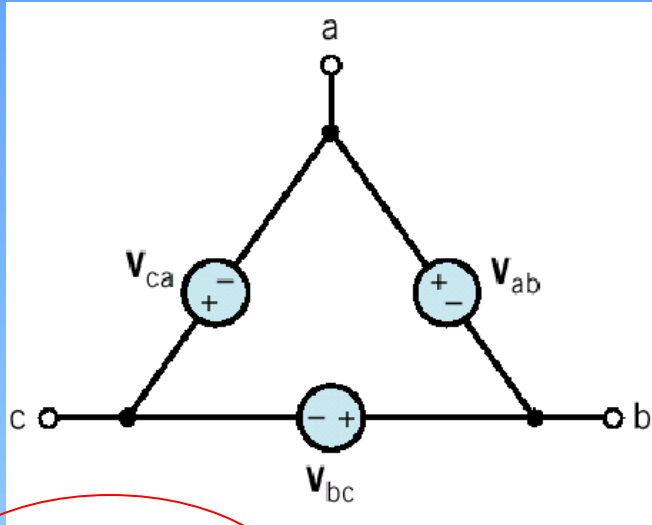
# Delta Connection

$$V_L = V_P$$

$$I_L = \sqrt{3} \times I_P$$



# The $\Delta$ -Connected Source and Load



$$V_{ab} = 120 \angle 0^\circ \quad V_{\text{rms}}$$

$$V_{bc} = 120.1 \angle -121^\circ \quad V_{\text{rms}}$$

$$V_{ca} = 120.2 \angle 121^\circ \quad V_{\text{rms}}$$

Circulating  
current

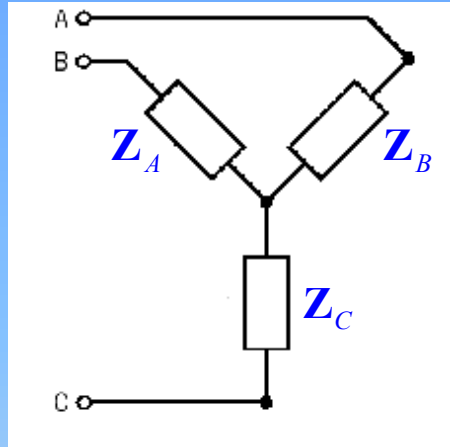
$$\mathbf{I} = \frac{(V_{ab} + V_{bc} + V_{ca})}{1} \approx -3.75 \text{ A}$$

**Unacceptable**

Total resistance around the loop

Therefore the  $\Delta$  sources connection is seldom used in practice.

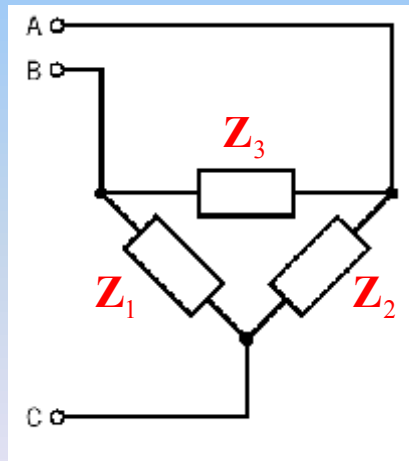
# The $\Delta$ -Y and Y- $\Delta$ Transformation



$$Z_A = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3}$$

$$Z_B = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3}$$

$$Z_C = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$$

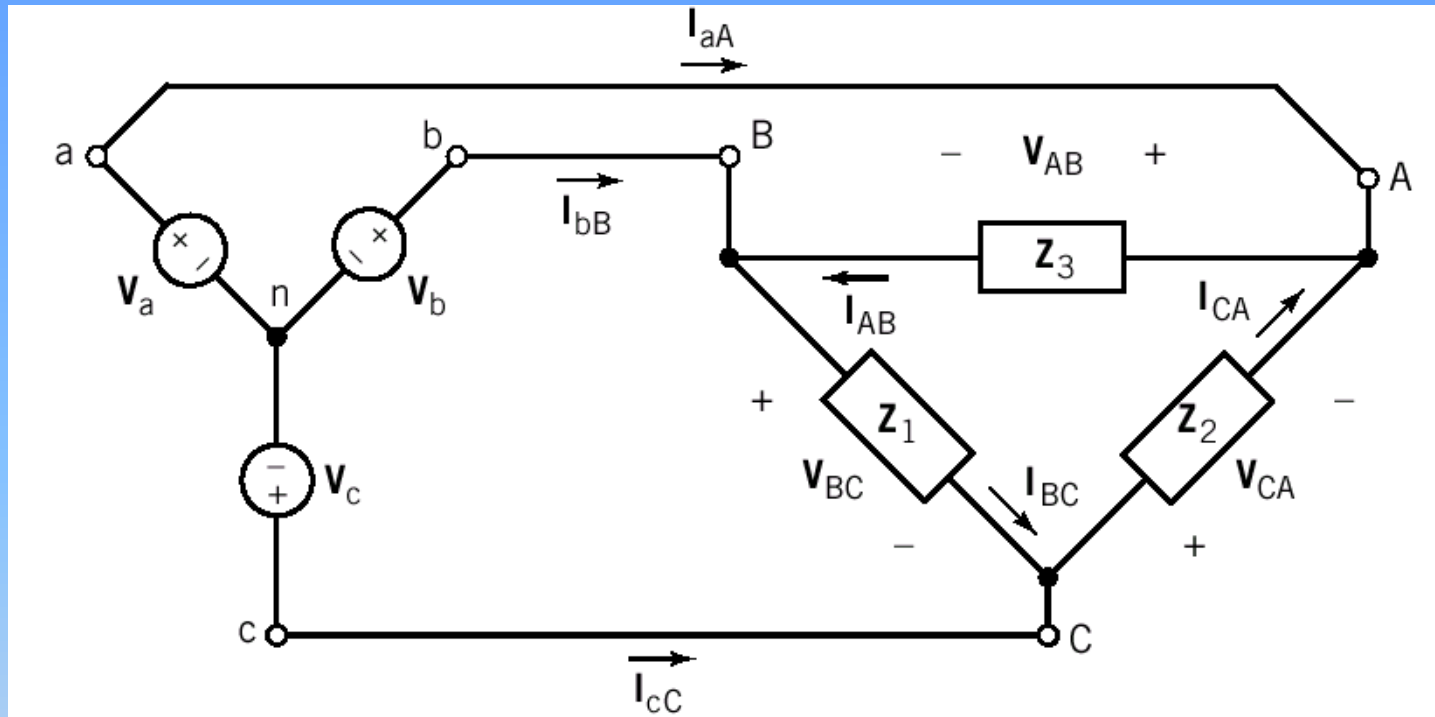


$$Z_1 = \frac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_B}$$

$$Z_2 = \frac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_A}$$

$$Z_3 = \frac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_C}$$

# The Y- $\Delta$ Circuits



$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$

$$\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB}$$

$$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

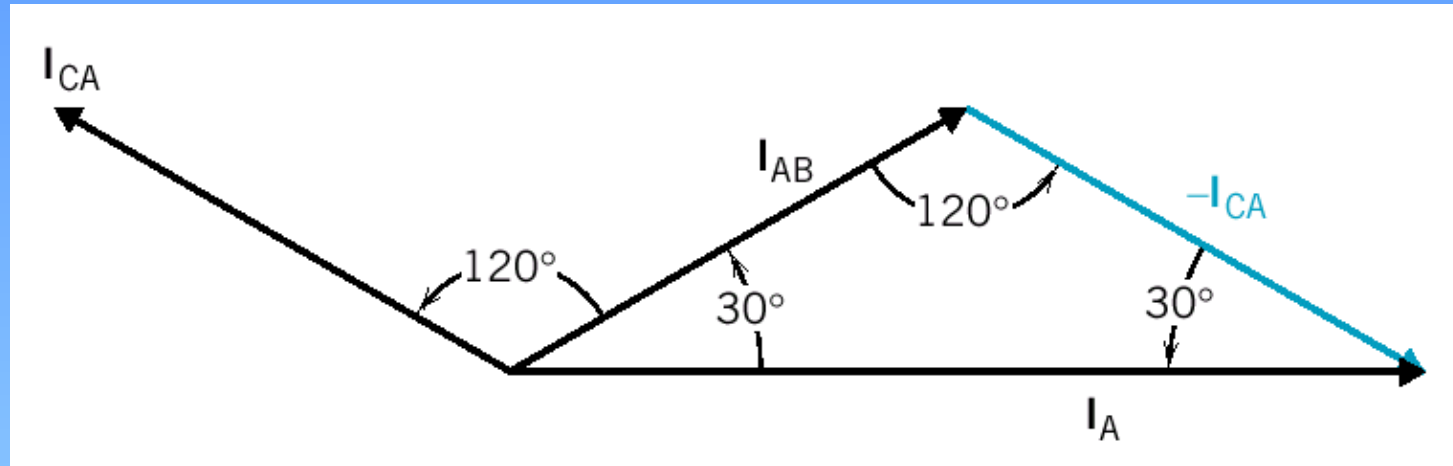
where

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_3}$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_1}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_2}$$

## The Y- $\Delta$ Circuits (cont.)

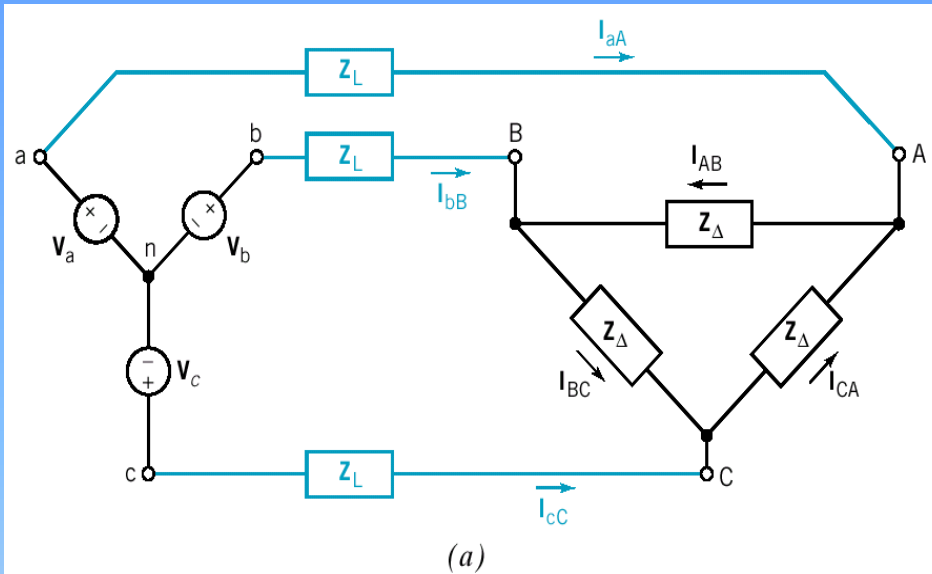


$$\begin{aligned}\mathbf{I}_{aA} &= \mathbf{I}_{AB} - \mathbf{I}_{CA} \\ &= I \cos \phi + j \sin \phi - I \cos(\phi + 120^\circ) - j \sin(\phi + 120^\circ) \\ &= \sqrt{3}I \angle(\phi - 30^\circ)\end{aligned}$$

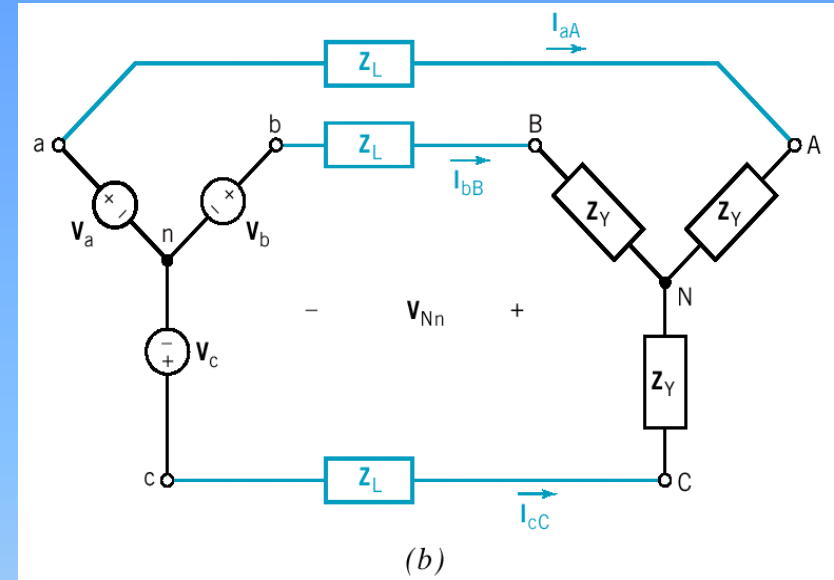
or

$$|\mathbf{I}_{aA}| = \sqrt{3}|I| \quad \Rightarrow \quad I_L = \sqrt{3}I_p$$

# The Balanced Three-Phase Circuits

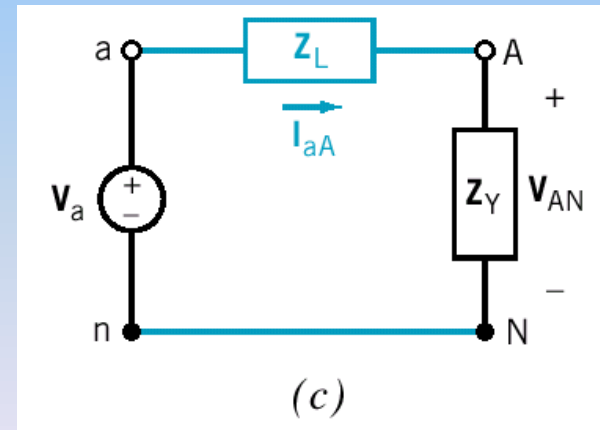


Y-to-Δ circuit



equivalent Y-to-Y circuit

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$$



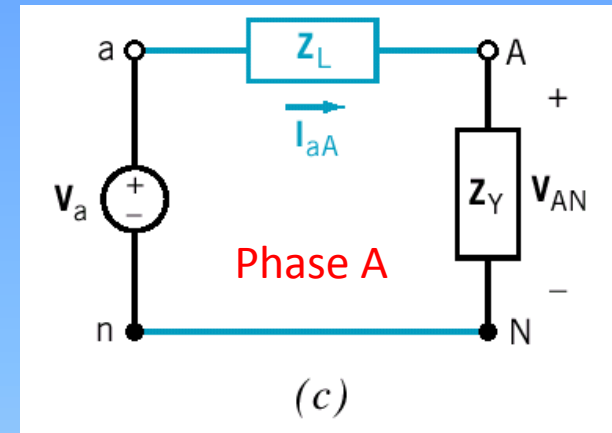
per-phase equivalent circuit



# Instantaneous and Average Power

The total average power delivered to the balanced Y-connected load is

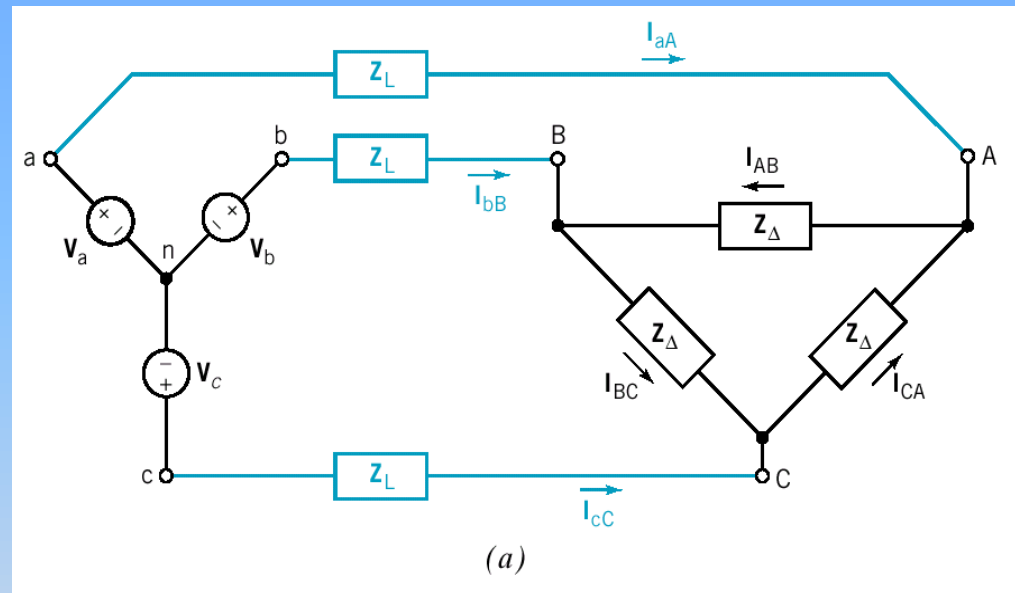
$$\mathbf{I}_{aA} = I_L \angle \theta_{AI}, \mathbf{V}_{AN} = V_P \angle \theta_{AV}$$



$$\begin{aligned} P_Y &= 3P_A = 3V_P I_L \cos(\theta_{AV} - \theta_{AI}) \\ &= 3V_P I_L \cos(\theta) \\ &= 3 \frac{V_L}{\sqrt{3}} I_L \cos(\theta) \\ &= \sqrt{3} V_L I_L \cos(\theta) \end{aligned}$$

# Instantaneous and Average Power in Balanced Three Phase Circuits

The total average power delivered to the balanced  $\Delta$ -connected load is



$$P = 3P_{AB} = 3V_{AB} \times I_{AB} \times \cos \theta$$

$$P = 3 \times V_P \times I_P \times \cos \theta$$

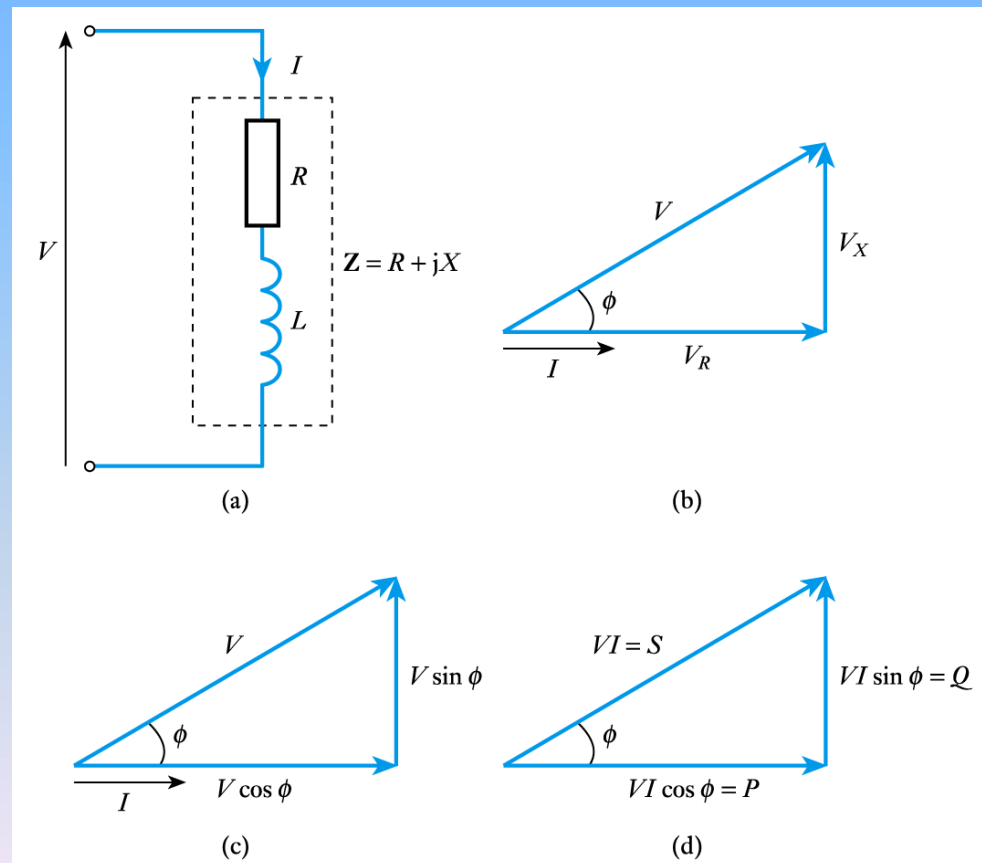
$$P = \sqrt{3} \times V_L \times I_L \times \cos \theta$$

# Active and Reactive Power

- When a circuit has resistive and reactive parts, the resultant power has 2 parts:
  - The first is *dissipated* in the resistive element. This is the **active power,  $P$**
  - The second is *stored and returned* by the reactive element. This is the **reactive power,  $Q$** , which has units of **volt amperes reactive** or **var**
- While reactive power is not dissipated it does have an effect on the system
  - for example, it increases the current that must be supplied and increases losses with cables

# Active and Reactive Power

- Consider an RL circuit
  - the relationship between the various forms of power can be illustrated using a power triangle



# Three Phase Power

- Single phase power

$$P = V \times I \times \cos \theta$$

- In a balanced 3 phase system the total power is equal to three times the power in any one phase

$$P = 3 \times V_P \times I_P \times \cos \theta$$

- In star and delta, 3 phase power can also be calculated using this formula

$$P = \sqrt{3} \times V_L \times I_L \times \cos \theta$$

- Note that this is the same on wye or delta systems

# Active and Reactive Power

- For Single Phase:

Active Power      $P = V \times I \times \cos \theta$      watts

Reactive Power      $Q = V \times I \times \sin \theta$      var

Apparent Power      $S = V I^*$      VA

$$S^2 = P^2 + Q^2$$

# Active and Reactive Power

- For 3-Phase:

Active Power	$P = 3 \times V_p \times I_p \times \cos \theta$	watts
--------------	--	-------

Reactive Power	$Q = 3 \times V_p \times I_p \times \sin \theta$	var
----------------	--	-----

Apparent Power	$S = 3 \times V_p \times I_p^*$	VA
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$$S^2 = P^2 + Q^2$$

# Active and Reactive Power

- For 3-Phase:

Active Power  $P = \sqrt{3} \times V_L \times I_L \times \cos \theta$  watts

Reactive Power  $Q = \sqrt{3} \times V_L \times I_L \times \sin \theta$  var

Apparent Power  $S = \sqrt{3} \times V_L \times I_L^*$  VA

$$S^2 = P^2 + Q^2$$

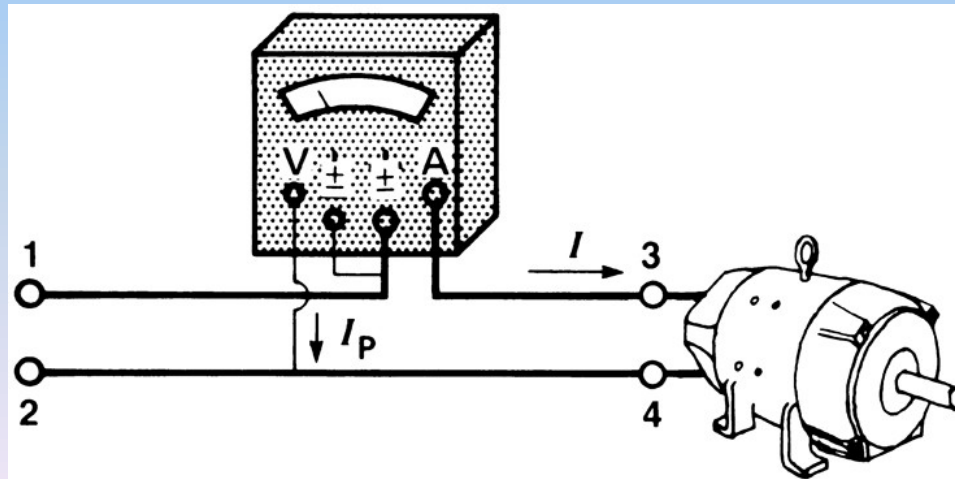


# Power Measurement

- When using AC, *power* is determined not only by the r.m.s. values of the **voltage** and **current**, but also by the **phase angle** (which determines the **power factor**)
  - consequently, you cannot determine the power from independent measurements of current and voltage

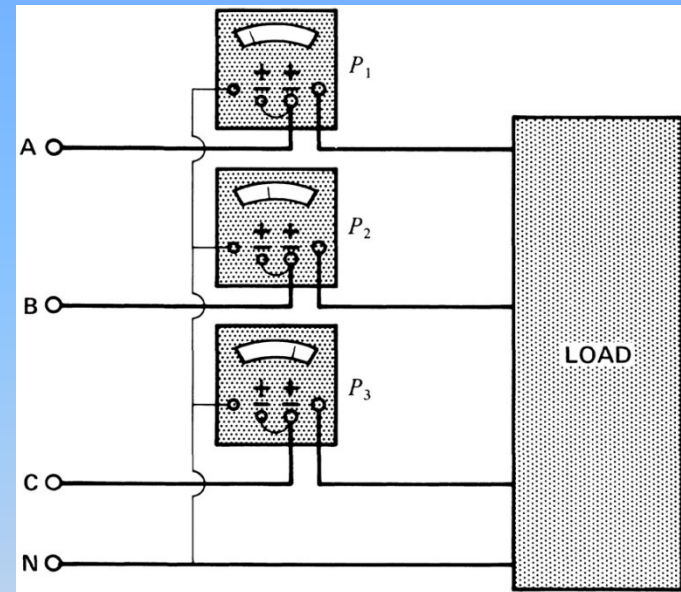
# Power Measurement

- In **single-phase systems** power is normally measured using a **watt-meter**
  - measures power directly using a single meter which effectively multiplies instantaneous current and voltage



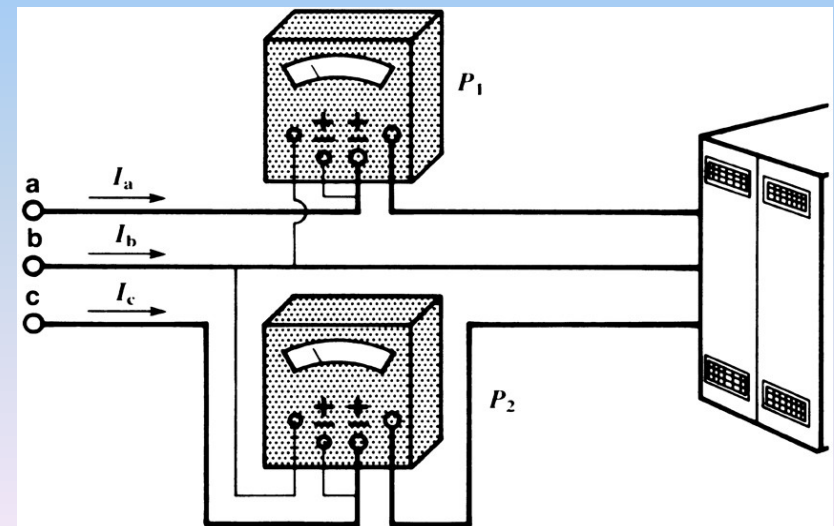
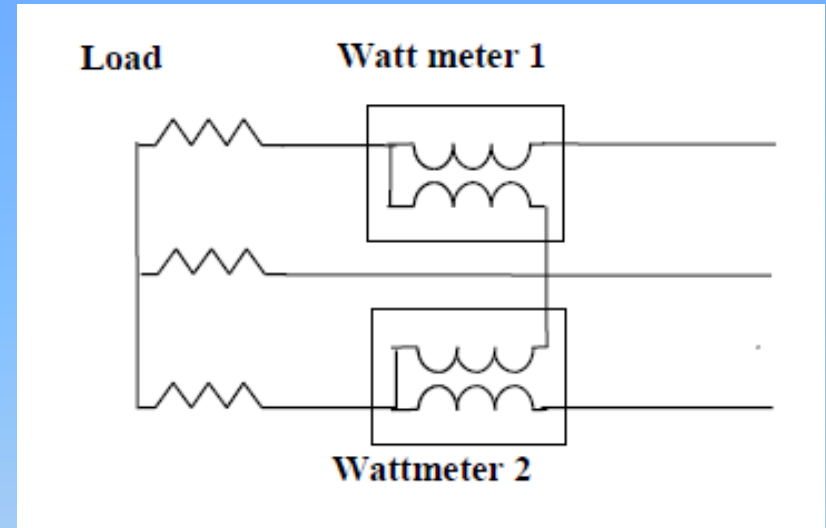
# Power Measurement

- In **three-phase systems** we need to sum the power taken from the various phases
  - in a four-wire system it may be necessary to use 3 single-phase watt-meters
  - in balanced systems (systems that take equal power from each phase) a single wattmeter can be used, its reading being multiplied by 3 to get the total power

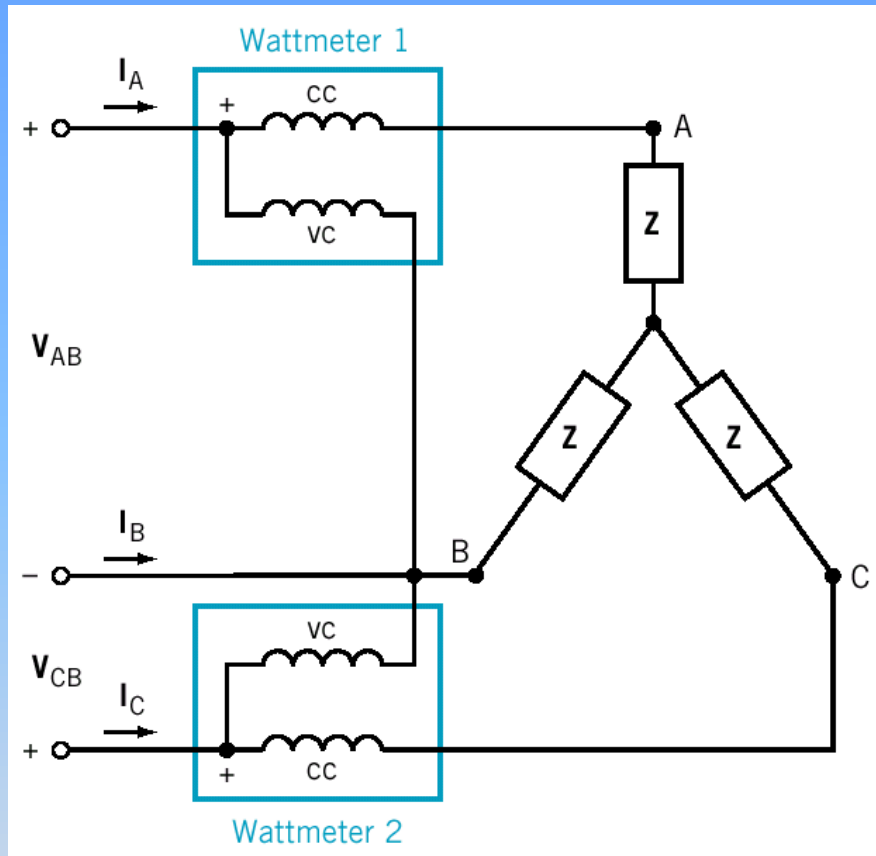


# Power Measurement

- In three-wire arrangements we can deduce the total power from measurements using 2 single-phase watt-meters
- The watt-meters are supplied by the line current and the line-to-line voltage.
- The total power is the algebraic sum of the two watt-meters reading.



# Two-Wattmeter Power Measurement



cc = current coil  
vc = voltage coil

W1 read

$$P_1 = V_{AB} I_A \cos \theta_1$$

W2 read

$$P_2 = V_{CB} I_C \cos \theta_2$$

For balanced load with  $abc$  phase sequence

$$\theta_1 = \theta_a + 30^\circ \quad \text{and} \quad \theta_2 = \theta_a - 30^\circ$$

$\theta_a$  is the angle between phase current and phase voltage of phase  $a$

## Two-Wattmeter Power Measurement (cont.)

$$\begin{aligned}P &= P_1 + P_2 \\&= 2V_L I_L \cos \theta \cos 30^\circ \\&= \sqrt{3} V_L I_L \cos \theta\end{aligned}$$

To determine the power factor angle

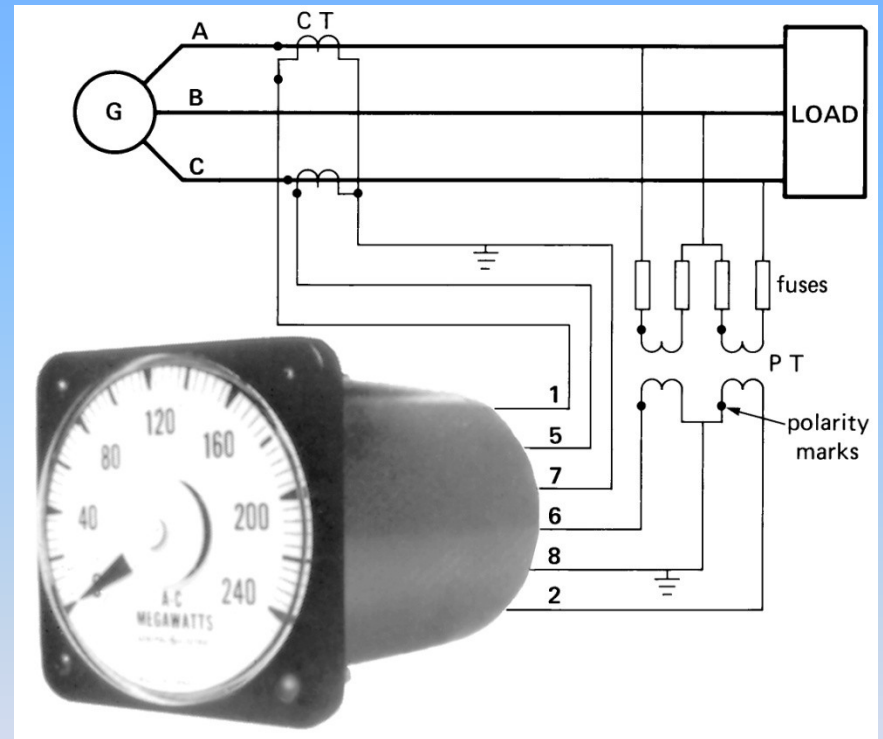
$$\begin{aligned}P_1 + P_2 &= V_L I_L 2 \cos \theta \cos 30^\circ \\P_1 - P_2 &= V_L I_L (-2 \sin \theta \sin 30^\circ)\end{aligned}$$

$$\frac{P_1 + P_2}{P_1 - P_2} = \frac{V_L I_L 2 \cos \theta \cos 30^\circ}{V_L I_L (-2 \sin \theta \sin 30^\circ)} = \frac{-\sqrt{3}}{\tan \theta}$$

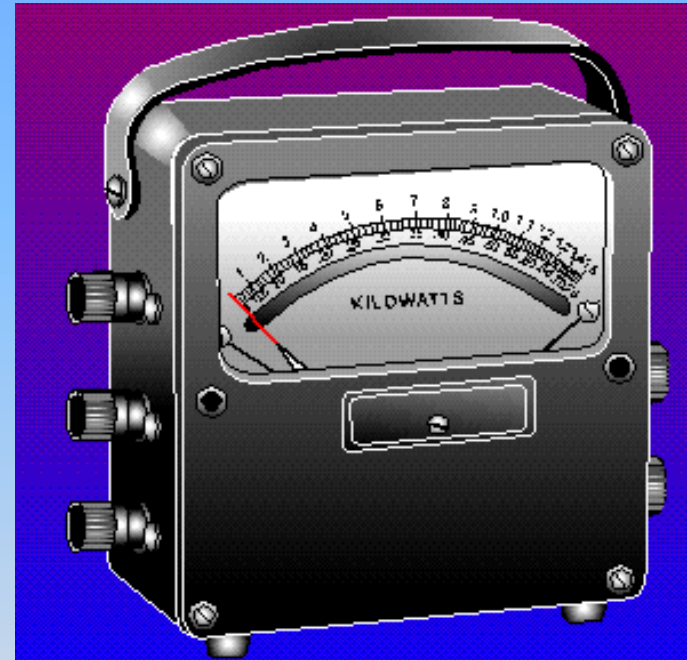
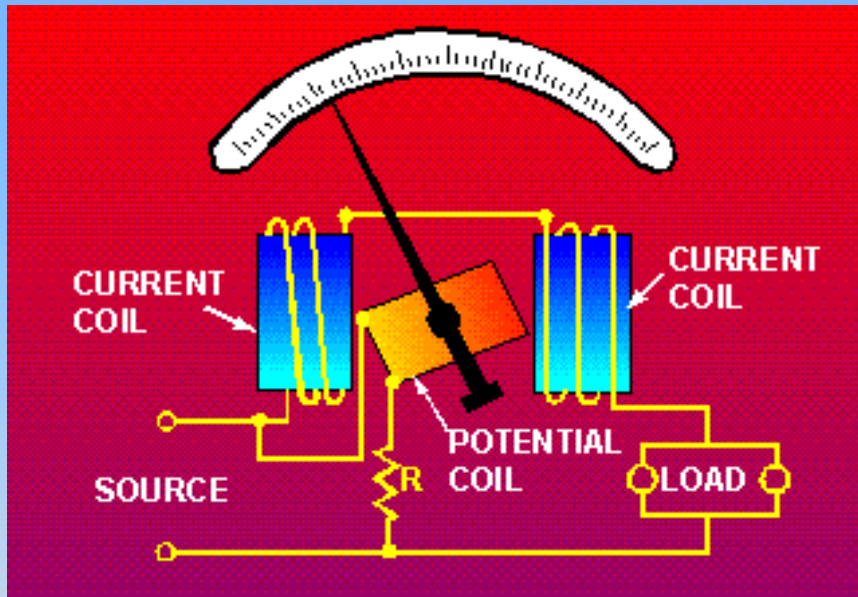
$$\therefore \tan \theta = \sqrt{3} \frac{P_1 - P_2}{P_1 + P_2} \quad \text{or} \quad \theta = \tan^{-1} \left( \sqrt{3} \frac{P_1 - P_2}{P_1 + P_2} \right)$$

# Power Measurement

- Some wattmeters, such as those used on switchboards, are specially designed to give a direct read out of the 3-phase power. The Figure shows a megawatt-range wattmeter circuit that measures the power in a generating station. The current transformers (CT) and potential transformers (PT) step down the line currents and voltages to values compatible with the instrument rating.



# Electrodynamic Wattmeter







**Digital Power Meter**



**VAR Meter**



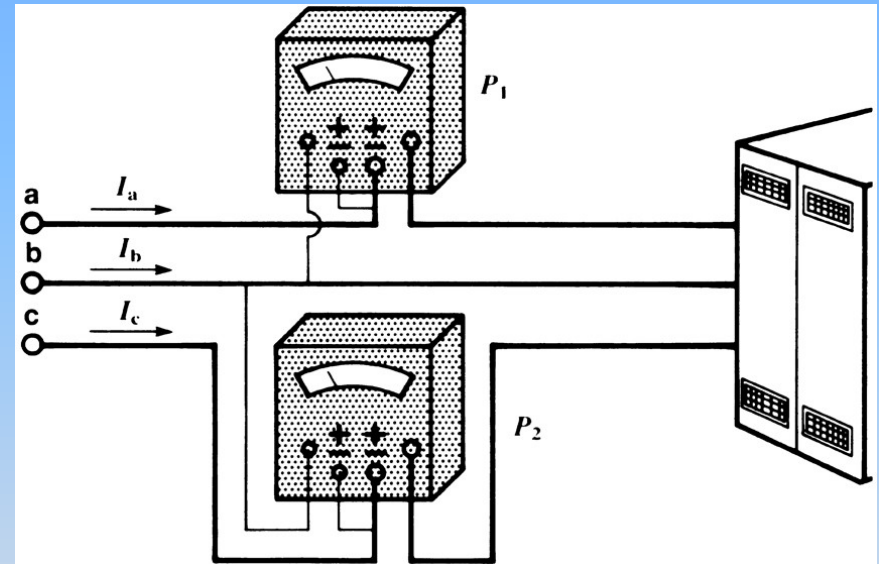
***pf* Meter**

# VARmeter

- A VARmeter indicates the reactive power in a circuit. It is built the same way as a wattmeter is, but an internal circuit shifts the line voltage by  $90^\circ$  before it is applied to the potential coil.
- VARmeters are mainly employed in the control rooms of generating stations and the substations of electrical utilities and large industrial consumers.

# VARmeter

- In 3-phase, 3-wire balanced circuits, we can calculate the reactive power from the two wattmeter readings by simply multiplying the difference of the two readings by  $\sqrt{3}$ .
- For example, if the two wattmeters indicate +5950 W and +2380 W respectively, the reactive power is  $(5950 - 2380) \times \sqrt{3} = 6176$  VAR.
- Note that this method of VAR measurement is only valid for balanced 3-phase circuits.



# Example 1

- A balanced star connected 3 phase load of  $10\Omega$  per phase is supplied from a 400V 50 Hz mains supply at unity power factor
- Calculate the phase voltage, the line current and the total power consumed

# Example 1

- A balanced star connected 3 phase load of  $10\Omega$  per phase is supplied from a 400V 50 Hz mains supply at unity power factor
- Calculate the phase voltage, the line current and the total power consumed

$$I_L = I_p$$

$$V_L = \sqrt{3} \times V_p$$

$$V_p = \frac{V_L}{\sqrt{3}}$$

$$V_p = \frac{400}{\sqrt{3}} = 230.9V$$

# Example 1

- A balanced star connected 3 phase load of  $10\Omega$  per phase is supplied from a 400V 50 Hz mains supply at unity power factor
- Calculate the phase voltage, the line current and the total power consumed

$$I_L = I_p$$

$$I = \frac{V}{R}$$

$$I_L = I_P = \frac{V_P}{R_P}$$

$$I_L = I_P = \frac{230}{10} = 23A$$

# Example 1

- A balanced star connected 3 phase load of  $10\Omega$  per phase is supplied from a 400V 50Hz mains supply at unity power factor
- Calculate the phase voltage, the line current and the total power consumed

$$P = \sqrt{3} \times V_L \times I_L \times \cos \theta$$

$$P = \sqrt{3} \times 400 \times 23 \times 1 = 16kW$$

# Example 2

- A 20 kW 400V balanced delta connected load has a power factor of 0.8
- Calculate the line current and the phase current



## Example 2

- A 20 kW 400V balanced delta connected load has a power factor of 0.8
- Calculate the line current and the phase current

$$P = \sqrt{3} \times V_L \times I_L \times \cos \theta$$

$$I_L = \frac{P}{\sqrt{3} \times V_L \times \cos \theta}$$

$$I_L = \frac{20000(w)}{\sqrt{3} \times 400 \times 0.8}$$

$$I_L = 36A$$

## Example 2

- A 20 kW 400V balanced delta connected load has a power factor of 0.8
- Calculate the line current and the phase current

$$I_L = 36A$$

Delta connection therefore

$$I_L = \sqrt{3} \times I_P$$

$$I_P = \frac{I_L}{\sqrt{3}}$$

$$I_P = \frac{36}{\sqrt{3}}$$

$$I_P = 20.78A$$

# Example 3

- A 3Ph, 60 Hz wye connected generator generates a line to-line voltage of 23,900 V. Calculate
  - a. The line-to-neutral voltage
  - b. The voltage induced in the individual windings
  - c. The time interval between the positive peak voltage of phase A and the positive peak of phase B
  - d. The peak value of the line voltage

# Example 3

- A 3Ph, 60 Hz wye connected generator generates a line to-line voltage of 23,900 V.

Calculate

- a. The line-to-neutral voltage
- b. The voltage induced in the individual windings
- c. The time interval between the positive peak voltage of phase A and the positive peak of phase B
- d. The peak value of the line voltage

(a) *The line-to-neutral voltage is:*

$$E_p = E_L / \sqrt{3} = 23,900 / \sqrt{3} = 13,800 \text{ V}$$

(b) *The windings are connected in wye, consequently, the voltage induced in each winding is 13,800 V.*

(c) *One complete cycle (360°) corresponds to 1/60 s. Consequently, a phase angle of 120° corresponds to an interval of*

$$T = 120 / 360 \times 1 / 60 = 5.55 \text{ ms}$$

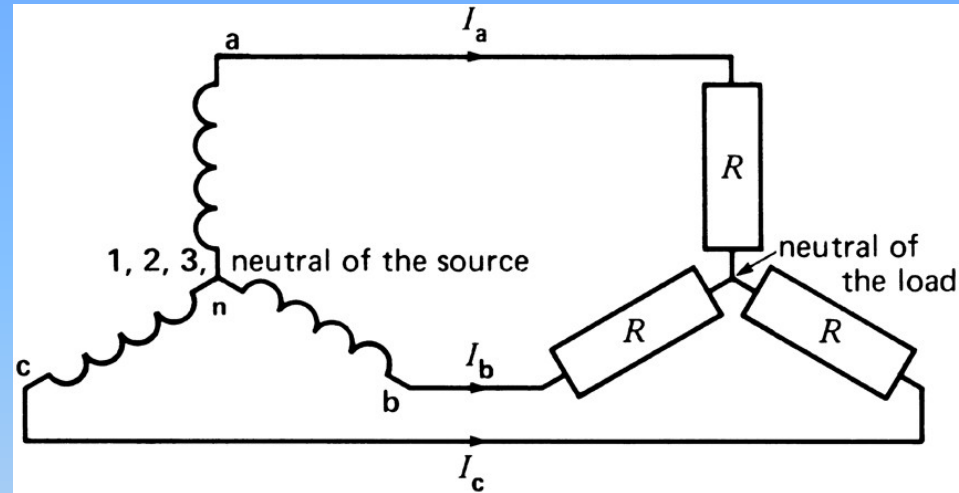
(d) *The peak line voltage is:*

$$E_{l(\text{peak})} = \sqrt{2} E_L = 23,900 \sqrt{2} = 33,800 \text{ V}$$

# Example 4

- The generator in the Figure generates a line voltage of 865 V, and each load resistor has an impedance of  $50\ \Omega$ . Calculate

- The voltage across each resistor
- The current in each resistor
- The total power output of the generator



# Example 4

- The generator in the Figure generates a line voltage of 865 V, and each load resistor has an impedance of 50  $\Omega$ . Calculate

- a. The voltage across each resistor
- b. The current in each resistor
- c. The total power output of the generator

*(a) The voltage across each resistor is:*

$$E_p = E_L / \sqrt{3} = 865 / \sqrt{3} = 500 \text{ V}$$

*(b) The current in each resistor is:*

$$I_p = E_p / R = 500 / 50 = 10 \text{ A}$$

*All the line currents are equal to 10 A.*

*(c) Power absorbed by each resistor is:*

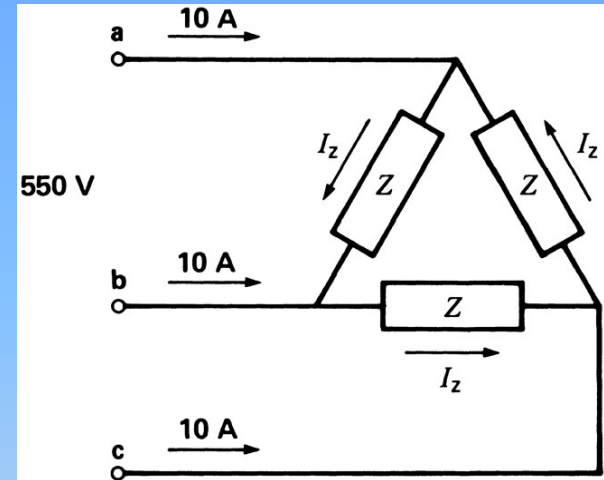
$$P = E_p I_p = 500 \times 10 = 5000 \text{ W}$$

*The power delivered by the generator to all three resistors is:*

$$P = 3 \times 5000 = 15 \text{ kW}$$

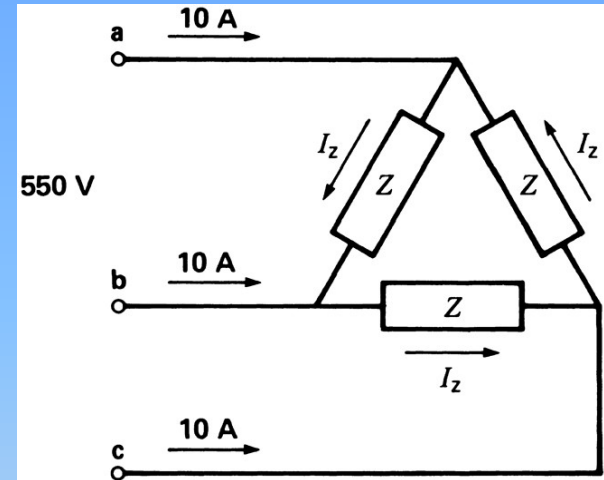
# Example 5

- Three identical impedances are connected in delta across a 3-phase, 550 V line. If the line current is 10 A, calculate the following:
  - The current in each impedance
  - The value of each impedance



# Example 5

- Three identical impedances are connected in delta across a 3-phase, 550 V line. If the line current is 10 A, calculate the following:
  - The current in each impedance
  - The value of each impedance



(a) The current in each impedance is:

$$I_z = 10 / \sqrt{3} = 5.77 \text{ A}$$

(b) The voltage across each impedance is 550 V. Consequently:

$$Z = E / I_z = 550 / 5.77 = 95 \Omega$$

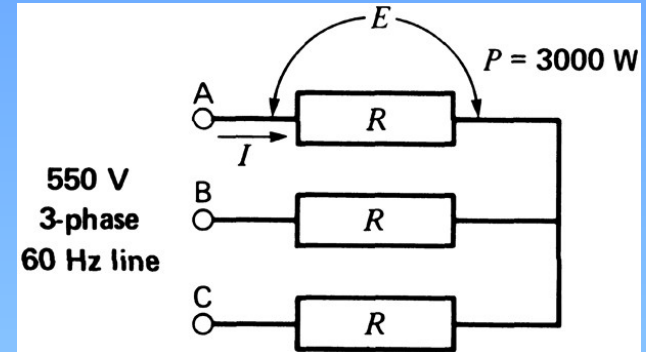


# Example 6

- A 3-phase motor, connected to a 440 V line, draws a line current of 5 A. If the power factor of the motor is 80 percent, calculate the following:
  - a. The total apparent power
  - b. The total active power
  - c. The total reactive power absorbed by the machine

# Example 7

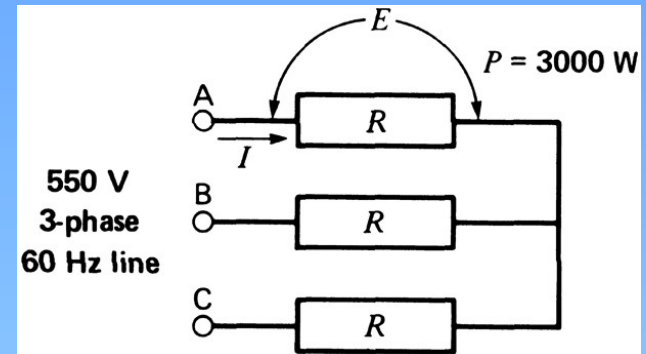
- Three identical resistors dissipating a total power of 3000 W are connected in wye across a 3-phase, 550 V line. Calculate:
  - The current in each line
  - The value of each resistor
  - In Figure shown, the phase sequence of the source is known to be A-C-B. Draw the phasor diagram of the line voltages.



# Example 7

- Three identical resistors dissipating a total power of 3000 W are connected in wye across a 3-phase, 550 V line. Calculate:

- The current in each line
- The value of each resistor



- (a) The power dissipated by each resistor is:

$$P = 3000 \text{ W} / 3 = 1000 \text{ W}$$

The voltage across the terminals of each resistor is:

$$E = 550 \text{ V} / \sqrt{3} = 318 \text{ V}$$

$$\text{The current in each resistor is: } I = P/E = 1000 \text{ W} / 318 \text{ V} = 3.15 \text{ A}$$

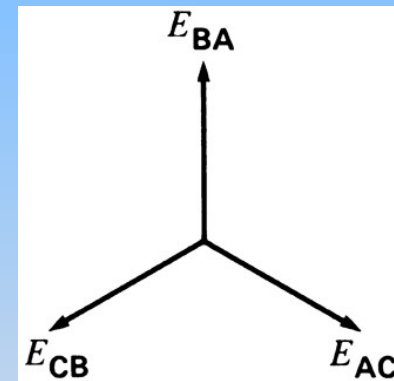
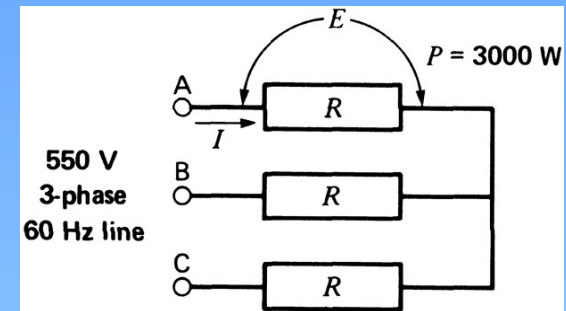
*The current in each line is also 3.15 A.*

- (b) The resistance of each element is:  $R = E / I = 318 / 3.15 = 101 \Omega$

# Example 7

- Three identical resistors dissipating a total power of 3000 W are connected in wye across a 3-phase, 550 V line. Calculate:

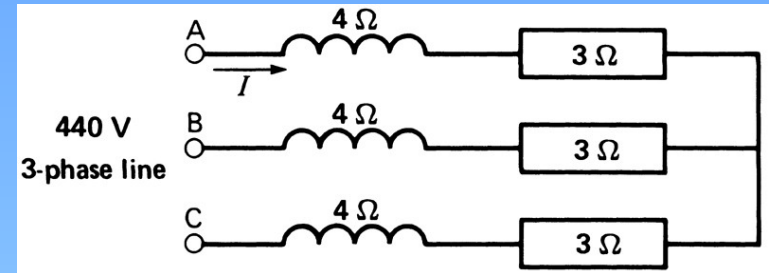
c. In Figure shown, the phase sequence of the source is known to be A-C-B. Draw the phasor diagram of the line voltages.



(c) The voltages follow the sequence A-C-B, which is the same as the sequence AC-CB-BA-AC.... Consequently, the line voltage sequence is  $E_{AC}$ - $E_{CB}$ -  $E_{BA}$  and the corresponding phasor diagram is shown. We can reverse the phase sequence of a 3-phase line by interchanging any two conductors.

# Example 8

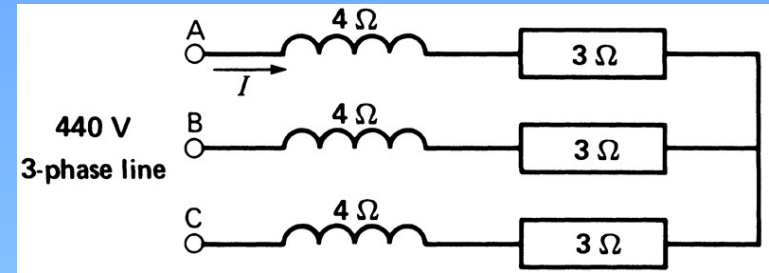
- In the circuit shown, calculate the following:
  - a. The current in each line
  - b. The voltage across the inductor terminals



# Example 8

- In the circuit shown, calculate the following:

- The current in each line
- The voltage across the inductor terminals



- (a) Each branch is composed of an inductive reactance  $X_L = 4 \Omega$  in series with a resistance  $R = 3 \Omega$ . Therefore, the impedance of each branch is  $Z_p = 5 \Omega$ .

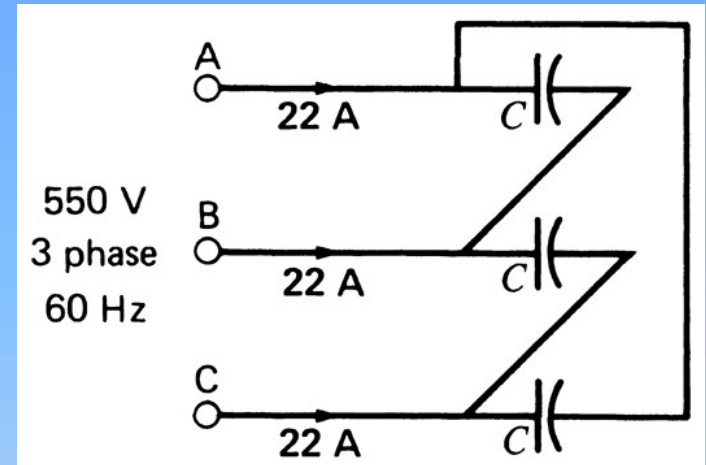
The voltage across each branch is:  $E_p = E_L / \sqrt{3} = 440 \text{ V} / \sqrt{3} = 254 \text{ V}$

The current in each circuit element is:  $I_p = E_p / Z_p = 254 / 5 = 50.8 \text{ A}$

- (b) The voltage across each inductor is:  $E = I X_L = 50.8 \times 4 = 203.2 \text{ V}$

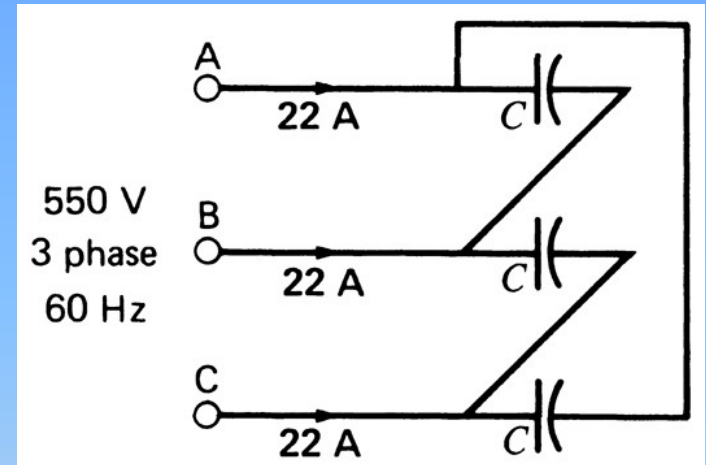
# Example 9

- A 3-phase, 550 V, 60 Hz line is connected to three identical capacitors connected in delta. If the line current is 22 A, calculate the capacitance of each capacitor.



# Example 9

- A 3-phase, 550 V, 60 Hz line is connected to three identical capacitors connected in delta. If the line current is 22 A, calculate the capacitance of each capacitor.

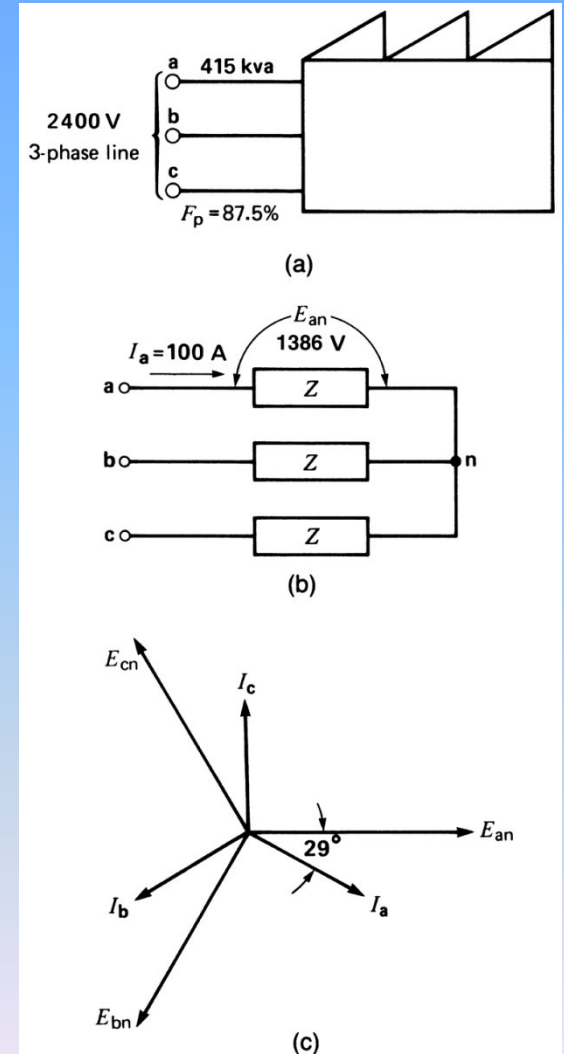


- (a) The current in each capacitor is:  $I_p = I_L / \sqrt{3} = 22 \text{ A} / \sqrt{3} = 12.7 \text{ A}$   
Voltage across each capacitor is:  $E_p = 550 \text{ V}$   
Capacitive reactance  $X_c$  of each capacitor is:  $X_c = E_p / I_p = 550 / 12.7 = 43.3 \Omega$ .
- (a) The capacitance of each capacitor is:  $C = 1 / (2 \pi f X_c) = 61.3 \mu\text{F}$



# Example 10

- A manufacturing plant draws a total of 415 kVA from a 2400 V (line-to-line), 3-phase line. If the plant power factor is 87.5 percent lagging, calculate:
  - The impedance of the plant, per phase
  - The phase angle between the line-to-neutral voltage and the line current
  - The complete phasor diagram for the plant



# Example 10

- A manufacturing plant draws a total of 415 kVA from a 2,400 V (line-to-line), 3-phase line. If the plant power factor is 87.5 percent lagging, calculate:

- a. The impedance of the plant, per phase
- b. The phase angle between the line-to-neutral voltage and the line current
- c. The complete phasor diagram for the plant

*(a) We assume a wye connection composed of three identical impedances  $Z$ .*

*The voltage per branch is:*

$$E_p = 2,400 / \sqrt{3} = 1,386\text{V}$$

*The current per branch is:  $I_p = S / (3 E_p)$*

$$I_p = 415,000 / (3 \times 1,386) = 100\text{ A}$$

*The impedance per branch is:*

$$Z = E_p / I_p = 1,386 / 100 = 13.9\ \Omega$$

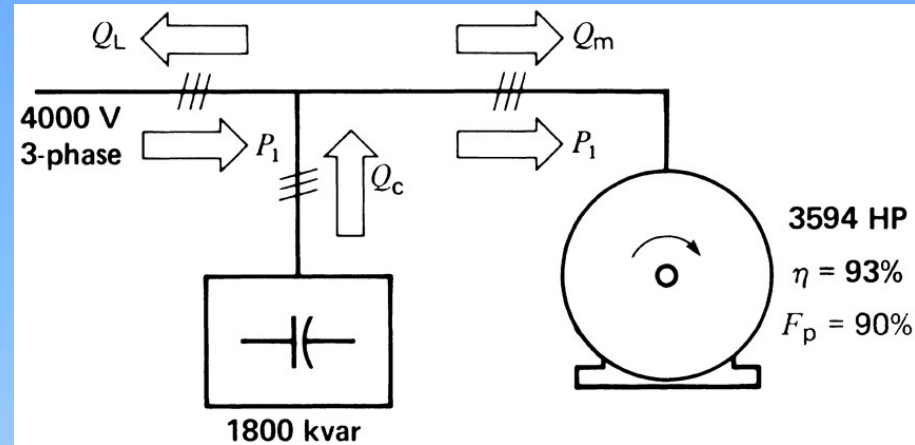
*(b) The phase angle between the line-to-neutral voltage (1386 V) and the corresponding line current (100 A) is given by:*

$$\cos \theta = 0.875 \rightarrow \theta = 29^\circ$$

*The current in each phase lags  $29^\circ$  behind the line-to-neutral voltage.*

# Example 11

- A 5000 hp, wye-connected motor is connected to a 4000 V, 3-phase, 60 Hz line. A delta-connected capacitor bank rated at 1800 kvar is also connected to the line. If the motor produces an output of 3594 hp at an efficiency of 93% and a power factor of 90% (lagging), calculate the following:
  - a. The active power absorbed by the motor
  - b. The reactive power absorbed by the motor
  - c. The reactive power supplied by the transmission line
  - d. The apparent power supplied by the transmission line
  - e. The transmission line current
  - f. The motor line current
  - g. Draw the complete phasor diagram for one phase



# Example 11

- A 5000 hp, wye-connected motor is connected to a 4000 V, 3-phase, 60 Hz line. A delta-connected capacitor bank rated at 1800 kvar is also connected to the line. If the motor produces an output of 3594 hp at an efficiency of 93% and a power factor of 90% (lagging), calculate the following:
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  - d. The apparent power supplied by the transmission line
  - e. The transmission line current
  - f. The motor line current
  - g. Draw the complete phasor diagram for one phase

*(a) Active power output is:*

$$P_2 = 3594 \text{ hp} \times 0.746 = 2681 \text{ kW}$$

*Active power input to motor:*

$$P_m = P_2 / \eta = 2681 / 0.93 = 2883 \text{ kW}$$

*(b) Apparent power absorbed by the motor:*

$$S_m = P_m / \cos \theta = 2883 / 0.90 = 3203 \text{ kVA}$$

*Reactive power absorbed by the motor:*

$$Q_m = \sqrt{(S_m^2 - P_m^2)} = 1395 \text{ kvar}$$

# Example 11

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  - d. The apparent power supplied by the transmission line
  - e. The transmission line current
  - f. The motor line current
  - g. Draw the complete phasor diagram for one phase

(c) *Reactive power supplied by the capacitor bank:  $Q_c = -1800$  kvar*

*Total reactive power absorbed by the load:*

$$Q_L = Q_c + Q_m = -1800 + 1395 = -405 \text{ kvar}$$

*This is an unusual situation because reactive power is being returned to the line. In most cases the capacitor bank furnishes no more than  $Q_m$  kilovars of reactive power.*

(d) *Active power supplied by the line is*

$$P_L = P_m = 2883 \text{ kW}$$

*Apparent power supplied by the line is:*

$$S_L = \sqrt{(P_L)^2 + (Q_L)^2} = 2911 \text{ kVA}$$

# Example 11

- A 5000 hp, wye-connected motor is connected to a 4000 V, 3-phase, 60 Hz line. A delta-connected capacitor bank rated at 1800 kvar is also connected to the line. If the motor produces an output of 3594 hp at an efficiency of 93% and a power factor of 90% (lagging), calculate the following:

- a. The active power absorbed by the motor
- b. The reactive power absorbed by the motor
- c. The reactive power supplied by the transmission line
- d. The apparent power supplied by the transmission line
- e. The transmission line current
- f. The motor line current
- g. Draw the complete phasor diagram for one phase

*(e) Transmission line current is:*

$$I_L = S_L / (E_L \times \sqrt{3}) = 420 \text{ A}$$

*(f) Motor line current is:*

$$I_m = S_m / (E_L \times \sqrt{3}) = 462 \text{ A}$$

*(g) The line-to-neutral voltage is*

$$E_p = 4000 / \sqrt{3} = 2309 \text{ V}$$

*Phase angle  $\theta$  between the motor current and the line-to-neutral voltage is:*

$$\cos \theta = 0.9 \rightarrow \theta = 25.8^\circ$$

*(The motor current lags  $25.8^\circ$  behind the voltage, as shown in the phasor diagram.)*

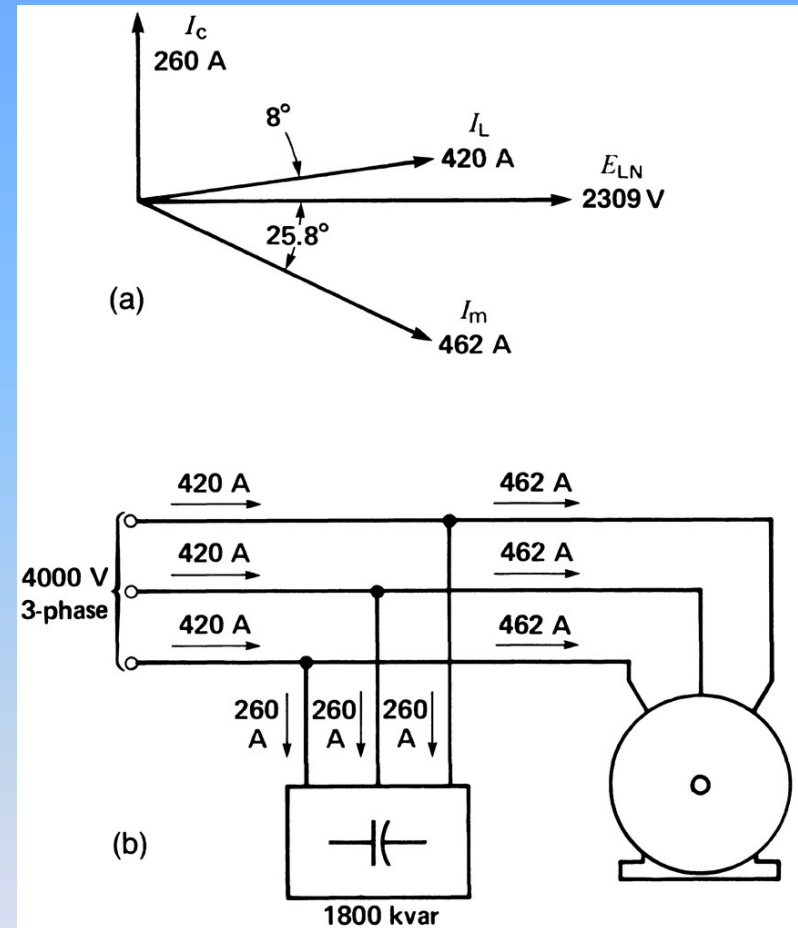
*Line current drawn by the capacitor bank is:*

$$I_c = Q_c / (E_L \times \sqrt{3}) = 1800000 / (4000 \times \sqrt{3}) = 260 \text{ A}$$

# Example 11

- A 5000 hp, wye-connected motor is connected to a 4000 V, 3-phase, 60 Hz line. A delta-connected capacitor bank rated at 1800 kvar is also connected to the line. If the motor produces an output of 3594 hp at an efficiency of 93% and a power factor of 90% (lagging), calculate the following:

- The active power absorbed by the motor
- The reactive power absorbed by the motor
- The reactive power supplied by the transmission line
- The apparent power supplied by the transmission line
- The transmission line current
- The motor line current
- Draw the complete phasor diagram for one phase



## Example #12

Calculate the total apparent power for three-phase Y-Y connected balanced system shown if the phase voltage is 110 V, and load impedance per phase is  $50 \angle 60^\circ \Omega$ .

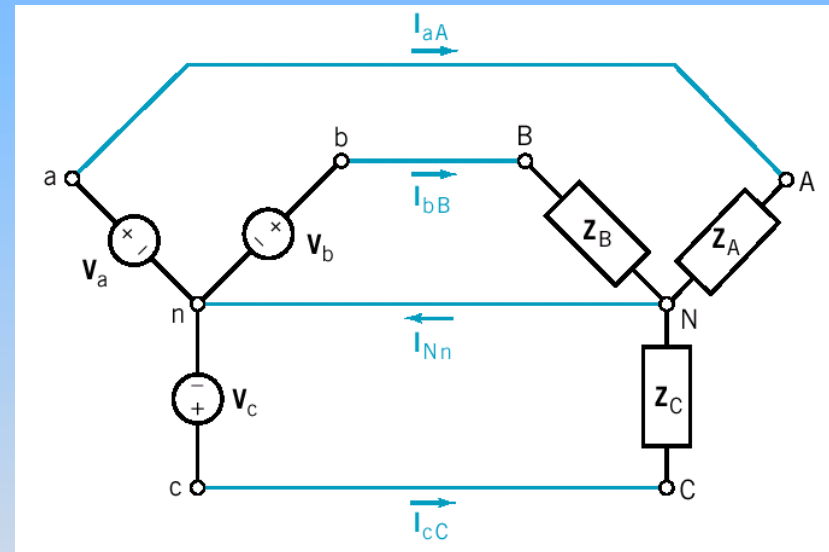
$$\mathbf{V}_a = 110 \angle 0^\circ \quad \mathbf{V}_{\text{rms}}$$

$$\mathbf{V}_b = 110 \angle -120^\circ \quad \mathbf{V}_{\text{rms}}$$

$$\mathbf{V}_c = 110 \angle 120^\circ \quad \mathbf{V}_{\text{rms}}$$

$$\mathbf{Z}_A = 50 \angle 60^\circ \Omega = \mathbf{Z}_B = \mathbf{Z}_C$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a}{\mathbf{Z}_A} = \frac{110 \angle 0^\circ}{50 \angle 60^\circ} = 2.2 \angle -60^\circ \text{ A}_{\text{rms}}$$





$$\mathbf{I}_{bB} = \frac{\mathbf{V}_b}{\mathbf{Z}_B} = \frac{110 \angle -120^\circ}{50 \angle 60^\circ} = 2.2 \angle -180^\circ \text{ A}_{\text{rms}}$$

$$\mathbf{I}_{cC} = \frac{\mathbf{V}_c}{\mathbf{Z}_C} = \frac{110 \angle 120^\circ}{50 \angle 60^\circ} = 2.2 \angle 60^\circ \text{ A}_{\text{rms}}$$

$$\begin{aligned} S &= 3 \text{ V } I^* = 3 \times 110 \angle 0^\circ \times 2.2 \angle 60^\circ \\ &= 726 \angle 60^\circ \text{ VA} \end{aligned}$$

➤ *Repeat the last example, if the source voltage is 230 V (rms) and the load impedance is  $100 \angle 40^\circ \Omega$  ?*

## Example #13

Calculate the total apparent power for three-phase 4-wire Y-Y connected shown in if the phase voltage is 110 V, and load impedances are:

$$\mathbf{Z}_A = 50 + j80 \quad \Omega$$

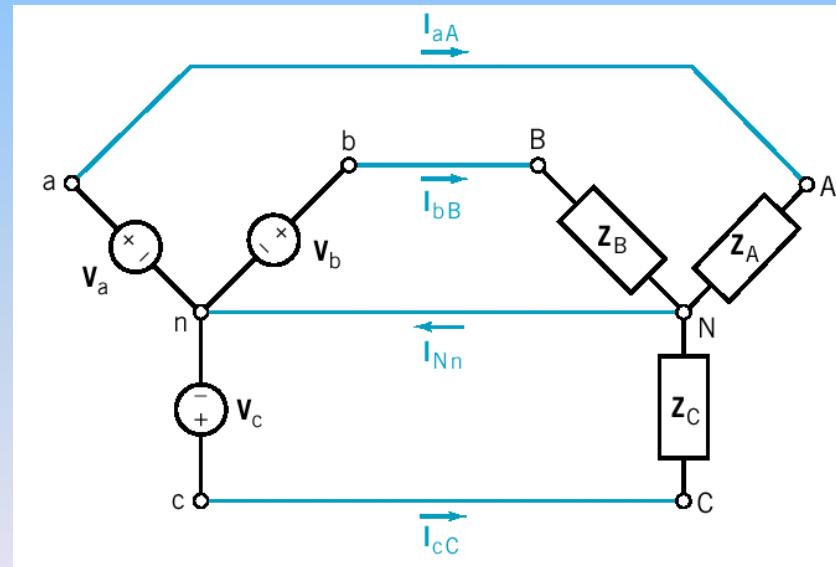
$$\mathbf{Z}_B = j50 \quad \Omega$$

$$\mathbf{Z}_C = 100 + j25 \quad \Omega$$

$$\mathbf{V}_a = 110\angle 0^\circ \quad \mathbf{V}_{\text{rms}}$$

$$\mathbf{V}_b = 110\angle -120^\circ \quad \mathbf{V}_{\text{rms}}$$

$$\mathbf{V}_c = 110\angle 120^\circ \quad \mathbf{V}_{\text{rms}}$$



## Unbalanced 4-wire

$$\begin{aligned}\mathbf{I}_{aA} &= \frac{\mathbf{V}_a}{\mathbf{Z}_A} = \frac{110\angle 0^\circ}{50 + j80} = 1.16\angle -58^\circ \quad \text{A}_{\text{rms}} \\ \mathbf{I}_{bB} &= \frac{\mathbf{V}_b}{\mathbf{Z}_B} = \frac{110\angle -120^\circ}{j50} = 2.2\angle 150^\circ \quad \text{A}_{\text{rms}} \\ \mathbf{I}_{cC} &= \frac{\mathbf{V}_c}{\mathbf{Z}_C} = \frac{110\angle 120^\circ}{100 + j25} = 1.07\angle 106^\circ \quad \text{A}_{\text{rms}}\end{aligned}$$

$$\mathbf{S}_A = \mathbf{I}_{aA}^* \mathbf{V}_a = 68 + j109 \quad \text{VA}$$

$$\mathbf{S}_B = \mathbf{I}_{bB}^* \mathbf{V}_b = j242 \quad \text{VA}$$

$$\mathbf{S}_C = \mathbf{I}_{cC}^* \mathbf{V}_c = 114 + j28 \quad \text{VA}$$

$$\mathbf{S} = \mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C = 182 + j379 \quad \text{VA}$$

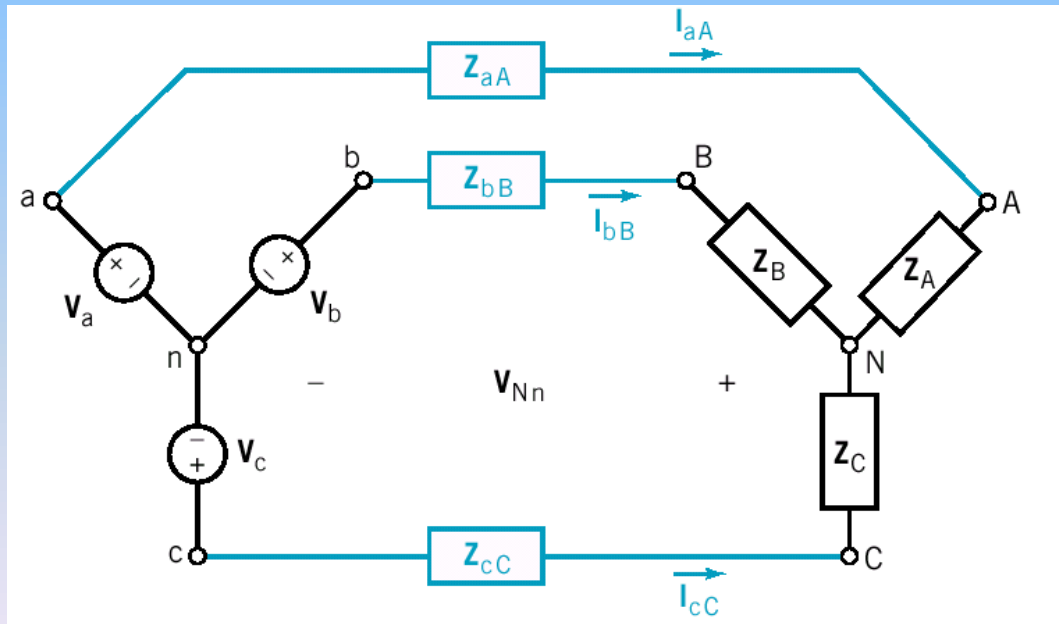
## Example #14

Calculate the total apparent power for three-phase 3-wire Y-Y connected shown if the phase voltage is 110 V, and load impedances are:

$$\mathbf{Z}_A = 50 + j80 \quad \Omega$$

$$\mathbf{Z}_B = j50 \quad \Omega$$

$$\mathbf{Z}_C = 100 + j25 \quad \Omega$$



$$\mathbf{V}_{Nn} = \frac{(110\angle -120^\circ)\mathbf{Z}_A\mathbf{Z}_C + 110\angle 120^\circ\mathbf{Z}_A\mathbf{Z}_B + 110\angle 0^\circ\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A\mathbf{Z}_C + \mathbf{Z}_A\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_C}$$

$$= 56\angle -151^\circ \quad \mathbf{V}_{\text{rms}}$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a - \mathbf{V}_{Nn}}{\mathbf{Z}_A}, \mathbf{I}_{bB} = \frac{\mathbf{V}_b - \mathbf{V}_{Nn}}{\mathbf{Z}_B}, \text{ and } \mathbf{I}_{cC} = \frac{\mathbf{V}_c - \mathbf{V}_{Nn}}{\mathbf{Z}_C}$$

$$\mathbf{I}_{aA} = 1.71\angle -48^\circ, \mathbf{I}_{bB} = 1.37\angle 74^\circ, \text{ and } \mathbf{I}_{cC} = 1.19\angle 79^\circ$$

$$\mathbf{S}_A = \mathbf{I}_{aA}^* \mathbf{V}_a = \mathbf{I}_{aA}^* (\mathbf{I}_{aA} \mathbf{Z}_A) = 146 + j234 \quad \text{VA}$$

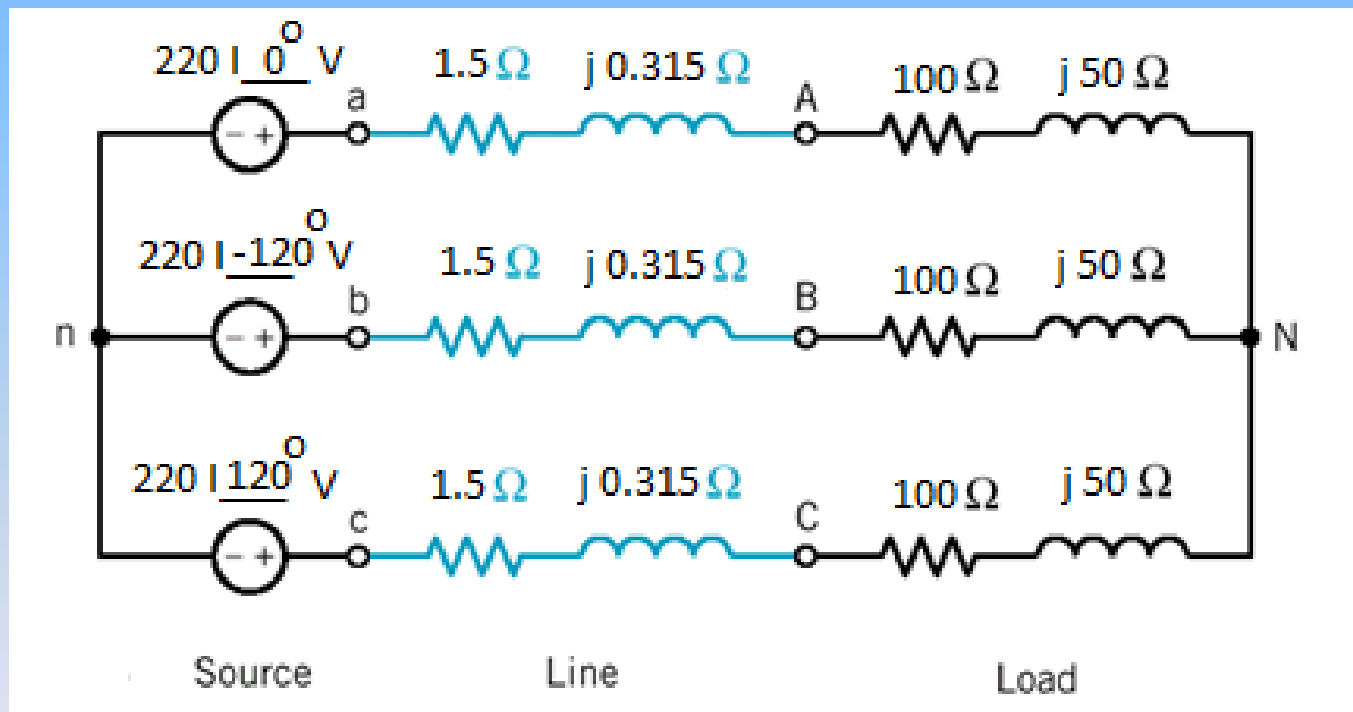
$$\mathbf{S}_B = \mathbf{I}_{bB}^* \mathbf{V}_b = \mathbf{I}_{bB}^* (\mathbf{I}_{bB} \mathbf{Z}_B) = j94 \quad \text{VA}$$

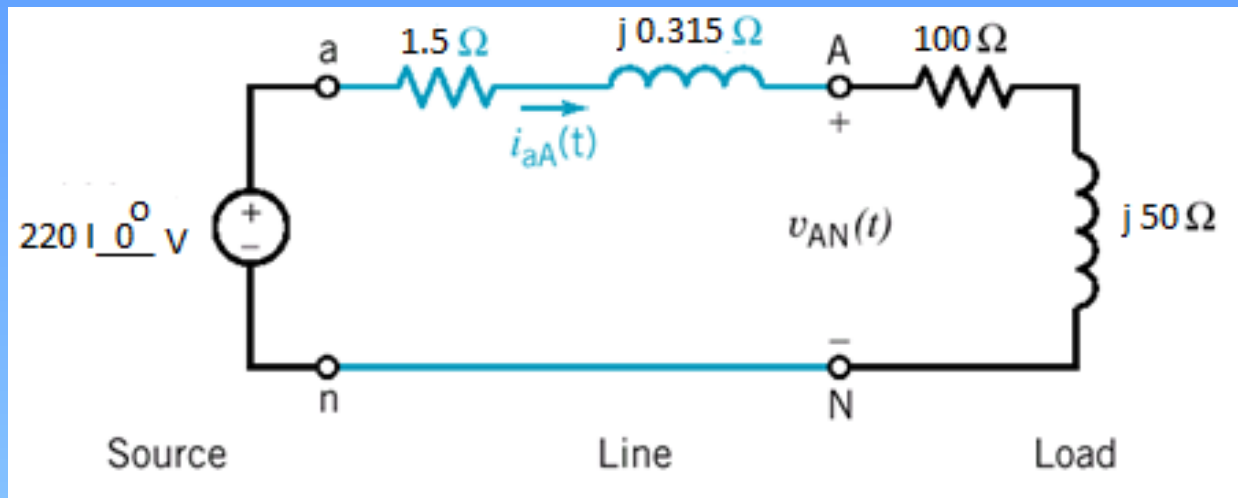
$$\mathbf{S}_C = \mathbf{I}_{cC}^* \mathbf{V}_c = \mathbf{I}_{cC}^* (\mathbf{I}_{cC} \mathbf{Z}_C) = 141 + j35 \quad \text{VA}$$

$$\mathbf{S} = \mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C = 287 + j364 \quad \text{VA}$$

## Example #15

Calculate the total source real power, the total power delivered to the load, and the total power loss in the transmission line for three-phase 3-wire Y-Y connected shown in the Figure.





$$I_{aA} = \frac{V_a}{Z_{Line} + Z_{load-A}} = \frac{220 \angle 0^\circ}{(1.5 + j0.315) + (100 + j50)} = \underline{\hspace{2cm}} \angle$$

$$P_a = V_a \times I_{aA} \times \cos(\theta_v - \theta_I) = \underline{\hspace{2cm}} \text{ watt}$$

$$P_T = 3 \times P_a = \underline{\hspace{2cm}} \text{ watt}$$

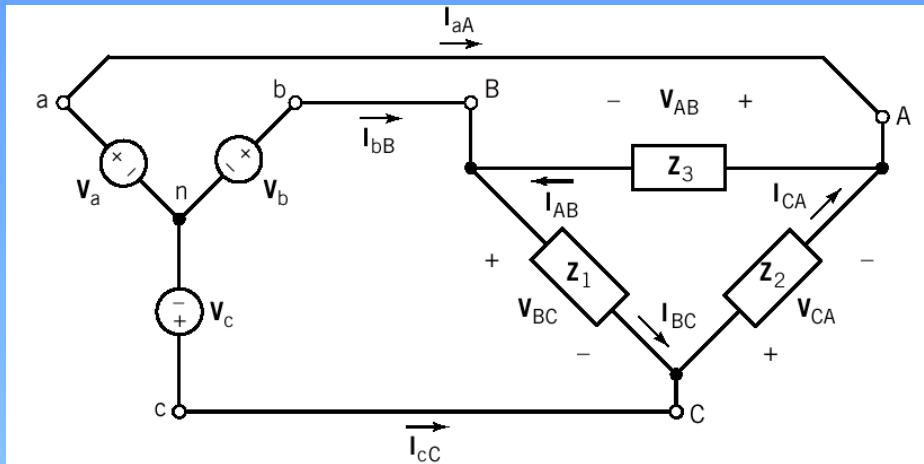
$$P_{Load-A} = |I_{aa}|^2 \times \text{Real}(Z_{L-A}) = \underline{\hspace{2cm}} \text{ watt}$$

$$P_{Total Load} = 3 \times P_{Load-A} = \underline{\hspace{2cm}} \text{ watt}$$

$$P_{Total Line loss} = 3 \times P_{Line loss-A} = \underline{\hspace{2cm}} \text{ watt}$$



## Example -16 $I_P = ?$ $I_L = ?$



$$\mathbf{V}_a = \frac{220}{\sqrt{3}} \angle -30^\circ \quad V_{\text{rms}}$$

$$\mathbf{V}_b = \frac{220}{\sqrt{3}} \angle -150^\circ \quad V_{\text{rms}}$$

$$\mathbf{V}_c = \frac{220}{\sqrt{3}} \angle 90^\circ \quad V_{\text{rms}}$$

$$\mathbf{Z}_\Delta = 10 \angle 50^\circ$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = 22 \angle 50^\circ \quad A_{\text{rms}}$$

$$\Rightarrow \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} = 22 \angle -70^\circ \quad A_{\text{rms}}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} = 22 \angle -190^\circ \quad A_{\text{rms}}$$

The  $\Delta$ -connected load is balanced with

$$\mathbf{V}_{AB} = \mathbf{V}_a - \mathbf{V}_b = 220 \angle 0^\circ \quad V_{\text{rms}}$$

$$\mathbf{V}_{BC} = \mathbf{V}_b - \mathbf{V}_c = 220 \angle -120^\circ \quad V_{\text{rms}}$$

$$\mathbf{V}_{CA} = \mathbf{V}_c - \mathbf{V}_a = 220 \angle -240^\circ \quad V_{\text{rms}}$$

The line currents are

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = 22\sqrt{3} \angle 20^\circ, \mathbf{I}_{bB} = 22\sqrt{3} \angle -100^\circ, \mathbf{I}_{cC} = 22\sqrt{3} \angle -220^\circ$$

## Example - 17 $I_p = ?$

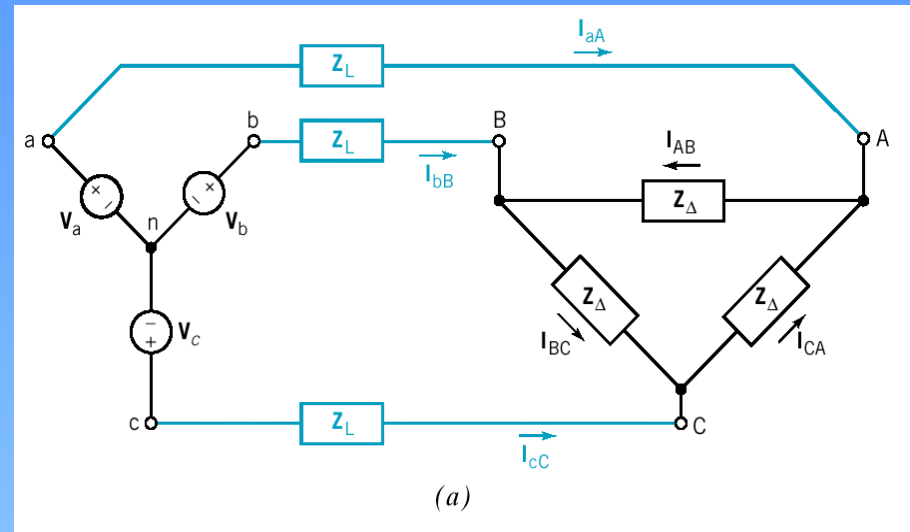
$$V_a = 110 \angle 0^\circ \quad V_{\text{rms}}$$

$$V_b = 110 \angle -120^\circ \quad V_{\text{rms}}$$

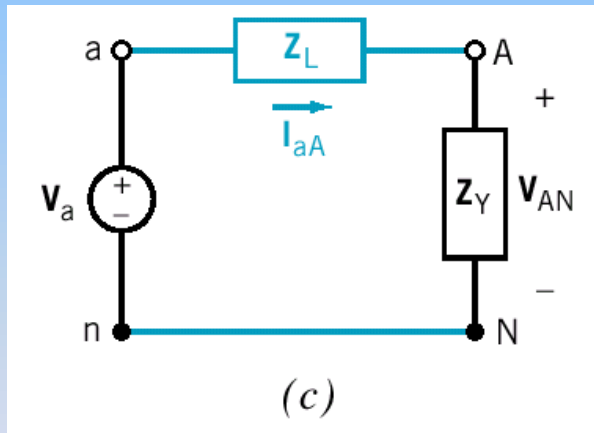
$$V_c = 110 \angle 120^\circ \quad V_{\text{rms}}$$

$$Z_L = 10 + j5 \quad \Omega$$

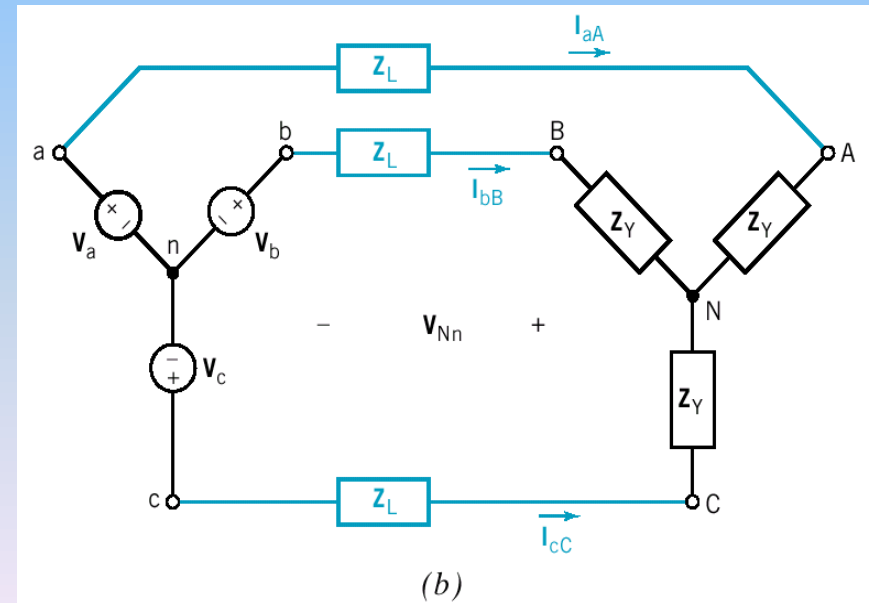
$$Z_\Delta = 75 + j225 \quad \Omega$$



$$Z_Y = \frac{Z_\Delta}{3} = 25 + j75 \quad \Omega$$



$$I_{aA} = \frac{V_a}{Z_L + Z_Y} = 1.26 \angle -66^\circ \text{ A}_{\text{rms}}$$



$$\mathbf{I}_{bB} = 1.26 \angle -186^\circ \text{ A}_{\text{rms}} \quad \text{and} \quad \mathbf{I}_{cC} = 1.26 \angle -54^\circ \text{ A}_{\text{rms}}$$

The voltages in the per-phase equivalent circuit are

$$\mathbf{V}_{AN} = \mathbf{I}_{aA} \mathbf{Z}_Y = 99.6 \angle 5^\circ \text{ V}_{\text{rms}}$$

$$\mathbf{V}_{BN} = 99.6 \angle -115^\circ \text{ V}_{\text{rms}}$$

$$\mathbf{V}_{CN} = 99.6 \angle 125^\circ \text{ V}_{\text{rms}}$$

The line-to-line voltages are

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = 172 \angle 35^\circ \text{ V}_{\text{rms}}$$

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN} = 172 \angle -85^\circ \text{ V}_{\text{rms}}$$

$$\mathbf{V}_{CA} = \mathbf{V}_{CN} - \mathbf{V}_{AN} = 172 \angle 155^\circ \text{ V}_{\text{rms}}$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = 0.727 \angle -36^\circ \text{ A}_{\text{rms}}$$

$$\Rightarrow \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} = 0.727 \angle -156^\circ \text{ A}_{\text{rms}}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} = 0.727 \angle 84^\circ \text{ A}_{\text{rms}}$$

## Example 18 $P = ?$

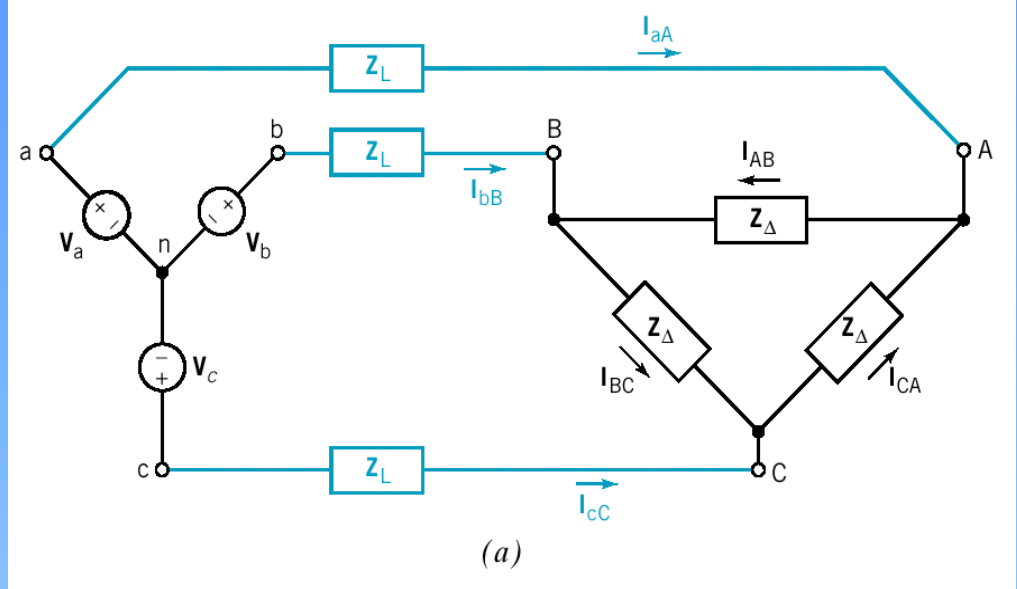
$$\mathbf{V}_a = 110\angle 0^\circ \quad \mathbf{V}_{\text{rms}}$$

$$\mathbf{V}_b = 110\angle -120^\circ \quad \mathbf{V}_{\text{rms}}$$

$$\mathbf{V}_c = 110\angle 120^\circ \quad \mathbf{V}_{\text{rms}}$$

$$\mathbf{Z}_L = 10 + j5 \quad \Omega$$

$$\mathbf{Z}_\Delta = 75 + j225 \quad \Omega$$



$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a}{\mathbf{Z}_L + \mathbf{Z}_Y} = 1.26\angle -66^\circ \text{ A}_{\text{rms}}$$

$$\mathbf{V}_{AN} = \mathbf{I}_{aA} \mathbf{Z}_Y = 99.6\angle 5^\circ \text{ V}_{\text{rms}}$$

$$P = 3(99.6)(1.26)\cos(5^\circ - (-66^\circ)) = 122.6 \text{ W}$$

### Example 19 $P = ?$

$$\mathbf{Z} = 10\angle 45^\circ$$

line-to-line voltage = 220Vrms

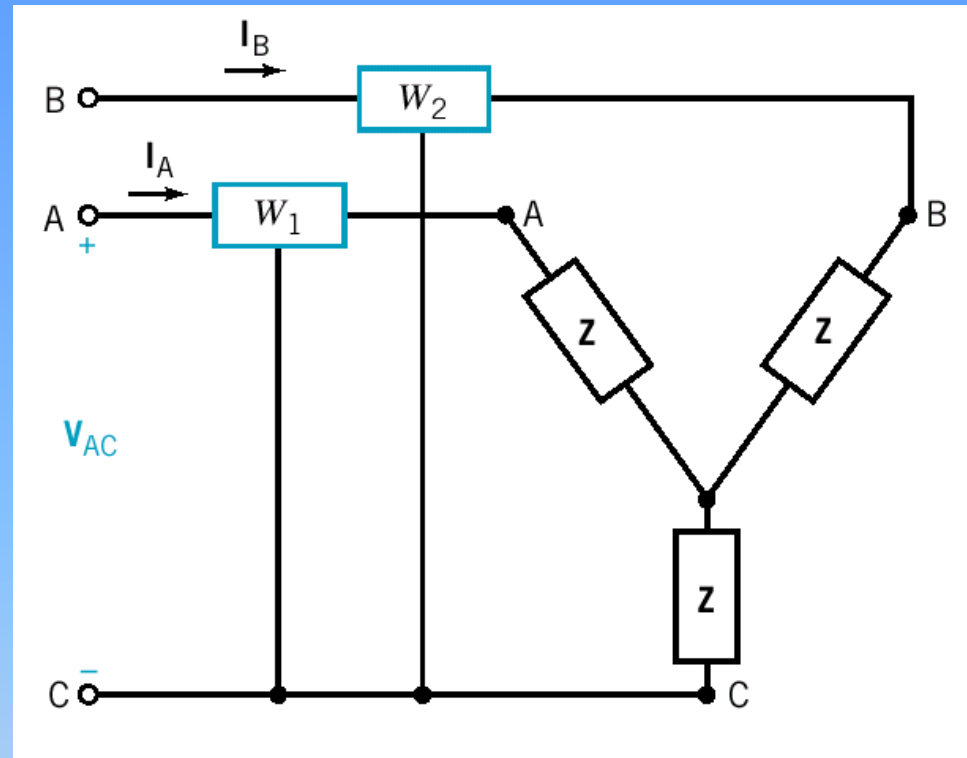
The phase voltage

$$\mathbf{V}_A = \frac{220}{\sqrt{3}} \angle -30^\circ$$

The line current

$$\mathbf{I}_A = \frac{\mathbf{V}_A}{\mathbf{Z}} = \frac{220\angle -30^\circ}{10\sqrt{3}\angle 45^\circ} = 12.7\angle -75^\circ \text{ and } \mathbf{I}_B = 12.7\angle -195^\circ$$

$$P_1 = V_{AC} I_A \cos \theta_1 = 2698 \text{ W} \quad P_2 = V_{BC} I_B \cos \theta_2 = 723 \text{ W} \quad \Rightarrow \quad P = P_1 + P_2 = 3421 \text{ W}$$



# Example 20

- A full-load test on a 10 hp, 3-phase motor yields the following results:  $P_1 = + 5,950 \text{ W}$ ;  $P_2 = + 2,380 \text{ W}$ ; the current in each of the three lines is 10 A; and the line voltage is 600 V. Calculate the power factor of the motor.

# Example 20

- A full-load test on a 10 hp, 3-phase motor yields the following results:  $P_1 = + 5,950 \text{ W}$ ;  $P_2 = + 2,380 \text{ W}$ ; the current in each of the three lines is 10 A; and the line voltage is 600 V. Calculate the power factor of the motor.

Apparent power supplied to the motor is:

$$S_L = \sqrt{3} \times E_L \times I_L = \sqrt{3} \times 600 \times 10 = 10,390 \text{ VA}$$

Active power supplied to the motor is:

$$P = 5,950 + 2,380 = 8,330 \text{ W}$$

$$P.F. = \cos \theta = P / S = 8,330 / 10,390 = 0.80, \text{ or } 80\%$$

# Example 21

- When the motor in the previous example runs at no-load, the line current drops to 3.6 A and the wattmeter readings are  $P_1 = +1295$  W;  $P_2 = -845$  W.

Calculate the no-load losses and power factor.



# Example 21

- When the motor in the previous example runs at no-load, the line current drops to 3.6 A and the wattmeter readings are  $P_1 = +1295 \text{ W}$ ;  $P_2 = -845 \text{ W}$ .

Calculate the no-load losses and power factor.

Apparent power supplied to the motor is:

$$S_L = \sqrt{3} \times E_L \times I_L = \sqrt{3} \times 600 \times 3.6 = 3,741 \text{ VA}$$

No-load losses are:

$$P = P_1 + P_2 = 1,295 - 845 = 450 \text{ W}$$

$$P.F. = \cos \theta = P / S = 450 / 3741 = 0.12 = 12\%$$